On the Convergence of Collocation Schemes for BVPs in DAEs with Singularities

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We investigate the convergence of collocation for index-1 DAEs, where the *inherent ODE* may have a singularity of the first kind. We analyze the error of collocation methods applied to a linear system of DAEs given in the following form:

$$A(t)(D(t)x(t))' + B(t)x(t) = g(t), \quad t \in (0,1], \tag{1}$$

where $A(t) \in \mathbb{R}^{m\times n},\ D(t) \in \mathbb{R}^{n\times m},\ B(t) \in \mathbb{R}^{m\times m},\ g(t),\ x(t) \in \mathbb{R}^m$, with $n \leq m$. Here, we focus on the DAEs with the so-called *properly stated leading term*. In order to describe the boundary conditions which are necessary and sufficient for (1) to be well-posed, we use the decoupling technique developed in [1].

For the numerical solution of (1) we apply polynomial collocation to the enlarged system,

$$A(t)u'(t) + B(t)x(t) = g(t), \tag{2}$$
$$D(t)x(t) - u(t) = 0, \quad t \in (0,1]. \tag{3}$$

We give a proof for the convergence order of the scheme and illustrate the convergence behavior by means of experiments for model problems exhibiting different difficulties. In the proof we again utilize the above decoupling technique and use the convergence results for the collocation applied to solve singular ODEs, developed in [2]. It turns out that the collocation method retains the stage order in case of sufficiently smooth problem and shows order reductions otherwise. Superconvergence cannot be expected to hold in general.

References

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