Local Estimates of the Time-Stepping Error for High-Order Splitting Methods

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We discuss the structure of the local error of high-order split-step time integrators for nonlinear evolution equations of Schrödinger type in both a semi-discrete and fully discretized setting,

$$\dot{u} = F(u), \quad F(u) = Au + B(u),$$

where $A = \mathrm{i}\Delta$ and B is a generally unbounded, nonlinear operator. Based on a rigorous analysis of the error structure which is detailed for a Laguerre–Fourier–Hermite spatial discretization for a rotating Bose–Einstein condensate [1], we introduce estimators for the local error and prove their asymptotical correctness. The estimators are based on embedded formulae for the method coefficients [2] or alternatively on the defect correction principle [3, 4]. The resulting time-stepping strategies are demonstrated to reflect the solution behavior well. Finally we assess the strategies' efficiency by numerical comparisons.

References

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