Analysis of a Dynamical Low Rank Approximation for Tensors

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joint work with Ch. Lubich

For the low rank approximation of time-dependent data tensors and of solutions to tensor differential equations, for example resulting from space semi-discretization of partial differential equations, an increment based computational approach is proposed and analyzed. In this variational method, the derivative is projected onto the tangent space of the manifold of low rank tensors at the current approximation. This yields nonlinear ordinary differential equations that are well-suited for efficient numerical integration. The error analysis compares the result with a suitable approximation in the manifold of low rank tensors, for example a pointwise best approximation computed by higher order singular value decomposition. It is proven that the approach gives locally quasi-optimal low rank approximations and under additional assumptions the error grows only linearly in t also over long time intervals. The implications for variational approximations in quantum dynamics are indicated. Numerical experiments illustrate the theoretical results.

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