Software for Singular Boundary Value Problems

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A MATLAB code for the solution of singular boundary value problems

$$z'(t) = \frac{1}{t^\alpha} f(t, z(t)), \quad t \in (0, 1], \quad \alpha \geq 1 \tag{1}$$

is presented. This solver is based on polynomial collocation, equipped with a posteriori estimates of the global error designed especially for the solution of singular ODEs, and features an adaptive mesh selection procedure based on the equidistribution of the global error. It has been demonstrated previously that the code is successful for problems with a singularity of the first kind [1], where $\alpha = 1$ in (1). By designing a new error estimate for problems with an essential singularity ($\alpha > 1$ in (1)), this problem class can also be solved efficiently. Thus, boundary value problems on a (semi-)infinite interval can be treated by transforming to a singular problem on a finite interval [2]. We demonstrate the advantages of this approach by various examples from recent applications. These comprise the computation of self-similar blow-up solution profiles of nonlinear PDEs [3], and a problem in hydrodynamics [4].

References

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