Pathfollowing for Multi-Bump Solutions of the Complex Ginzburg-Landau Equation

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Singular Boundary Value Problems and Related Topics

We consider a pathfollowing strategy based on pseudo-arclength parametrization applied for the computation of solution branches with turning points of parameter-dependent ordinary differential equations with an essential singularity,

$$\begin{aligned} &z'(t) = 1/t^{\alpha} f(t, z(t), \lambda), \quad t \in (0, 1], \\ &B_0 z(0) + B_1 z(1) = \beta. \end{aligned}$$

We show that the pathfollowing procedure is well-defined under realistic assumptions, and a numerical solution is possible with a stable, high-order discretization method. Our method of choice for the solution of the associated boundary value problems is polynomial collocation. This basic solution procedure in conjunction with an error estimate and mesh adaptation strategy suited especially for the present problem class was implemented in a standard MATLAB code which represents the core of our pathfollowing procedure. We demonstrate the reliability and efficiency of our code by computing solution branches for the complex Ginzburg-Landau (CGL) equation

$$\mathrm{i}\frac{\partial u}{\partial t} + (1 - \mathrm{i}\varepsilon)\Delta u + (1 + \mathrm{i}\delta)|u|^2 u = 0, \quad t > 0,$$

which arises as a model in a variety of problems from physics, biology and chemistry. These include nonlinear optics, models of turbulence, Rayleigh-Benard convection, superconductivity, superfluidity, Taylor-Couette flow and reaction-diffusion systems. After a similarity reduction, we compute solution branches for the second-order ordinary differential equation

$$(1 - \mathrm{i}\varepsilon)\left(z''(\tau) + \frac{2}{\tau} z'(\tau)\right) - z(\tau) + \mathrm{i}a(\tau z(\tau))' + (1 + \mathrm{i}\delta)|z(\tau)|^2 z(\tau) = 0,$$

where we vary the parameters ε and δ.