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Acceleration Techniques for Singular Initial Value Problems

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Contents

Abstract

We consider the numerical solution of singular initial and terminal value problems using various low-order Runge-Kutta methods. With these basic solutions, we investigate the acceleration technique known as Iterated Defect Correction (IDeC). We show that the performance depends crucially on the asymptotic expansions of the global error. The results are compared with the asymptotic properties of extrapolation.

1   Introduction

We consider the numerical solution of *singular initial value problems* of the form[1]

$$z'(t) = \frac{M(t)}{t} z(t) + f(t, z(t)), \quad t \in (0, 1], \tag{1.1a}$$

$$B_0 z(0) = \beta, \tag{1.1b}$$

$$z \in C[0, 1], \tag{1.1c}$$

where z and f are n-dimensional vector-valued functions, M is an $n \times n$ matrix, and $B_0 \in \mathbb{R}^{r \times n}$ and $\beta \in \mathbb{R}^r$ are chosen in such a way that the problem (1.1) is well-posed. It turns out that the smoothness conditions (1.1c) can be equivalently expressed as a set of $n - r$ homogeneous initial conditions and the set (1.1b) is augmented by these conditions for the numerical treatment, cf. §2.

Moreover, *singular terminal value problems* are investigated,

$$z'(t) = \frac{M(t)}{t} z(t) + f(t, z(t)), \quad t \in (0, 1], \tag{1.2a}$$

$$B_1 z(1) = \beta, \tag{1.2b}$$

$$z \in C[0, 1], \tag{1.2c}$$

where again the conditions (1.2b) yield a well-posed problem.

The analytical properties of (1.1) and (1.2) are discussed in §2.

Singular ordinary differential equations with boundary conditions posed at one point arise for example in the context of the solution of two-point boundary value problems using shooting methods. Singular two-point boundary value problems in turn often describe symmetric solutions of partial differential equations from applications in physics (see for example [4]), chemistry (cf. [22]) or mechanics (buckling of spherical shells, [5]). Moreover, there are some models in ecology (avalanche run-out, [21]) that are posed as singular initial value problems of the form (1.1).

For the numerical solution of (1.1) various schemes have been proposed. It turns out, however, that many high-order methods show order reductions when applied to singular problems. Explicit Runge-Kutta methods for example show

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[1]This type of singularity is called a *singularity of the first kind.*

a reduction down to order 2 in general, see [13], and multi-step methods deviate from their classical convergence order by a logarithmic term, cf. [12].

Another way to obtain a numerical solution with high accuracy is to solve the problems (1.1) and (1.2) with a basic method of low order and then use one of the well-known acceleration techniques. To this aim we consider *Iterated Defect Correction (IDeC)* which is described in §3.2, and compare the results with the performance of extrapolation.

We show that IDeC based on the implicit Euler method works for (1.1) as expected for regular problems. This is explained in more detail in §4.1. Then we demonstrate that higher order methods like the box scheme or the trapezoidal rule cannot be used to obtain a basic solution for IDeC and it is also explained why the iteration fails to yield higher convergence orders. These considerations can be found in §4.2 and §4.3. Moreover, the (computationally cheap) explicit Euler method is tested. However, this method does not yield the desired results either, see §4.4.

For the problem (1.2) the picture is similar. In this case, however, we face the additional problem that the smoothness of the solution depends on the eigenvalues of $M(0)$, a fact which will become clearer in §2. This of course influences the performance of IDeC with the explicit Euler rule as basic method, which is explained in §5.1. Moreover, we show that the results for the box scheme applied to (1.2) are similar to those for (1.1). Finally, we demonstrate that implicit methods cannot be used for (1.2) reliably, cf. §5.3.

1.1 Notation

Throughout the paper, the following notation is used. We denote by $\mathbb{C}^n$ the space of complex-valued vectors of dimension n and use $|\cdot|$,

$$|x| = |(x_1, x_2, \ldots, x_n)| := \max_{1 \leq i \leq n} |x_i|,$$

to denote the maximum norm in $\mathbb{C}^n$. $C_n^p[0,1]$ is the space of complex vector-valued functions which are p times continuously differentiable on $[0,1]$. For every function $y \in C_n^0[0,1]$ we define the maximum norm,

$$\|y\| := \max_{0 \leq t \leq 1} |y(t)|.$$

$C_{n \times n}^p[0,1]$ is the space of complex-valued $n \times n$ matrices with columns in $C_n^p[0,1]$. For a matrix $A = (a_{ij})_{i,j=1}^n$, $A \in C_{n \times n}^0[0,1]$, $\|A\|$ is the induced norm,

$$\|A\| := \max_{0 \leq t \leq 1} |A(t)| = \max_{0 \leq t \leq 1} \left(\max_{1 \leq i \leq n} \sum_{j=1}^n |a_{ij}(t)| \right).$$

Where there is no confusion we will delete the subscripts n and $n \times n$ and call $C = C[0,1] = C^0[0,1]$.

For a constant matrix A the kernel of A is denoted by $\ker(A)$ and we write I_n for the identity matrix in $\mathbb{R}^{n \times n}$.

For the numerical analysis, we define equidistant grids of the form

$$\Delta_h := (t_0, t_1, \ldots, t_N),$$

where $t_i = ih$, $i = 0, \ldots, N$, $h = \frac{1}{N}$, and grid vectors

$$u_h := (u_0, \ldots, u_N).$$

The norm on the space of grid vectors is given by

$$\|u_h\|_h := \max_{0 \le k \le N} |u_k|.$$

For a continuous function $x(t) \in C[0,1]$, we denote by R_h the projection onto the space of grid vectors,

$$R_h(x) := (x(t_0), \ldots, x(t_N)).$$

2 Analysis of Singular Ordinary Differential Equations

2.1 Initial Value Problems at $t = 0$

Consider the singular initial value problem of the first order

$$z'(t) = \frac{M(t)}{t} z(t) + f(t, z(t)), \quad t \in (0, 1], \tag{2.1a}$$

$$B_0 z(0) = \beta, \tag{2.1b}$$

$$M(0) z(0) = 0, \tag{2.1c}$$

where z, f are n-dimensional vector-valued functions, M is an $n \times n$ matrix, B_0 is an $r \times n$ matrix and β is an r-dimensional vector, $n \geq r := \dim \ker(M(0))$. We assume $M(t) \in C^1[0, 1]$ which means that we can write

$$M(t) = M(0) + tC(t) \tag{2.2}$$

with $C(t) \in C[0, 1]$. For the numerical treatment we always assume $B_0 = I_n$ and $\beta \in \ker(M(0))$. For suitable $\beta = \tilde{E} \beta_r$, where $\tilde{E}$ is a basis of the kernel of $M(0)$, this choice provides r linearly independent initial conditions equivalent to (2.1b).

The analytical properties of (2.1) have been investigated in full detail in [17], [15], and [16]. For the analysis we assume that $f(t, z)$ is continuous and satisfies a Lipschitz-condition with respect to z on $[0, 1] \times \mathbb{R}^n$. It can be shown that a restriction on the spectrum of $M(0)$, namely the absence of purely imaginary eigenvalues or eigenvalues with positive real parts, is necessary in order to formulate an initial value problem of the form (1.1) having a unique, continuous solution $y(t)$. In this case, the condition $M(0)z(0) = 0$ is necessary and sufficient for $z \in C[0, 1]$ and provides $n - r$ linearly independent conditions which the initial value $z(0)$ has to satisfy. This solution is unique iff the $r \times r$ matrix $B_0 \tilde{E}$ is nonsingular. If f is k times continuously differentiable and $M \in C^{k+1}[0, 1]$, then the solution satisfies $z \in C^{k+1}[0, 1]$. The solution of (2.1) is the unique solution of the equivalent integral equation

$$z(t) = z(0) + t \int_0^1 \tau^{-M(0)} C(\tau t) z(\tau t) \, d\tau + t \int_0^1 \tau^{-M(0)} f(\tau t, z(\tau t)) \, d\tau. \tag{2.3}$$

For sufficiently small $\delta > 0$ the right-hand side of this equation is a contraction for $t \in [0, \delta]$.

2.2 Terminal Value Problems at $t = 1$

Here we consider the singular terminal value problem

$$z'(t) = \frac{M(t)}{t} z(t) + f(t, z(t)), \quad t \in (0, 1], \tag{2.4a}$$

$$B_1 z(1) = \beta, \tag{2.4b}$$

where in contrast to (2.1), $B_1 \in \mathbb{R}^{n \times n}$ and $\beta \in \mathbb{R}^n$. Again we assume that $M(t)$ can be written in the form (2.2) and that $f(t, z)$ is continuous and satisfies a Lipschitz-condition with respect to z on $[0, 1] \times \mathbb{R}^n$.

Let all the eigenvalues of $M(0)$ have either a positive real part (the smallest of these is denoted by σ_+) or if the eigenvalue 0 occurs, the associated block in the Jordan canonical form is diagonal. It was shown in [15] that this condition is necessary and sufficient for the existence of a continuous solution of (2.4). Thus, we require B_1 to be nonsingular to obtain a unique solution. If f is k times continuously differentiable, $M \in C^{k+1}[0, 1]$ and $\sigma_+ > k + 1$, then $z \in C^{k+1}[0, 1]$. This solution is the unique solution of

$$z(t) = \int_\delta^t \left(\frac{t}{\tau} \right)^{M(0)} (C(\tau)y(\tau) + f(\tau, y(\tau))) \, d\tau + \left(\frac{t}{\delta} \right)^{M(0)} y(\delta). \qquad (2.5)$$

For $t \in [0, \delta]$, the right-hand side of this equation is a contraction for sufficiently small $\delta > 0$.

2.3   Test Problems

Example 1.1a. This linear problem with constant coefficient matrix is taken from [2]. The eigenvalues are $\lambda = -3, -5$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ -15 & -8 \end{pmatrix} \cdot z(t) + e^{2t} t \begin{pmatrix} 0 \\ 4t^2 + 26t + 35 \end{pmatrix}, \quad t \in (0, 1],$$
$$z(0) = (0, 0).$$

The exact solution is
$$z(t) = (t^2 e^{2t}, 2(t+1)t^2 e^{2t}).$$

Example 1.2a. This linear example with variable coefficient matrix is taken from [3]. The eigenvalues are $\lambda = -1, -2$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ -2 - 8t + t^2 & -3 \end{pmatrix} \cdot v(t) + t \begin{pmatrix} 0 \\ 12 e^{1-t} \end{pmatrix}, \quad t \in (0, 1],$$
$$z(0) = (0, 0).$$

The exact solution is

$$z(t) = (t^2 e^{1-t}, (2 - t)t^2 e^{1-t}).$$

Example 2.1a. This linear problem with constant coefficient matrix is taken from [26]. The eigenvalues are $\lambda = 0, 1 + \sigma, \sigma \le -1$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ 0 & 1 + \sigma \end{pmatrix} \cdot z(t) + t \begin{pmatrix} 0 \\ -9 \cos(3t) + 3 \frac{\sigma}{t} \sin(3t) \end{pmatrix}, \quad t \in (0, 1],$$
$$z(0) = (1, 0).$$

The exact solution is

$$z(t) = (\cos(3t), -3t \sin(3t)).$$

Here we choose $\sigma = -2$.

Example 2.2a. This linear problem with variable coefficient matrix is taken from [23]. The eigenvalues are $\lambda = 0, -1$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} \cdot z(t) + t \begin{pmatrix} 0 \\ 4z_1(t) - 2 \end{pmatrix}, \quad t \in (0, 1],$$

$$z(0) = \left(\frac{1}{2} + \frac{10}{\sinh(2)}, 0 \right).$$

The exact solution is

$$z(t) = \left(\frac{1}{2} + \frac{5\sinh(2t)}{t\sinh(2)}, \frac{10t\cosh(2t) - 5\sinh(2t)}{t\sinh(2)} \right).$$

Example 2.3a. This nonlinear problem, the so-called *Emden Differential Equation*, is taken from [11]. The eigenvalues of $M(0)$ are $\lambda = 0, -1$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} \cdot z(t) - t \begin{pmatrix} 0 \\ z_1^5(t) \end{pmatrix}, \quad t \in (0, 1],$$

$$z(0) = (1, 0).$$

The exact solution is

$$z(t) = \left(\frac{1}{\sqrt{1 + t^2/3}}, -\frac{t^2}{3\sqrt{(1 + t^2/3)^3}} \right).$$

Example 3.1a. This is the same problem as 2.1a, but with $\sigma = -1$.

Example 3.1b. This linear problem with constant coefficient matrix is taken from [20]. The eigenvalues are $\lambda = 0, 0$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \cdot z(t) + \begin{pmatrix} 0 \\ t^{k-1}e^t(t^2 + t(2k + 1) + k^2) \end{pmatrix}, \quad t \in (0, 1],$$

$$z(0) = (0, 0).$$

The exact solution is

$$z(t) = (t^k e^t, (t + k)t^k e^t).$$

Here, we choose $k = 2$.

Example 3.1c. This is the same problem as 3.1b but with $k = 5$.

Example 3.2a. This linear problem with variable coefficient matrix is taken from [20]. The eigenvalues are $\lambda = 0, 0$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ t(4+\cosh(t))+t^2 & 1-\cosh(t) \end{pmatrix} \cdot z(t) + t \begin{pmatrix} 0 \\ 2(1+\cosh(t))e^t \end{pmatrix},$$

$$z(0) = (0, 0).$$

The exact solution is

$$z(t) = (t^2 e^t, (t + 2)t^2 e^t).$$

Example 3.3a. This nonlinear problem is taken from [13]. The eigenvalues are $\lambda = 0, 0$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \cdot z(t) + t \begin{pmatrix} 0 \\ z_1(t)^3 - 3z_1(t)^5 \end{pmatrix}, \quad t \in (0, 1],$$

$$z(0) = (1, 0).$$

The exact solution is

$$z(t) = (\frac{1}{\sqrt{1 + t^2}}, -\frac{t^2}{\sqrt{(1 + t^2)^3}}).$$

Example 4.1a. This homogeneous linear problem with constant coefficient matrix was also considered in [10]. The eigenvalues are $\lambda = 0, \sigma, \sigma > 0$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ 0 & \sigma \end{pmatrix} \cdot z(t), \quad t \in (0, 1],$$

$$z(1) = (-1, \sigma).$$

The exact solution is

$$z(t) = (t^\sigma - 2, \sigma t^\sigma).$$

Here we choose $\sigma = 1$.

Example 4.1b. This is the same problem as 4.1a, but with $\sigma = 3$.

Example 4.1c. The same as 4.1a, $\sigma = 10$.

Example 4.1d. This linear problem with constant coefficient matrix is taken from [9]. The eigenvalues are $\lambda = 0, \sigma, \sigma > 0$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ 0 & \sigma \end{pmatrix} \cdot z(t) + t e^t \begin{pmatrix} 0 \\ t^2 + t(5 - \sigma) + 4 - 2\sigma \end{pmatrix}, \quad t \in (0, 1],$$

$$z(1) = (e - 1, 3e + \sigma).$$

The exact solution is

$$z(t) = (t^2 e^t + t^\sigma - 2, (2 + t)t^2 e^t + \sigma t^\sigma).$$

Here we choose $\sigma = 1$.

Example 4.1e. The same as 4.1d, $\sigma = 3$.

Example 4.1f. The same as 4.1d, $\sigma = 10$.

Example 4.2a. This linear problem with variable coefficient matrix is taken from [3]. The eigenvalues are $\lambda = 0, 2$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix} \cdot z(t) + 10 \begin{pmatrix} 0 \\ -10t z_1(t) + 100t^2 + \cos(10t) - 1 \end{pmatrix},$$

$$z(1) = (10 - \sin(10), 10 - 10\cos(10)).$$

The exact solution is

$$z(t) = (10t - \sin(10t), 10t - 10t\cos(10t)).$$

Example 5.2a. This linear problem with variable coefficient matrix is taken from [3]. The eigenvalues are $\lambda = 2, 3$.

$$z'(t) = \frac{1}{t} \begin{pmatrix} 0 & 1 \\ -6 & 5 \end{pmatrix} \cdot z(t) + t \begin{pmatrix} 0 \\ z_1(t) \end{pmatrix}, \quad t \in (0, 1],$$
$$z(1) = (1, 1).$$

The exact solution is

$$z(t) = (t^2 e^{1-t}, (2-t)t^2 e^{1-t}).$$

3   Numerical Solution of Ordinary Differential Equations

In this section we briefly discuss the numerical solution of initial value problems

$$z'(t) = F(t, z(t)), \quad t \in [a, b], \tag{3.1a}$$
$$z(a) = \beta. \tag{3.1b}$$

A *numerical method* φ_h for (3.1) on a grid[2] $\Delta_h = (t_0, \ldots, t_N)$, $t_{i+1} - t_i =: h$ is a function on the space of grid vectors $z_h = (z_0, \ldots, z_N) \in \mathbb{R}^{n(N+1)}$. The *numerical solution* z_h associated with φ_h is the solution of

$$\varphi_h(z_h) = 0 \in \mathbb{R}^{n(N+1)}. \tag{3.2}$$

The numerical method (or, equivalently, the numerical solution) is said to be *convergent of order p* for (3.1) if

$$\|R_h(z) - z_h\|_h = O(h^p) \text{ for } h \to 0. \tag{3.3}$$

3.1   One-step Methods

The following numerical methods, which are so-called *one-step methods*, will be the main scope of this report.

- **The Explicit (or Forward) Euler Method.** For this method,

$$\varphi_h(z_h) = \begin{pmatrix} z_j - z_{j-1} - hF(t_{j-1}, z_{j-1}), \ j = 1, \ldots, N \\ z_0 - \beta \end{pmatrix}. \tag{3.4}$$

   For regular problems (3.1) which are sufficiently smooth[3], classical theory yields that the explicit Euler method is convergent of order 1, cf. for example [27].

- **The Implicit (or Backward) Euler Method.** For this method,

$$\varphi_h(z_h) = \begin{pmatrix} z_j - z_{j-1} - hF(t_j, z_j), \ j = 1, \ldots, N \\ z_0 - \beta \end{pmatrix}, \tag{3.5}$$

   and convergence order 1 can be shown, [27].

- **The Trapezoidal Rule.** In this case,

$$\varphi_h(z_h) = \begin{pmatrix} z_j - z_{j-1} - \frac{h}{2}(F(t_{j-1}, z_{j-1}) + F(t_j, z_j)), \ j = 1, \ldots, N \\ z_0 - \beta \end{pmatrix}. \tag{3.6}$$

   For this method, convergence order 2 holds for $z \in C^3$, cf. [27].

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[2]For notational convenience we consider equidistant grids.
[3]That is, there exists a unique solution $z \in C^2[0, 1]$ of (3.1).

- **The Box Scheme**[4] Here,

$$\varphi_h(z_h) = \left(\begin{array}{l} z_j - z_{j-1} - hF\left(t_{j-1} + \frac{h}{2}, \frac{z_{j-1}+z_j}{2}\right), \ j = 1, \ldots, N \\ z_0 - \beta \end{array} \right), \quad (3.7)$$

and again the convergence order is 2, [1].

For the use of acceleration techniques like Iterated Defect Correction (IDeC), see §3.2, or Richardson extrapolation, §3.3, it is important to know about the structure of the global error of the basic numerical method. For sufficiently smooth problems, it can be shown for rather general numerical methods of order p that *asymptotic expansions of their global errors* exist, see for example [24]. These have the form

$$z_h - R_h(z) = \sum_{j=p}^{m} h^j R_h(e_j) + r_h, \quad (3.8)$$

where e_j are smooth functions which are defined recursively by the *variational equations*[5], and

$$\|r_h\|_h = O(h^{m+1}).$$

For regular problems, such expansions can be derived for the explicit and the implicit Euler method with $p = 1$. For the trapezoidal rule and the box scheme, the expansions are *in even powers of h*, that is

$$z_h - R_h(z) = \sum_{j=1}^{m} h^{2j} R_h(e_{2j}) + r_h, \quad (3.9)$$

with $\|r_h\|_h = O(h^{2m+2})$.

3.2 Iterated Defect Correction

The IDeC procedure based on Zadunaisky's idea, see [28], has been successfully applied to solve classical second order boundary value problems and its performance in this context has been theoretically investigated in [6], [7]. For initial value problems, the applicability of Runge-Kutta methods was shown in [8]. These results do not carry over to the case of singular problems directly, however.

Here, we briefly discuss some important features of this acceleration technique. We consider initial value problems of the form[6] (3.1). We assume to know the approximate solution, $z_h^{[0]} := z_h$, obtained by some numerical method φ_h, and denote by $p^{[0]}(t)$ the polynomial of degree N interpolating the values of $z_h^{[0]}$,

$$p^{[0]}(t_i) = z_i^{[0]}, \quad i = 0, \ldots, N.$$

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[4]In the literature this method is also referred to as *(implicit) mid-point rule* or *centered Euler method.*

[5]The variational equations are linearized versions of (3.1).

[6]In the case of singular problems, we may think of $F(t, z(t))$ as being the right-hand side of (1.1a). (1.2) is treated analogously.

Using this polynomial we construct a neighboring problem associated with (3.1) and solved by $p^{[0]}(t)$,

$$z'(t) = F(t, z(t)) + d^{[0]}(t), \quad t \in [a, b], \tag{3.10a}$$
$$z(a) = p^{[0]}(a) = \beta, \tag{3.10b}$$

where

$$d^{[0]}(t) := \left(p^{[0]}\right)'(t) - F(t, p^{[0]}(t)).$$

We now solve (3.10) by the same numerical method φ_h and obtain an approximate solution $p_h^{[0]}$ for $p^{[0]}(t)$. This means that for the solution of the neighboring problem (3.10) we know the global error which we can use to estimate the unknown error of the original problem (3.1),

$$\varepsilon_h = R_h(z) - z_h \approx \delta_h^{[0]} := R_h(p^{[0]}) - p_h^{[0]} = z_h^{[0]} - p_h^{[0]}. \tag{3.11}$$

Zadunaisky gave the following heuristic argument for his method to work: If the values z_h are good approximations for the values of the solution $R_h(z)$ at the grid points, then the polynomial $p^{[0]}(t)$ is a good approximation for the solution $z(t)$ itself. Consequently, the defect $d^{[0]}(t)$ is small and hence the neighboring problem (3.10) and the original problem (3.1) are closely related. This implies that the global error of the solution of (3.10) is closely related to the global error of the solution of (3.1), and therefore the estimate (3.11) shall provide some dependable information about its size.

Having the estimate for the global error of the solution $z_h^{[0]}$ we are able to improve this solution by setting

$$z_h^{[1]} := z_h^{[0]} + \delta_h^{[0]} = z_h^{[0]} + \left(R_h(p^{[0]}) - p_h^{[0]}\right).$$

We use these values to define a new interpolating polynomial $p^{[1]}(t)$ by requiring $p^{[1]}(t_i) = z_i^{[1]}$, $i = 0, \ldots, N$, and the associated defect

$$d^{[1]}(t) := \left(p^{[1]}\right)'(t) - F(t, p^{[1]}(t)).$$

Clearly, the next neighboring problem reads

$$z'(t) = F(t, z(t)) + d^{[1]}(t), \quad t \in [a, b], \tag{3.12a}$$
$$z(a) = \beta, \tag{3.12b}$$

and we solve it with φ_h to obtain the approximation $p_h^{[1]}$ which is used to correct the basic solution again,

$$z_h^{[2]} := z_h^{[0]} + \delta_h^{[1]} = z_h^{[0]} + \left(R_h(p^{[1]}) - p_h^{[1]}\right).$$

Clearly, the procedure can be continued in the above manner.

For obvious reasons one does not use one interpolating polynomial for the whole interval in practice. Instead, a piecewise polynomial function, composed of polynomials of (moderate) degree m is defined to specify the neighboring problem. Due to classical theory, see [6], [7] or [8] for example, the maximal level of accuracy $O(h^{m_{\max}})$ which can be obtained by the IDeC iteration, depends on both the polynomial degree m and on the length of an asymptotic error expansion of the form (3.8).

3.3   Richardson Extrapolation

Consider a family of discretization methods φ_h, $h \leq h_0$. The existence of an asymptotic error expansion like (3.8) or (3.9) makes it possible to use extrapolation to improve the numerical solution z_h of an analytical problem (3.1). To explain this acceleration technique, we consider the solution at a (fixed) point $t \in [a,b]$. For every[7] h, denote by $T(h)$ the approximate value obtained with step-size h. Assume that an asymptotic error expansion for $T(h)$ exists and has the form

$$T(h) = \sum_{j=0}^{m} \tau_j h^{j\gamma} + \tau_{m+1}(h) h^{(m+1)\gamma}, \tag{3.13}$$

where $\gamma > 0$ and τ_{m+1} is bounded for $h \to 0$. $\tau_0 = \lim_{h \to 0} T(h)$ is the value of the true solution, $z(t)$. Now, we choose a sequence of step-sizes $h_0 > h_1 > \ldots$ and compute $T(h_i)$, $i = 0, 1, \ldots$.

For $i \geq k$ we denote by $\tilde{T}_{ik}(h)$ the interpolating polynomials of the form

$$\tilde{T}_{ik}(h) = \sum_{j=0}^{k} b_{i,j} h^{j\gamma}$$

which satisfy

$$\tilde{T}_{ik}(h_j) = T(h_j), \quad j = i - k, \ldots, i,$$

and use the values

$$T_{ik} := \tilde{T}_{ik}(0)$$

as an approximation for τ_0. Now, for every fixed $k \leq m$,

$$T_{ik} - \tau_0 = O(h_{i-k}^{(k+1)\gamma}) \tag{3.14}$$

holds asymptotically for $i \to \infty$. For a proof see for example [25].

It is clear that (3.14) holds for every $t \in [a,b]$ if φ_h admits an asymptotic expansion of the global error of the form (3.8) or (3.9). Thus, the asymptotic property analogous to (3.14) holds for the maximal error on $[a,b]$ of the solution obtained by the extrapolation method. For the methods from §3.1, $\gamma = 1$ holds for the explicit and the implicit Euler method, and $\gamma = 2$ for the box scheme and the trapezoidal rule.

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[7]In this setting, it is conceptually simpler to choose step-sizes appropriate so as to reach the point t without modifications of the algorithm.

4   The Numerical Solution of Singular Initial Value Problems at $t = 0$

We now turn to the discussion of the application of IDeC to the singular initial value problem

$$z'(t) = \frac{M(t)}{t} z(t) + f(t, z(t)), \quad t \in (0, 1], \tag{4.1a}$$

$$z(0) = \beta \in \ker(M(0)). \tag{4.1b}$$

First, we investigate the properties of the IDeC procedure when the implicit Euler rule is used as a basic method. Then, higher order methods and the computationally cheaper explicit Euler method are investigated. It turns out that only the implicit Euler method yields the desired results, whereas for the other methods IDeC fails. The reasons for the break-down of the iteration are discussed for some test examples.

4.1   The Implicit Euler Method

If the implicit Euler method is used as the basic method for the IDeC iteration for the solution of (4.1), the classical order sequence (cf. [8]) for the respective iterates can be observed. This was found experimentally in [2] and [16], and only recently shown theoretically in [19]. The main result of this paper is given in the following

Theorem 4.1 *Consider the IDeC method based on the implicit Euler rule and on piecewise interpolation with polynomials of degree m for the numerical solution of problem (4.1). For the approximations obtained in the course of the iteration,*

$$\|z_h^{[j]} - R_h(z)\|_h = O(h^{j+1}) \tag{4.2}$$

holds for $j = 0, \ldots, m-1$, provided that f and M are sufficiently smooth. In this case (polynomials of degree m are used for the interpolation), further iteration does not increase the asymptotic order of the approximation in general.

The main idea of the proof is to use an asymptotic expansion of the global error of the implicit Euler method. For sufficiently smooth data f and M we can prove the existence of an expansion of the form

$$z_h - R_h(z) = \sum_{j=1}^{m} h^j R_h(e_j) + r_h, \tag{4.3}$$

for any m, where e_j, $j = 1, \ldots, m$, are smooth functions defined by the variational equations. These have the form

$$e_j'(t) = \frac{M(t)}{t} e_j(t) + \frac{\partial f(t, z)}{\partial z}(t, z(t)) e_j(t)$$

$$+ \sum_{\mu=2}^{j} \frac{\partial^\mu f(t,z)}{\partial z^\mu}(t,z(t)) \sum_{\substack{\mu_1 + \cdots + \mu_j = \mu \\ \mu_i \geq 0,\ i=1,\ldots,j \\ \sum_{\nu=1}^{j} \nu\mu_\nu = j}} \frac{1}{\mu_1! \cdots \mu_j!} \prod_{\kappa=1}^{j} e_\kappa^{\mu_\kappa}(t)$$

$$+ \sum_{l=1}^{j-1} \frac{(-1)^{j+1-l}}{(j+1-l)!} e_l^{(j+1-l)}(t) + \frac{(-1)^{j+1}}{(j+1)!} z^{(j+1)}(t)$$

$$=: \frac{M(0)}{t} e_j(t) + S_j\left(t, e_1, \ldots, e_1^{(j)}, \ldots, e_{j-1}, \ldots, e_{j-1}'', e_j, z, z^{(j+1)}\right),$$

$$e_j(0) = 0.$$

The remainder term r_h satisfies the recursion

$$\frac{r_{i+1} - r_i}{h} = \frac{M(t_{i+1})}{t_{i+1}} r_{i+1} + g(t_{i+1}, r_{i+1}) + l_i, \quad i = 0, \ldots, N-1,$$

$$r_0 = 0$$

with

$$g(t_i, r_i) := \int_0^1 \frac{\partial f(t,z)}{\partial z}\left(t_i, z(t_i) + \sum_{j=1}^{m} h^j e_j(t_i) + \tau r_i\right) d\tau \cdot r_i,$$

$$l_i := h^{m+1} \sum_{l=1}^{m} \frac{(-1)^{m-l}}{(m+1-l)!} \int_0^1 e_l^{(m+2-l)}(t_{i+1} - \tau h)(1-\tau)^{m+1-l}\, d\tau$$

$$+ h^{m+1} \frac{(-1)^m}{(m+1)!} \int_0^1 z^{(m+2)}(t_{i+1} - \tau h)(1-\tau)^{m+1}\, d\tau$$

$$+ \sum_{\mu=1}^{m} \frac{\partial^\mu f(t,z)}{\partial z^\mu}(t_{i+1}, z(t_{i+1})) \sum_{\substack{\mu_1 + \cdots + \mu_m = \mu \\ \mu_i \geq 0,\ i=1,\ldots,m \\ \sum_{\nu=1}^{m} \nu\mu_\nu > m}} \frac{1}{\mu_1! \cdots \mu_m!} \prod_{\kappa=1}^{m} e_\kappa^{\mu_\kappa}(t_{i+1}) h^{\kappa\mu_\kappa}.$$

The estimates $\|l_h\|_h = O(h^{m+1})$ and consequently

$$\|r_h\|_h = O(h^{m+1})$$

hold.

The above variational equations and the difference equation can be derived by substituting the ansatz (4.3) into the defining equation $\varphi_h(z_h) = 0$ of the implicit Euler method, using Taylor expansion of the involved quantities and equating coefficients of the same powers of h. It turns out that for smooth $M(t)$ and $f(t, z(t))$, $e_j(t)$ are smooth as well.

Under the same assumptions, similar expansions exist for the neighboring problems analogous to (3.10) and equally for $p_h^{[l]}$, $l = 1, \ldots, m-2$. Since the neighboring problems depend on h, so do the associated error expansions. Written in terms of a step-size $\bar{h} = \frac{h}{\nu}$, $\nu \in \mathbb{N}$, they read

$$p_{\bar{h}}^{[l]} - R_{\bar{h}}\left(p^{[l]}\right) = \sum_{j=1}^{m} \bar{h}^j R_{\bar{h}}\left(e_j^{[l]}\right) + r_{\bar{h}}^{[l]}. \tag{4.4}$$

The functions $e_j^{[l]}$ may have jump discontinuities in the first derivatives at the switch points between the polynomials defining the neighboring problems, but in the interior of these intervals they have the same smoothness properties as e_j from (4.3). For the remainder term, a similar estimate (independent of h) holds.

Let $\bar{h} := h$. Then from the existence of the error expansions for the original and the neighboring problems we can conclude for $l = 0, \ldots, m - 2$

$$R_h(z) - z_h^{[l+1]} = \sum_{j=1}^{m} h^j \left(R_h(e_j) - R_h \left(e_j^{[l]} \right) \right) + r_h - r_h^{[l]}. \qquad (4.5)$$

Consequently, the proof is completed by showing that

$$\| R_h(e_j) - R_h \left(e_j^{[l]} \right) \|_h = O(h^{2+l-j}) \qquad (4.6)$$

holds for $j = 1, \ldots, m, \ l = 0, \ldots, m - 2$. For the technical details of the proof refer to [19].

An experimental result for $m = 5$ is displayed in Table 4.1 which illustrates the theory, a comprehensive collection of test results is contained in [2], see also [16], [19]. In this table as well as for the rest of this report if not explicitly stated otherwise, δ denotes the maximal absolute error of the numerical solution for a given step-size h on the whole interval $[0, 1]$, p the empirical convergence order and c the error constant[8]. All computations in this report were performed in quadruple precision, where for the maximal relative round-off error EPS $= 2^{-107} \approx 6.16 \cdot 10^{-33}$ holds.

Thus, for smooth data f and M, the IDeC method based on the implicit Euler scheme can potentially yield approximations to singular initial value problems of an arbitrary order.

This fact was for example used successfully in a shooting code for singular boundary value problems[9]. For numerical results of this approach see [2], for a theoretical investigation of shooting methods cf. [18].

The existence of an error expansion (4.3) also makes the use of extrapolation methods possible. Thus, the classical order sequence $O(h)$, $O(h^2)$, ... can be observed for Richardson extrapolation based on the implicit Euler method. Tables 4.2 – 4.17 show the results for the linear examples from §2.3. For every test problem, 9 extrapolation steps were computed. Naturally, the empirical convergence orders may become blurred when very high precisions are achieved, but nevertheless values compatible with the theory are assumed. However, the results of the last extrapolation steps for Example 3.2a are omitted because the levels of accuracy obtained do not permit an unambiguous interpretation.

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[8]In many cases where c stabilizes at a moderate value, this quantity is omitted in the displayed tables.

[9]We have to restrict ourselves to BVPs where the IVPs associated with shooting are well-posed.

h	δ	p	c	δ	p	c
$1/5$	$7.9 \cdot 10^{-02}$	0.565	$-1.9 \cdot 10^{-01}$	$5.3 \cdot 10^{-02}$	1.334	$-4.5 \cdot 10^{-01}$
$1/5 \cdot 2^{-1}$	$5.3 \cdot 10^{-02}$	0.761	$-3.1 \cdot 10^{-01}$	$2.1 \cdot 10^{-02}$	1.661	$-9.6 \cdot 10^{-01}$
$1/5 \cdot 2^{-2}$	$3.1 \cdot 10^{-02}$	0.875	$-4.3 \cdot 10^{-01}$	$6.6 \cdot 10^{-03}$	1.830	$-1.6 \cdot 10^{+00}$
$1/5 \cdot 2^{-3}$	$1.7 \cdot 10^{-02}$	0.936	$-5.4 \cdot 10^{-01}$	$1.8 \cdot 10^{-03}$	1.915	$-2.1 \cdot 10^{+00}$
$1/5 \cdot 2^{-4}$	$9.0 \cdot 10^{-03}$	0.968	$-6.2 \cdot 10^{-01}$	$4.9 \cdot 10^{-04}$	1.957	$-2.6 \cdot 10^{+00}$
$1/5 \cdot 2^{-5}$	$4.6 \cdot 10^{-03}$	0.983	$-6.7 \cdot 10^{-01}$	$1.2 \cdot 10^{-04}$	1.978	$-2.9 \cdot 10^{+00}$
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-03}$	0.991	$-7.1 \cdot 10^{-01}$	$3.2 \cdot 10^{-05}$	1.989	$-3.1 \cdot 10^{+00}$
$1/5 \cdot 2^{-7}$	$1.1 \cdot 10^{-03}$	0.995	$-7.3 \cdot 10^{-01}$	$8.1 \cdot 10^{-06}$	1.994	$-3.2 \cdot 10^{+00}$
$1/5 \cdot 2^{-8}$	$5.8 \cdot 10^{-04}$	0.997	$-7.4 \cdot 10^{-01}$	$2.0 \cdot 10^{-06}$	1.997	$-3.3 \cdot 10^{+00}$
$1/5 \cdot 2^{-9}$	$2.9 \cdot 10^{-04}$	0.998	$-7.4 \cdot 10^{-01}$	$5.1 \cdot 10^{-07}$	1.998	$-3.3 \cdot 10^{+00}$
$1/5 \cdot 2^{-10}$	$1.4 \cdot 10^{-04}$	0.999	$-7.5 \cdot 10^{-01}$	$1.2 \cdot 10^{-07}$	1.999	$-3.3 \cdot 10^{+00}$
$1/5 \cdot 2^{-11}$	$7.3 \cdot 10^{-05}$	0.999	$-7.5 \cdot 10^{-01}$	$3.2 \cdot 10^{-08}$	1.999	$-3.3 \cdot 10^{+00}$
$1/5 \cdot 2^{-12}$	$3.6 \cdot 10^{-05}$	0.999	$-7.5 \cdot 10^{-01}$	$8.0 \cdot 10^{-09}$	1.999	$-3.3 \cdot 10^{+00}$
$1/5 \cdot 2^{-13}$	$1.8 \cdot 10^{-05}$	0.999	$-7.5 \cdot 10^{-01}$	$2.0 \cdot 10^{-09}$	1.999	$-3.3 \cdot 10^{+00}$
$1/5 \cdot 2^{-14}$	$9.2 \cdot 10^{-06}$			$5.0 \cdot 10^{-10}$		

h	δ	p	c	δ	p	c
$1/5$	$3.0 \cdot 10^{-02}$	2.486	$-1.6 \cdot 10^{+00}$	$1.1 \cdot 10^{-02}$	2.988	$-1.4 \cdot 10^{+00}$
$1/5 \cdot 2^{-1}$	$5.4 \cdot 10^{-03}$	2.992	$-5.3 \cdot 10^{+00}$	$1.4 \cdot 10^{-03}$	2.931	$-1.2 \cdot 10^{+00}$
$1/5 \cdot 2^{-2}$	$6.8 \cdot 10^{-04}$	3.158	$-8.7 \cdot 10^{+00}$	$1.9 \cdot 10^{-04}$	3.532	$-7.7 \cdot 10^{+00}$
$1/5 \cdot 2^{-3}$	$7.6 \cdot 10^{-05}$	3.137	$-8.0 \cdot 10^{+00}$	$1.6 \cdot 10^{-05}$	3.809	$-2.1 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$8.6 \cdot 10^{-06}$	3.013	$-4.7 \cdot 10^{+00}$	$1.2 \cdot 10^{-06}$	3.915	$-3.4 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-06}$	2.932	$-3.1 \cdot 10^{+00}$	$8.0 \cdot 10^{-08}$	3.960	$-4.3 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-07}$	2.966	$-3.8 \cdot 10^{+00}$	$5.1 \cdot 10^{-09}$	3.980	$-4.8 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$1.7 \cdot 10^{-08}$	2.983	$-4.2 \cdot 10^{+00}$	$3.2 \cdot 10^{-10}$	3.990	$-5.1 \cdot 10^{+01}$
$1/5 \cdot 2^{-8}$	$2.2 \cdot 10^{-09}$	2.991	$-4.4 \cdot 10^{+00}$	$2.0 \cdot 10^{-11}$	3.995	$-5.3 \cdot 10^{+01}$
$1/5 \cdot 2^{-9}$	$2.8 \cdot 10^{-10}$	2.995	$-4.6 \cdot 10^{+00}$	$1.2 \cdot 10^{-12}$	3.997	$-5.4 \cdot 10^{+01}$
$1/5 \cdot 2^{-10}$	$3.5 \cdot 10^{-11}$	2.997	$-4.7 \cdot 10^{+00}$	$8.0 \cdot 10^{-14}$	3.998	$-5.4 \cdot 10^{+01}$
$1/5 \cdot 2^{-11}$	$4.4 \cdot 10^{-12}$	2.999	$-4.7 \cdot 10^{+00}$	$5.0 \cdot 10^{-15}$	3.999	$-5.5 \cdot 10^{+01}$
$1/5 \cdot 2^{-12}$	$5.6 \cdot 10^{-13}$	2.999	$-4.7 \cdot 10^{+00}$	$3.1 \cdot 10^{-16}$	3.999	$-5.5 \cdot 10^{+01}$
$1/5 \cdot 2^{-13}$	$7.0 \cdot 10^{-14}$			$1.9 \cdot 10^{-17}$		

h	δ	p	c	δ	p	c
$1/5$	$7.1 \cdot 10^{-03}$	2.033	$-1.8 \cdot 10^{-01}$	$1.2 \cdot 10^{-02}$	3.227	$-2.1 \cdot 10^{+00}$
$1/5 \cdot 2^{-1}$	$1.7 \cdot 10^{-03}$	4.803	$-1.1 \cdot 10^{+02}$	$1.2 \cdot 10^{-03}$	6.104	$-1.6 \cdot 10^{+03}$
$1/5 \cdot 2^{-2}$	$6.2 \cdot 10^{-05}$	5.198	$-3.6 \cdot 10^{+02}$	$1.8 \cdot 10^{-05}$	4.802	$-3.3 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$1.7 \cdot 10^{-06}$	5.140	$-2.9 \cdot 10^{+02}$	$6.6 \cdot 10^{-07}$	5.269	$-1.8 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$4.8 \cdot 10^{-08}$	5.079	$-2.2 \cdot 10^{+02}$	$1.7 \cdot 10^{-08}$	5.324	$-2.3 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$1.4 \cdot 10^{-09}$	5.041	$-1.8 \cdot 10^{+02}$	$4.3 \cdot 10^{-10}$	5.250	$-1.6 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$4.3 \cdot 10^{-11}$	5.020	$-1.6 \cdot 10^{+02}$	$1.1 \cdot 10^{-11}$	5.165	$-9.9 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$1.3 \cdot 10^{-12}$	5.009	$-1.5 \cdot 10^{+02}$	$3.1 \cdot 10^{-13}$	5.099	$-6.4 \cdot 10^{+01}$
$1/5 \cdot 2^{-8}$	$4.1 \cdot 10^{-14}$	4.990	$-1.3 \cdot 10^{+02}$	$9.2 \cdot 10^{-15}$	5.056	$-4.7 \cdot 10^{+01}$
$1/5 \cdot 2^{-9}$	$1.3 \cdot 10^{-15}$	4.995	$-1.3 \cdot 10^{+02}$	$2.7 \cdot 10^{-16}$	5.031	$-3.9 \cdot 10^{+01}$
$1/5 \cdot 2^{-10}$	$4.1 \cdot 10^{-17}$	4.997	$-1.4 \cdot 10^{+02}$	$8.4 \cdot 10^{-18}$	5.016	$-3.4 \cdot 10^{+01}$
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-18}$	4.999	$-1.4 \cdot 10^{+02}$	$2.6 \cdot 10^{-19}$	5.009	$-3.2 \cdot 10^{+01}$
$1/5 \cdot 2^{-12}$	$4.0 \cdot 10^{-20}$			$8.1 \cdot 10^{-21}$		

Table 4.1: Implicit Euler, basic solution and IDeC iterates $1-5$ for 3.3a.

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.293 \cdot 10^{+01}$	1.00000	1.013	$0.150 \cdot 10^{+02}$
$1/5 \cdot 2^{-1}$	$0.145 \cdot 10^{+01}$	1.00000	1.011	$0.149 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.722 \cdot 10^{+00}$	1.00000	1.007	$0.147 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.359 \cdot 10^{+00}$	1.00000	1.004	$0.146 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.179 \cdot 10^{+00}$	1.00000	1.002	$0.144 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.895 \cdot 10^{-01}$	1.00000	1.001	$0.144 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.447 \cdot 10^{-01}$	1.00000	1.000	$0.143 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.223 \cdot 10^{-01}$	1.00000	1.000	$0.143 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.112 \cdot 10^{-01}$	1.00000	1.000	$0.143 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.558 \cdot 10^{-02}$	1.00000	1.000	$0.143 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.279 \cdot 10^{-02}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.425 \cdot 10^{+00}$	1.00000	3.119	$0.559 \cdot 10^{+03}$
$1/5 \cdot 2^{-2}$	$0.489 \cdot 10^{-01}$	0.80000	2.602	$0.119 \cdot 10^{+03}$
$1/5 \cdot 2^{-3}$	$0.805 \cdot 10^{-02}$	0.70000	1.969	$0.115 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.206 \cdot 10^{-02}$	0.70000	2.156	$0.260 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.461 \cdot 10^{-03}$	0.67500	2.079	$0.176 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.109 \cdot 10^{-03}$	0.66250	0.101	$0.196 \cdot 10^{-03}$
$1/5 \cdot 2^{-7}$	$0.117 \cdot 10^{-03}$	1.00000	1.999	$0.477 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.293 \cdot 10^{-04}$	1.00000	2.000	$0.479 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.733 \cdot 10^{-05}$	1.00000	2.000	$0.480 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.183 \cdot 10^{-05}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.104 \cdot 10^{-01}$	1.00000	3.225	$0.163 \cdot 10^{+03}$
$1/5 \cdot 2^{-3}$	$0.111 \cdot 10^{-02}$	1.00000	3.148	$0.122 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.125 \cdot 10^{-03}$	1.00000	3.085	$0.928 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.147 \cdot 10^{-04}$	1.00000	3.046	$0.761 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.178 \cdot 10^{-05}$	1.00000	3.024	$0.670 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.219 \cdot 10^{-06}$	1.00000	3.012	$0.621 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.272 \cdot 10^{-07}$	1.00000	3.006	$0.595 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.338 \cdot 10^{-08}$	1.00000	3.003	$0.581 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.422 \cdot 10^{-09}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.214 \cdot 10^{-03}$	1.00000	3.796	$0.258 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.154 \cdot 10^{-04}$	1.00000	4.503	$0.571 \cdot 10^{+04}$
$1/5 \cdot 2^{-5}$	$0.679 \cdot 10^{-06}$	0.90000	4.260	$0.166 \cdot 10^{+04}$
$1/5 \cdot 2^{-6}$	$0.354 \cdot 10^{-07}$	0.85000	5.220	$0.422 \cdot 10^{+06}$
$1/5 \cdot 2^{-7}$	$0.951 \cdot 10^{-09}$	0.66250	1.863	$0.161 \cdot 10^{-03}$
$1/5 \cdot 2^{-8}$	$0.262 \cdot 10^{-09}$	1.00000	3.995	$0.679 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.164 \cdot 10^{-10}$	1.00000	3.998	$0.692 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.103 \cdot 10^{-11}$	1.00000		

Table 4.2: Implicit Euler, basic solution and extrapolation steps $1-3$ for 1.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-4}$	$0.216 \cdot 10^{-05}$	1.00000	5.089	$0.105 \cdot 10^{+05}$
$1/5 \cdot 2^{-5}$	$0.636 \cdot 10^{-07}$	1.00000	5.059	$0.900 \cdot 10^{+04}$
$1/5 \cdot 2^{-6}$	$0.191 \cdot 10^{-08}$	1.00000	5.034	$0.777 \cdot 10^{+04}$
$1/5 \cdot 2^{-7}$	$0.583 \cdot 10^{-10}$	1.00000	5.018	$0.702 \cdot 10^{+04}$
$1/5 \cdot 2^{-8}$	$0.180 \cdot 10^{-11}$	1.00000	5.009	$0.660 \cdot 10^{+04}$
$1/5 \cdot 2^{-9}$	$0.558 \cdot 10^{-13}$	1.00000	5.005	$0.637 \cdot 10^{+04}$
$1/5 \cdot 2^{-10}$	$0.174 \cdot 10^{-14}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-5}$	$0.417 \cdot 10^{-08}$	1.00000	5.664	$0.127 \cdot 10^{+05}$
$1/5 \cdot 2^{-6}$	$0.821 \cdot 10^{-10}$	1.00000	6.850	$0.119 \cdot 10^{+08}$
$1/5 \cdot 2^{-7}$	$0.712 \cdot 10^{-12}$	0.85000	7.256	$0.163 \cdot 10^{+09}$
$1/5 \cdot 2^{-8}$	$0.466 \cdot 10^{-14}$	0.67500	3.660	$0.110 \cdot 10^{-02}$
$1/5 \cdot 2^{-9}$	$0.369 \cdot 10^{-15}$	1.00000	5.985	$0.922 \cdot 10^{+05}$
$1/5 \cdot 2^{-10}$	$0.582 \cdot 10^{-17}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-6}$	$0.173 \cdot 10^{-10}$	1.00000	7.031	$0.712 \cdot 10^{+07}$
$1/5 \cdot 2^{-7}$	$0.133 \cdot 10^{-12}$	1.00000	7.025	$0.685 \cdot 10^{+07}$
$1/5 \cdot 2^{-8}$	$0.102 \cdot 10^{-14}$	1.00000	7.015	$0.638 \cdot 10^{+07}$
$1/5 \cdot 2^{-9}$	$0.787 \cdot 10^{-17}$	1.00000	7.008	$0.604 \cdot 10^{+07}$
$1/5 \cdot 2^{-10}$	$0.611 \cdot 10^{-19}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-7}$	$0.291 \cdot 10^{-14}$	1.00000	8.236	$0.375 \cdot 10^{+09}$
$1/5 \cdot 2^{-8}$	$0.964 \cdot 10^{-17}$	0.90000	9.786	$0.247 \cdot 10^{+14}$
$1/5 \cdot 2^{-9}$	$0.109 \cdot 10^{-19}$	0.90000	4.968	$0.934 \cdot 10^{-03}$
$1/5 \cdot 2^{-10}$	$0.349 \cdot 10^{-21}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-8}$	$0.660 \cdot 10^{-17}$	1.00000	8.997	$0.597 \cdot 10^{+11}$
$1/5 \cdot 2^{-9}$	$0.129 \cdot 10^{-19}$	1.00000	9.006	$0.640 \cdot 10^{+11}$
$1/5 \cdot 2^{-10}$	$0.251 \cdot 10^{-22}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-9}$	$0.139 \cdot 10^{-21}$	1.00000	10.329	$0.222 \cdot 10^{+14}$
$1/5 \cdot 2^{-10}$	$0.108 \cdot 10^{-24}$	1.00000		

Table 4.3: Implicit Euler, extrapolation steps $4 - 9$ for 1.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.835 \cdot 10^{-01}$	0.60000	0.817	$0.311 \cdot 10^{+00}$
$1/5 \cdot 2^{-1}$	$0.474 \cdot 10^{-01}$	0.60000	0.915	$0.390 \cdot 10^{+00}$
$1/5 \cdot 2^{-2}$	$0.251 \cdot 10^{-01}$	0.60000	0.959	$0.445 \cdot 10^{+00}$
$1/5 \cdot 2^{-3}$	$0.129 \cdot 10^{-01}$	0.60000	0.974	$0.470 \cdot 10^{+00}$
$1/5 \cdot 2^{-4}$	$0.658 \cdot 10^{-02}$	0.61250	0.993	$0.511 \cdot 10^{+00}$
$1/5 \cdot 2^{-5}$	$0.331 \cdot 10^{-02}$	0.60625	0.995	$0.516 \cdot 10^{+00}$
$1/5 \cdot 2^{-6}$	$0.166 \cdot 10^{-02}$	0.60625	0.998	$0.524 \cdot 10^{+00}$
$1/5 \cdot 2^{-7}$	$0.831 \cdot 10^{-03}$	0.60625	0.999	$0.529 \cdot 10^{+00}$
$1/5 \cdot 2^{-8}$	$0.416 \cdot 10^{-03}$	0.60547	1.000	$0.530 \cdot 10^{+00}$
$1/5 \cdot 2^{-9}$	$0.208 \cdot 10^{-03}$	0.60508	1.000	$0.531 \cdot 10^{+00}$
$1/5 \cdot 2^{-10}$	$0.104 \cdot 10^{-03}$	0.60527		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.113 \cdot 10^{-01}$	0.60000	1.862	$0.821 \cdot 10^{+00}$
$1/5 \cdot 2^{-2}$	$0.310 \cdot 10^{-02}$	0.50000	1.964	$0.111 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.795 \cdot 10^{-03}$	0.50000	2.007	$0.131 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.198 \cdot 10^{-03}$	0.52500	1.981	$0.116 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.501 \cdot 10^{-04}$	0.51250	1.991	$0.122 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.126 \cdot 10^{-04}$	0.50625	2.002	$0.130 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.315 \cdot 10^{-05}$	0.50937	1.998	$0.127 \cdot 10^{+01}$
$1/5 \cdot 2^{-8}$	$0.788 \cdot 10^{-06}$	0.50781	2.000	$0.129 \cdot 10^{+01}$
$1/5 \cdot 2^{-9}$	$0.197 \cdot 10^{-06}$	0.50781	1.999	$0.128 \cdot 10^{+01}$
$1/5 \cdot 2^{-10}$	$0.493 \cdot 10^{-07}$	0.50742		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.513 \cdot 10^{-03}$	1.00000	2.757	$0.198 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.759 \cdot 10^{-04}$	1.00000	2.868	$0.298 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.104 \cdot 10^{-04}$	0.95000	2.934	$0.398 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.136 \cdot 10^{-05}$	0.92500	2.966	$0.468 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.174 \cdot 10^{-06}$	0.92500	2.983	$0.517 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.220 \cdot 10^{-07}$	0.92500	2.991	$0.546 \cdot 10^{+01}$
$1/5 \cdot 2^{-8}$	$0.277 \cdot 10^{-08}$	0.92500	2.996	$0.563 \cdot 10^{+01}$
$1/5 \cdot 2^{-9}$	$0.347 \cdot 10^{-09}$	0.92500	2.998	$0.573 \cdot 10^{+01}$
$1/5 \cdot 2^{-10}$	$0.435 \cdot 10^{-10}$	0.92500		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.150 \cdot 10^{-04}$	0.80000	2.266	$0.642 \cdot 10^{-01}$
$1/5 \cdot 2^{-4}$	$0.313 \cdot 10^{-05}$	1.00000	3.823	$0.589 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.221 \cdot 10^{-06}$	1.00000	3.910	$0.916 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.147 \cdot 10^{-07}$	1.00000	3.954	$0.119 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.949 \cdot 10^{-09}$	1.00000	3.977	$0.137 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.603 \cdot 10^{-10}$	1.00000	5.504	$0.763 \cdot 10^{+07}$
$1/5 \cdot 2^{-9}$	$0.133 \cdot 10^{-11}$	0.73750	2.479	$0.372 \cdot 10^{-03}$
$1/5 \cdot 2^{-10}$	$0.238 \cdot 10^{-12}$	1.00000		

Table 4.4: Implicit Euler, basic solution and extrapolation steps $1-3$ for 1.2a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-4}$	$0.703 \cdot 10^{-06}$	1.00000	4.687	$0.586 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.273 \cdot 10^{-07}$	1.00000	4.838	$0.126 \cdot 10^{+04}$
$1/5 \cdot 2^{-6}$	$0.953 \cdot 10^{-09}$	1.00000	4.918	$0.199 \cdot 10^{+04}$
$1/5 \cdot 2^{-7}$	$0.315 \cdot 10^{-10}$	1.00000	4.959	$0.259 \cdot 10^{+04}$
$1/5 \cdot 2^{-8}$	$0.101 \cdot 10^{-11}$	1.00000	4.979	$0.300 \cdot 10^{+04}$
$1/5 \cdot 2^{-9}$	$0.322 \cdot 10^{-13}$	1.00000	4.990	$0.326 \cdot 10^{+04}$
$1/5 \cdot 2^{-10}$	$0.101 \cdot 10^{-14}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-5}$	$0.548 \cdot 10^{-08}$	1.00000	5.716	$0.218 \cdot 10^{+05}$
$1/5 \cdot 2^{-6}$	$0.104 \cdot 10^{-09}$	1.00000	5.855	$0.486 \cdot 10^{+05}$
$1/5 \cdot 2^{-7}$	$0.180 \cdot 10^{-11}$	1.00000	5.927	$0.773 \cdot 10^{+05}$
$1/5 \cdot 2^{-8}$	$0.296 \cdot 10^{-13}$	1.00000	5.963	$0.100 \cdot 10^{+06}$
$1/5 \cdot 2^{-9}$	$0.474 \cdot 10^{-15}$	1.00000	5.982	$0.116 \cdot 10^{+06}$
$1/5 \cdot 2^{-10}$	$0.751 \cdot 10^{-17}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-6}$	$0.189 \cdot 10^{-10}$	1.00000	6.757	$0.160 \cdot 10^{+07}$
$1/5 \cdot 2^{-7}$	$0.175 \cdot 10^{-12}$	1.00000	6.881	$0.355 \cdot 10^{+07}$
$1/5 \cdot 2^{-8}$	$0.148 \cdot 10^{-14}$	1.00000	6.941	$0.549 \cdot 10^{+07}$
$1/5 \cdot 2^{-9}$	$0.121 \cdot 10^{-16}$	1.00000	6.971	$0.693 \cdot 10^{+07}$
$1/5 \cdot 2^{-10}$	$0.962 \cdot 10^{-19}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-7}$	$0.272 \cdot 10^{-13}$	1.00000	7.843	$0.278 \cdot 10^{+09}$
$1/5 \cdot 2^{-8}$	$0.119 \cdot 10^{-15}$	1.00000	7.938	$0.550 \cdot 10^{+09}$
$1/5 \cdot 2^{-9}$	$0.484 \cdot 10^{-18}$	1.00000	7.976	$0.737 \cdot 10^{+09}$
$1/5 \cdot 2^{-10}$	$0.192 \cdot 10^{-20}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-8}$	$0.203 \cdot 10^{-16}$	1.00000	8.554	$0.768 \cdot 10^{+10}$
$1/5 \cdot 2^{-9}$	$0.540 \cdot 10^{-19}$	1.00000	8.764	$0.401 \cdot 10^{+11}$
$1/5 \cdot 2^{-10}$	$0.124 \cdot 10^{-21}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-9}$	$0.144 \cdot 10^{-19}$	1.00000	9.584	$0.665 \cdot 10^{+13}$
$1/5 \cdot 2^{-10}$	$0.188 \cdot 10^{-22}$	1.00000		

Table 4.5: Implicit Euler, extrapolation steps $4 - 9$ for 1.2a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.472 \cdot 10^{+00}$	0.40000	0.811	$0.174 \cdot 10^{+01}$
$1/5 \cdot 2^{-1}$	$0.269 \cdot 10^{+00}$	0.50000	0.291	$0.525 \cdot 10^{+00}$
$1/5 \cdot 2^{-2}$	$0.220 \cdot 10^{+00}$	1.00000	0.991	$0.427 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.111 \cdot 10^{+00}$	1.00000	0.995	$0.435 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.555 \cdot 10^{-01}$	1.00000	0.998	$0.439 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.278 \cdot 10^{-01}$	1.00000	0.999	$0.442 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.139 \cdot 10^{-01}$	1.00000	0.999	$0.444 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.696 \cdot 10^{-02}$	1.00000	1.571	$0.178 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.234 \cdot 10^{-02}$	0.52344	0.429	$0.502 \cdot 10^{-01}$
$1/5 \cdot 2^{-9}$	$0.174 \cdot 10^{-02}$	1.00000	1.000	$0.445 \cdot 10^{+01}$
$1/5 \cdot 2^{-10}$	$0.870 \cdot 10^{-03}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.115 \cdot 10^{+00}$	1.00000	1.973	$0.108 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.294 \cdot 10^{-01}$	1.00000	1.982	$0.111 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.743 \cdot 10^{-02}$	1.00000	1.990	$0.115 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.187 \cdot 10^{-02}$	1.00000	1.995	$0.117 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.470 \cdot 10^{-03}$	1.00000	1.997	$0.119 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.118 \cdot 10^{-03}$	1.00000	1.999	$0.119 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.294 \cdot 10^{-04}$	1.00000	1.999	$0.120 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.736 \cdot 10^{-05}$	1.00000	2.000	$0.120 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.184 \cdot 10^{-05}$	1.00000	2.000	$0.120 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.460 \cdot 10^{-06}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.726 \cdot 10^{-03}$	1.00000	2.571	$0.161 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.122 \cdot 10^{-03}$	1.00000	2.821	$0.404 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.173 \cdot 10^{-04}$	1.00000	2.917	$0.615 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.229 \cdot 10^{-05}$	1.00000	2.960	$0.764 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.294 \cdot 10^{-06}$	1.00000	2.965	$0.788 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.377 \cdot 10^{-07}$	1.00000	2.998	$0.973 \cdot 10^{+01}$
$1/5 \cdot 2^{-8}$	$0.472 \cdot 10^{-08}$	1.00000	3.003	$0.101 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.588 \cdot 10^{-09}$	1.00000	2.996	$0.954 \cdot 10^{+01}$
$1/5 \cdot 2^{-10}$	$0.738 \cdot 10^{-10}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.359 \cdot 10^{-04}$	1.00000	3.963	$0.802 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.230 \cdot 10^{-05}$	1.00000	3.975	$0.843 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.146 \cdot 10^{-06}$	1.00000	3.986	$0.892 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.924 \cdot 10^{-08}$	1.00000	3.992	$0.927 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.580 \cdot 10^{-09}$	1.00000	3.996	$0.950 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.364 \cdot 10^{-10}$	1.00000	3.998	$0.963 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.228 \cdot 10^{-11}$	1.00000	3.999	$0.970 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.142 \cdot 10^{-12}$	1.00000		

Table 4.6: Implicit Euler, basic solution and extrapolation steps $1 - 3$ for 2.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-4}$	$0.100 \cdot 10^{-06}$	0.40000	5.125	$0.567 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.287 \cdot 10^{-08}$	0.40000	4.876	$0.161 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.977 \cdot 10^{-10}$	1.00000	4.908	$0.192 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.326 \cdot 10^{-11}$	1.00000	4.922	$0.211 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.107 \cdot 10^{-12}$	1.00000	4.995	$0.357 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.337 \cdot 10^{-14}$	1.00000	5.006	$0.388 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.105 \cdot 10^{-15}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-5}$	$0.819 \cdot 10^{-09}$	1.00000	5.965	$0.115 \cdot 10^{+05}$
$1/5 \cdot 2^{-6}$	$0.131 \cdot 10^{-10}$	1.00000	5.975	$0.122 \cdot 10^{+05}$
$1/5 \cdot 2^{-7}$	$0.209 \cdot 10^{-12}$	1.00000	5.986	$0.131 \cdot 10^{+05}$
$1/5 \cdot 2^{-8}$	$0.329 \cdot 10^{-14}$	1.00000	5.992	$0.137 \cdot 10^{+05}$
$1/5 \cdot 2^{-9}$	$0.517 \cdot 10^{-16}$	1.00000	5.996	$0.141 \cdot 10^{+05}$
$1/5 \cdot 2^{-10}$	$0.810 \cdot 10^{-18}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-6}$	$0.755 \cdot 10^{-12}$	0.40000	7.071	$0.390 \cdot 10^{+06}$
$1/5 \cdot 2^{-7}$	$0.562 \cdot 10^{-14}$	0.50000	7.031	$0.301 \cdot 10^{+06}$
$1/5 \cdot 2^{-8}$	$0.429 \cdot 10^{-16}$	0.45000	7.019	$0.278 \cdot 10^{+06}$
$1/5 \cdot 2^{-9}$	$0.331 \cdot 10^{-18}$	0.45000	7.009	$0.256 \cdot 10^{+06}$
$1/5 \cdot 2^{-10}$	$0.257 \cdot 10^{-20}$	0.45000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-7}$	$0.113 \cdot 10^{-14}$	1.00000	7.968	$0.260 \cdot 10^{+08}$
$1/5 \cdot 2^{-8}$	$0.453 \cdot 10^{-17}$	1.00000	7.977	$0.276 \cdot 10^{+08}$
$1/5 \cdot 2^{-9}$	$0.180 \cdot 10^{-19}$	1.00000	7.987	$0.298 \cdot 10^{+08}$
$1/5 \cdot 2^{-10}$	$0.708 \cdot 10^{-22}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-8}$	$0.320 \cdot 10^{-18}$	0.40000	9.028	$0.361 \cdot 10^{+10}$
$1/5 \cdot 2^{-9}$	$0.613 \cdot 10^{-21}$	0.50000	9.031	$0.369 \cdot 10^{+10}$
$1/5 \cdot 2^{-10}$	$0.117 \cdot 10^{-23}$	0.45000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-9}$	$0.945 \cdot 10^{-22}$	1.00000	9.973	$0.922 \cdot 10^{+12}$
$1/5 \cdot 2^{-10}$	$0.940 \cdot 10^{-25}$	1.00000		

Table 4.7: Implicit Euler, extrapolation steps $4 - 9$ for 2.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.296 \cdot 10^{+01}$	1.00000	1.277	$0.231 \cdot 10^{+02}$
$1/5 \cdot 2^{-1}$	$0.122 \cdot 10^{+01}$	1.00000	1.119	$0.160 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.561 \cdot 10^{+00}$	1.00000	1.056	$0.133 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.270 \cdot 10^{+00}$	1.00000	1.027	$0.119 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.133 \cdot 10^{+00}$	1.00000	1.013	$0.112 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.657 \cdot 10^{-01}$	1.00000	1.007	$0.109 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.327 \cdot 10^{-01}$	1.00000	1.003	$0.107 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.163 \cdot 10^{-01}$	1.00000	1.002	$0.105 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.814 \cdot 10^{-02}$	1.00000	1.001	$0.105 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.407 \cdot 10^{-02}$	1.00000	1.000	$0.104 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.203 \cdot 10^{-02}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.517 \cdot 10^{+00}$	1.00000	2.419	$0.136 \cdot 10^{+03}$
$1/5 \cdot 2^{-2}$	$0.967 \cdot 10^{-01}$	1.00000	2.185	$0.673 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.213 \cdot 10^{-01}$	1.00000	2.087	$0.470 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.500 \cdot 10^{-02}$	1.00000	2.043	$0.386 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.121 \cdot 10^{-02}$	1.00000	2.021	$0.346 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.299 \cdot 10^{-03}$	1.00000	2.010	$0.325 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.743 \cdot 10^{-04}$	1.00000	2.005	$0.315 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.185 \cdot 10^{-04}$	1.00000	2.003	$0.309 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.462 \cdot 10^{-05}$	1.00000	2.001	$0.306 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.115 \cdot 10^{-05}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.434 \cdot 10^{-01}$	1.00000	3.485	$0.148 \cdot 10^{+04}$
$1/5 \cdot 2^{-3}$	$0.388 \cdot 10^{-02}$	1.00000	3.218	$0.554 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.417 \cdot 10^{-03}$	1.00000	3.104	$0.336 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.485 \cdot 10^{-04}$	1.00000	3.051	$0.257 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.585 \cdot 10^{-05}$	1.00000	3.025	$0.222 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.719 \cdot 10^{-06}$	1.00000	3.012	$0.204 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.891 \cdot 10^{-07}$	1.00000	3.006	$0.195 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.111 \cdot 10^{-07}$	1.00000	3.003	$0.191 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.138 \cdot 10^{-08}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.177 \cdot 10^{-02}$	1.00000	4.512	$0.299 \cdot 10^{+05}$
$1/5 \cdot 2^{-4}$	$0.775 \cdot 10^{-04}$	1.00000	4.231	$0.873 \cdot 10^{+04}$
$1/5 \cdot 2^{-5}$	$0.413 \cdot 10^{-05}$	1.00000	4.110	$0.474 \cdot 10^{+04}$
$1/5 \cdot 2^{-6}$	$0.239 \cdot 10^{-06}$	1.00000	4.054	$0.342 \cdot 10^{+04}$
$1/5 \cdot 2^{-7}$	$0.144 \cdot 10^{-07}$	1.00000	4.027	$0.287 \cdot 10^{+04}$
$1/5 \cdot 2^{-8}$	$0.883 \cdot 10^{-09}$	1.00000	4.013	$0.261 \cdot 10^{+04}$
$1/5 \cdot 2^{-9}$	$0.547 \cdot 10^{-10}$	1.00000	4.007	$0.247 \cdot 10^{+04}$
$1/5 \cdot 2^{-10}$	$0.340 \cdot 10^{-11}$	1.00000		

Table 4.8: Implicit Euler, basic solution and extrapolation steps $1 - 3$ for 2.2a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-4}$	$0.352 \cdot 10^{-04}$	1.00000	5.526	$0.115 \cdot 10^{+07}$
$1/5 \cdot 2^{-5}$	$0.764 \cdot 10^{-06}$	1.00000	5.238	$0.268 \cdot 10^{+06}$
$1/5 \cdot 2^{-6}$	$0.203 \cdot 10^{-07}$	1.00000	5.114	$0.131 \cdot 10^{+06}$
$1/5 \cdot 2^{-7}$	$0.585 \cdot 10^{-09}$	1.00000	5.056	$0.900 \cdot 10^{+05}$
$1/5 \cdot 2^{-8}$	$0.176 \cdot 10^{-10}$	1.00000	5.028	$0.736 \cdot 10^{+05}$
$1/5 \cdot 2^{-9}$	$0.539 \cdot 10^{-12}$	1.00000	5.014	$0.660 \cdot 10^{+05}$
$1/5 \cdot 2^{-10}$	$0.167 \cdot 10^{-13}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-5}$	$0.347 \cdot 10^{-06}$	1.00000	6.532	$0.867 \cdot 10^{+08}$
$1/5 \cdot 2^{-6}$	$0.375 \cdot 10^{-08}$	1.00000	6.241	$0.162 \cdot 10^{+08}$
$1/5 \cdot 2^{-7}$	$0.495 \cdot 10^{-10}$	1.00000	6.115	$0.717 \cdot 10^{+07}$
$1/5 \cdot 2^{-8}$	$0.714 \cdot 10^{-12}$	1.00000	6.057	$0.471 \cdot 10^{+07}$
$1/5 \cdot 2^{-9}$	$0.107 \cdot 10^{-13}$	1.00000	6.028	$0.376 \cdot 10^{+07}$
$1/5 \cdot 2^{-10}$	$0.164 \cdot 10^{-15}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-6}$	$0.170 \cdot 10^{-08}$	1.00000	7.535	$0.128 \cdot 10^{+11}$
$1/5 \cdot 2^{-7}$	$0.915 \cdot 10^{-11}$	1.00000	7.243	$0.193 \cdot 10^{+10}$
$1/5 \cdot 2^{-8}$	$0.604 \cdot 10^{-13}$	1.00000	7.116	$0.781 \cdot 10^{+09}$
$1/5 \cdot 2^{-9}$	$0.436 \cdot 10^{-15}$	1.00000	7.057	$0.491 \cdot 10^{+09}$
$1/5 \cdot 2^{-10}$	$0.327 \cdot 10^{-17}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-7}$	$0.414 \cdot 10^{-11}$	1.00000	8.537	$0.374 \cdot 10^{+13}$
$1/5 \cdot 2^{-8}$	$0.112 \cdot 10^{-13}$	1.00000	8.243	$0.459 \cdot 10^{+12}$
$1/5 \cdot 2^{-9}$	$0.368 \cdot 10^{-16}$	1.00000	8.117	$0.170 \cdot 10^{+12}$
$1/5 \cdot 2^{-10}$	$0.133 \cdot 10^{-18}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-8}$	$0.505 \cdot 10^{-14}$	1.00000	9.537	$0.218 \cdot 10^{+16}$
$1/5 \cdot 2^{-9}$	$0.679 \cdot 10^{-17}$	1.00000	9.244	$0.217 \cdot 10^{+15}$
$1/5 \cdot 2^{-10}$	$0.112 \cdot 10^{-19}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-9}$	$0.307 \cdot 10^{-17}$	1.00000	10.538	$0.253 \cdot 10^{+19}$
$1/5 \cdot 2^{-10}$	$0.207 \cdot 10^{-20}$	1.00000		

Table 4.9: Implicit Euler, extrapolation steps $4 - 9$ for 2.2a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.735 \cdot 10^{+00}$	0.60000	0.718	$0.233 \cdot 10^{+01}$
$1/5 \cdot 2^{-1}$	$0.447 \cdot 10^{+00}$	0.60000	0.886	$0.344 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.242 \cdot 10^{+00}$	0.60000	0.949	$0.415 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.125 \cdot 10^{+00}$	0.60000	0.976	$0.458 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.637 \cdot 10^{-01}$	0.60000	0.988	$0.483 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.321 \cdot 10^{-01}$	0.60625	0.994	$0.498 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.161 \cdot 10^{-01}$	0.60938	0.997	$0.507 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.808 \cdot 10^{-02}$	0.61094	0.998	$0.512 \cdot 10^{+01}$
$1/5 \cdot 2^{-8}$	$0.404 \cdot 10^{-02}$	0.61172	0.999	$0.515 \cdot 10^{+01}$
$1/5 \cdot 2^{-9}$	$0.202 \cdot 10^{-02}$	0.61211	1.000	$0.516 \cdot 10^{+01}$
$1/5 \cdot 2^{-10}$	$0.101 \cdot 10^{-02}$	0.61211		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.251 \cdot 10^{+00}$	1.00000	2.035	$0.272 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.613 \cdot 10^{-01}$	1.00000	2.018	$0.259 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.151 \cdot 10^{-01}$	1.00000	2.010	$0.251 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.376 \cdot 10^{-02}$	1.00000	2.005	$0.246 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.936 \cdot 10^{-03}$	1.00000	2.002	$0.243 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.234 \cdot 10^{-03}$	1.00000	2.001	$0.241 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.584 \cdot 10^{-04}$	1.00000	2.001	$0.240 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.146 \cdot 10^{-04}$	1.00000	2.000	$0.239 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.364 \cdot 10^{-05}$	1.00000	2.000	$0.239 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.911 \cdot 10^{-06}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.411 \cdot 10^{-02}$	0.60000	2.989	$0.318 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.517 \cdot 10^{-03}$	0.60000	2.990	$0.319 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.651 \cdot 10^{-04}$	0.60000	2.994	$0.325 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.818 \cdot 10^{-05}$	0.60000	2.996	$0.329 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.102 \cdot 10^{-05}$	0.58750	2.998	$0.332 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.128 \cdot 10^{-06}$	0.59375	2.999	$0.334 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.160 \cdot 10^{-07}$	0.59375	3.000	$0.335 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.201 \cdot 10^{-08}$	0.59219	3.000	$0.336 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.251 \cdot 10^{-09}$	0.59219		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.243 \cdot 10^{-04}$	1.00000	4.033	$0.703 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.148 \cdot 10^{-05}$	1.00000	4.008	$0.631 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.923 \cdot 10^{-07}$	1.00000	4.002	$0.611 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.576 \cdot 10^{-08}$	1.00000	4.001	$0.606 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.360 \cdot 10^{-09}$	1.00000	3.993	$0.577 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.226 \cdot 10^{-10}$	0.99375	4.007	$0.638 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.141 \cdot 10^{-11}$	1.00000	3.934	$0.359 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.920 \cdot 10^{-13}$	0.88437		

Table 4.10: Implicit Euler, basic solution and extrapolation steps $1 - 3$ for 3.1a

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-4}$	$0.242 \cdot 10^{-06}$	0.60000	4.959	$0.664 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.778 \cdot 10^{-08}$	0.60000	4.973	$0.712 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.248 \cdot 10^{-09}$	0.60000	4.984	$0.758 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.782 \cdot 10^{-11}$	0.62500	4.992	$0.796 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.246 \cdot 10^{-12}$	0.61250	4.996	$0.819 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.771 \cdot 10^{-14}$	0.61875	4.998	$0.833 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.241 \cdot 10^{-15}$	0.61875		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-5}$	$0.600 \cdot 10^{-09}$	1.00000	6.033	$0.119 \cdot 10^{+05}$
$1/5 \cdot 2^{-6}$	$0.917 \cdot 10^{-11}$	1.00000	6.008	$0.103 \cdot 10^{+05}$
$1/5 \cdot 2^{-7}$	$0.142 \cdot 10^{-12}$	1.00000	5.969	$0.802 \cdot 10^{+04}$
$1/5 \cdot 2^{-8}$	$0.227 \cdot 10^{-14}$	1.00000	5.982	$0.882 \cdot 10^{+04}$
$1/5 \cdot 2^{-9}$	$0.359 \cdot 10^{-16}$	1.00000	5.991	$0.944 \cdot 10^{+04}$
$1/5 \cdot 2^{-10}$	$0.565 \cdot 10^{-18}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-6}$	$0.124 \cdot 10^{-11}$	0.60000	6.935	$0.292 \cdot 10^{+06}$
$1/5 \cdot 2^{-7}$	$0.101 \cdot 10^{-13}$	0.60000	6.959	$0.340 \cdot 10^{+06}$
$1/5 \cdot 2^{-8}$	$0.813 \cdot 10^{-16}$	0.65000	6.977	$0.387 \cdot 10^{+06}$
$1/5 \cdot 2^{-9}$	$0.645 \cdot 10^{-18}$	0.65000	6.988	$0.422 \cdot 10^{+06}$
$1/5 \cdot 2^{-10}$	$0.508 \cdot 10^{-20}$	0.63750		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-7}$	$0.938 \cdot 10^{-15}$	1.00000	7.908	$0.146 \cdot 10^{+08}$
$1/5 \cdot 2^{-8}$	$0.391 \cdot 10^{-17}$	1.00000	7.948	$0.194 \cdot 10^{+08}$
$1/5 \cdot 2^{-9}$	$0.158 \cdot 10^{-19}$	1.00000	7.972	$0.235 \cdot 10^{+08}$
$1/5 \cdot 2^{-10}$	$0.630 \cdot 10^{-22}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-8}$	$0.402 \cdot 10^{-18}$	0.60000	8.915	$0.202 \cdot 10^{+10}$
$1/5 \cdot 2^{-9}$	$0.832 \cdot 10^{-21}$	0.60000	8.944	$0.253 \cdot 10^{+10}$
$1/5 \cdot 2^{-10}$	$0.169 \cdot 10^{-23}$	0.65000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-9}$	$0.952 \cdot 10^{-22}$	1.00000	9.925	$0.640 \cdot 10^{+12}$
$1/5 \cdot 2^{-10}$	$0.979 \cdot 10^{-25}$	1.00000		

Table 4.11: Implicit Euler, extrapolation steps $4-9$ for 3.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.295 \cdot 10^{+01}$	1.00000	1.057	$0.162 \cdot 10^{+02}$
$1/5 \cdot 2^{-1}$	$0.142 \cdot 10^{+01}$	1.00000	1.030	$0.152 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.694 \cdot 10^{+00}$	1.00000	1.015	$0.145 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.343 \cdot 10^{+00}$	1.00000	1.008	$0.141 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.171 \cdot 10^{+00}$	1.00000	1.004	$0.139 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.852 \cdot 10^{-01}$	1.00000	1.002	$0.138 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.425 \cdot 10^{-01}$	1.00000	1.001	$0.137 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.213 \cdot 10^{-01}$	1.00000	1.000	$0.136 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.106 \cdot 10^{-01}$	1.00000	1.000	$0.136 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.531 \cdot 10^{-02}$	1.00000	1.000	$0.136 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.265 \cdot 10^{-02}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.230 \cdot 10^{+00}$	1.00000	2.072	$0.272 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.547 \cdot 10^{-01}$	1.00000	2.037	$0.245 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.133 \cdot 10^{-01}$	1.00000	2.019	$0.229 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.329 \cdot 10^{-02}$	1.00000	2.010	$0.219 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.817 \cdot 10^{-03}$	1.00000	2.005	$0.214 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.204 \cdot 10^{-03}$	1.00000	2.002	$0.211 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.508 \cdot 10^{-04}$	1.00000	2.001	$0.210 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.127 \cdot 10^{-04}$	1.00000	2.001	$0.209 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.317 \cdot 10^{-05}$	1.00000	2.000	$0.208 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.792 \cdot 10^{-06}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.374 \cdot 10^{-02}$	1.00000	3.008	$0.307 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.465 \cdot 10^{-03}$	1.00000	3.005	$0.304 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.579 \cdot 10^{-04}$	1.00000	3.003	$0.300 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.723 \cdot 10^{-05}$	1.00000	3.002	$0.298 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.902 \cdot 10^{-06}$	1.00000	3.001	$0.297 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.113 \cdot 10^{-06}$	1.00000	3.000	$0.296 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.141 \cdot 10^{-07}$	1.00000	3.000	$0.296 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.176 \cdot 10^{-08}$	1.00000	3.000	$0.296 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.220 \cdot 10^{-09}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.913 \cdot 10^{-05}$	1.00000	3.994	$0.228 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.573 \cdot 10^{-06}$	1.00000	3.998	$0.233 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.359 \cdot 10^{-07}$	1.00000	4.000	$0.235 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.224 \cdot 10^{-08}$	1.00000	4.000	$0.235 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.140 \cdot 10^{-09}$	1.00000	4.000	$0.235 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.876 \cdot 10^{-11}$	1.00000	4.000	$0.235 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.548 \cdot 10^{-12}$	1.00000	4.000	$0.235 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.342 \cdot 10^{-13}$	1.00000		

Table 4.12: Implicit Euler, basic solution and extrapolation steps $1-3$ for 3.1b

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-4}$	$0.539 \cdot 10^{-07}$	1.00000	5.022	$0.195 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.166 \cdot 10^{-08}$	1.00000	5.013	$0.185 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.514 \cdot 10^{-10}$	1.00000	5.007	$0.179 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.160 \cdot 10^{-11}$	1.00000	5.003	$0.176 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.498 \cdot 10^{-13}$	1.00000	5.002	$0.173 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.156 \cdot 10^{-14}$	1.00000	5.001	$0.172 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.486 \cdot 10^{-16}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-5}$	$0.434 \cdot 10^{-10}$	1.00000	5.994	$0.708 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.682 \cdot 10^{-12}$	1.00000	5.999	$0.726 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.107 \cdot 10^{-13}$	1.00000	6.000	$0.731 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.167 \cdot 10^{-15}$	1.00000	6.000	$0.732 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.260 \cdot 10^{-17}$	1.00000	6.000	$0.733 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.407 \cdot 10^{-19}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-6}$	$0.539 \cdot 10^{-13}$	1.00000	7.031	$0.222 \cdot 10^{+05}$
$1/5 \cdot 2^{-7}$	$0.412 \cdot 10^{-15}$	1.00000	7.017	$0.202 \cdot 10^{+05}$
$1/5 \cdot 2^{-8}$	$0.318 \cdot 10^{-17}$	1.00000	7.009	$0.191 \cdot 10^{+05}$
$1/5 \cdot 2^{-9}$	$0.247 \cdot 10^{-19}$	1.00000	7.005	$0.185 \cdot 10^{+05}$
$1/5 \cdot 2^{-10}$	$0.192 \cdot 10^{-21}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-7}$	$0.109 \cdot 10^{-16}$	1.00000	7.995	$0.296 \cdot 10^{+06}$
$1/5 \cdot 2^{-8}$	$0.427 \cdot 10^{-19}$	1.00000	7.999	$0.305 \cdot 10^{+06}$
$1/5 \cdot 2^{-9}$	$0.167 \cdot 10^{-21}$	1.00000	8.000	$0.307 \cdot 10^{+06}$
$1/5 \cdot 2^{-10}$	$0.652 \cdot 10^{-24}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-8}$	$0.303 \cdot 10^{-20}$	1.00000	9.038	$0.366 \cdot 10^{+08}$
$1/5 \cdot 2^{-9}$	$0.577 \cdot 10^{-23}$	1.00000	9.020	$0.320 \cdot 10^{+08}$
$1/5 \cdot 2^{-10}$	$0.111 \cdot 10^{-25}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-9}$	$0.153 \cdot 10^{-24}$	1.00000	9.923	$0.101 \cdot 10^{+10}$
$1/5 \cdot 2^{-10}$	$0.158 \cdot 10^{-27}$	1.00000		

Table 4.13: Implicit Euler, extrapolation steps $4 - 9$ for 3.1b

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.118 \cdot 10^{+02}$	1.00000	1.111	$0.708 \cdot 10^{+02}$
$1/5 \cdot 2^{-1}$	$0.548 \cdot 10^{+01}$	1.00000	1.060	$0.629 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.263 \cdot 10^{+01}$	1.00000	1.031	$0.577 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.129 \cdot 10^{+01}$	1.00000	1.016	$0.545 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.636 \cdot 10^{+00}$	1.00000	1.008	$0.527 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.316 \cdot 10^{+00}$	1.00000	1.004	$0.516 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.158 \cdot 10^{+00}$	1.00000	1.002	$0.510 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.787 \cdot 10^{-01}$	1.00000	1.001	$0.507 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.393 \cdot 10^{-01}$	1.00000	1.001	$0.505 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.197 \cdot 10^{-01}$	1.00000	1.000	$0.504 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.982 \cdot 10^{-02}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.110 \cdot 10^{+01}$	1.00000	2.176	$0.165 \cdot 10^{+03}$
$1/5 \cdot 2^{-2}$	$0.244 \cdot 10^{+00}$	1.00000	2.095	$0.130 \cdot 10^{+03}$
$1/5 \cdot 2^{-3}$	$0.571 \cdot 10^{-01}$	1.00000	2.027	$0.101 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.140 \cdot 10^{-01}$	1.00000	2.000	$0.895 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.350 \cdot 10^{-02}$	1.00000	2.000	$0.897 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.876 \cdot 10^{-03}$	1.00000	2.000	$0.897 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.219 \cdot 10^{-03}$	1.00000	2.000	$0.897 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.548 \cdot 10^{-04}$	1.00000	2.000	$0.897 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.137 \cdot 10^{-04}$	1.00000	2.000	$0.897 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.342 \cdot 10^{-05}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.421 \cdot 10^{-01}$	1.00000	3.017	$0.355 \cdot 10^{+03}$
$1/5 \cdot 2^{-3}$	$0.520 \cdot 10^{-02}$	1.00000	3.015	$0.351 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.644 \cdot 10^{-03}$	1.00000	3.009	$0.342 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.800 \cdot 10^{-04}$	1.00000	3.005	$0.336 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.996 \cdot 10^{-05}$	1.00000	3.002	$0.331 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.124 \cdot 10^{-05}$	1.00000	3.001	$0.329 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.155 \cdot 10^{-06}$	1.00000	3.001	$0.327 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.194 \cdot 10^{-07}$	1.00000	3.000	$0.326 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.242 \cdot 10^{-08}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.328 \cdot 10^{-03}$	1.00000	3.977	$0.773 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.208 \cdot 10^{-04}$	1.00000	3.994	$0.833 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.131 \cdot 10^{-05}$	1.00000	3.999	$0.851 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.818 \cdot 10^{-07}$	1.00000	4.000	$0.856 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.511 \cdot 10^{-08}$	1.00000	4.000	$0.857 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.320 \cdot 10^{-09}$	1.00000	4.000	$0.858 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.200 \cdot 10^{-10}$	1.00000	4.000	$0.858 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.125 \cdot 10^{-11}$	1.00000		

Table 4.14: Implicit Euler, basic solution and extrapolation steps $1-3$ for 3.1c

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-4}$	$0.319 \cdot 10^{-05}$	1.00000	5.041	$0.126 \cdot 10^{+05}$
$1/5 \cdot 2^{-5}$	$0.970 \cdot 10^{-07}$	1.00000	5.026	$0.116 \cdot 10^{+05}$
$1/5 \cdot 2^{-6}$	$0.298 \cdot 10^{-08}$	1.00000	5.014	$0.108 \cdot 10^{+05}$
$1/5 \cdot 2^{-7}$	$0.921 \cdot 10^{-10}$	1.00000	5.007	$0.104 \cdot 10^{+05}$
$1/5 \cdot 2^{-8}$	$0.286 \cdot 10^{-11}$	1.00000	5.004	$0.101 \cdot 10^{+05}$
$1/5 \cdot 2^{-9}$	$0.893 \cdot 10^{-13}$	1.00000	5.002	$0.996 \cdot 10^{+04}$
$1/5 \cdot 2^{-10}$	$0.279 \cdot 10^{-14}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-5}$	$0.554 \cdot 10^{-08}$	1.00000	5.985	$0.862 \cdot 10^{+05}$
$1/5 \cdot 2^{-6}$	$0.875 \cdot 10^{-10}$	1.00000	5.996	$0.919 \cdot 10^{+05}$
$1/5 \cdot 2^{-7}$	$0.137 \cdot 10^{-11}$	1.00000	5.999	$0.936 \cdot 10^{+05}$
$1/5 \cdot 2^{-8}$	$0.214 \cdot 10^{-13}$	1.00000	6.000	$0.941 \cdot 10^{+05}$
$1/5 \cdot 2^{-9}$	$0.335 \cdot 10^{-15}$	1.00000	6.000	$0.942 \cdot 10^{+05}$
$1/5 \cdot 2^{-10}$	$0.523 \cdot 10^{-17}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-6}$	$0.103 \cdot 10^{-10}$	1.00000	7.050	$0.474 \cdot 10^{+07}$
$1/5 \cdot 2^{-7}$	$0.780 \cdot 10^{-13}$	1.00000	7.029	$0.413 \cdot 10^{+07}$
$1/5 \cdot 2^{-8}$	$0.597 \cdot 10^{-15}$	1.00000	7.016	$0.375 \cdot 10^{+07}$
$1/5 \cdot 2^{-9}$	$0.461 \cdot 10^{-17}$	1.00000	7.008	$0.354 \cdot 10^{+07}$
$1/5 \cdot 2^{-10}$	$0.358 \cdot 10^{-19}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-7}$	$0.358 \cdot 10^{-14}$	1.00000	7.989	$0.941 \cdot 10^{+08}$
$1/5 \cdot 2^{-8}$	$0.141 \cdot 10^{-16}$	1.00000	7.997	$0.997 \cdot 10^{+08}$
$1/5 \cdot 2^{-9}$	$0.552 \cdot 10^{-19}$	1.00000	7.999	$0.101 \cdot 10^{+09}$
$1/5 \cdot 2^{-10}$	$0.216 \cdot 10^{-21}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-8}$	$0.138 \cdot 10^{-17}$	1.00000	9.055	$0.189 \cdot 10^{+11}$
$1/5 \cdot 2^{-9}$	$0.260 \cdot 10^{-20}$	1.00000	9.031	$0.156 \cdot 10^{+11}$
$1/5 \cdot 2^{-10}$	$0.498 \cdot 10^{-23}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-9}$	$0.103 \cdot 10^{-21}$	1.00000	9.915	$0.640 \cdot 10^{+12}$
$1/5 \cdot 2^{-10}$	$0.107 \cdot 10^{-24}$	1.00000		

Table 4.15: Implicit Euler, extrapolation steps $4-9$ for 3.1c

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.413 \cdot 10^{+03}$	1.00000	5.392	$0.242 \cdot 10^{+07}$
$1/5 \cdot 2^{-1}$	$0.983 \cdot 10^{+01}$	1.00000	1.607	$0.398 \cdot 10^{+03}$
$1/5 \cdot 2^{-2}$	$0.323 \cdot 10^{+01}$	1.00000	1.228	$0.128 \cdot 10^{+03}$
$1/5 \cdot 2^{-3}$	$0.138 \cdot 10^{+01}$	1.00000	1.102	$0.802 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.642 \cdot 10^{+00}$	1.00000	1.048	$0.635 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.310 \cdot 10^{+00}$	1.00000	1.024	$0.560 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.153 \cdot 10^{+00}$	1.00000	1.012	$0.523 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.758 \cdot 10^{-01}$	1.00000	1.006	$0.503 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.377 \cdot 10^{-01}$	1.00000	1.003	$0.493 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.188 \cdot 10^{-01}$	1.00000	1.001	$0.487 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.940 \cdot 10^{-02}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.432 \cdot 10^{+03}$	1.00000	7.001	$0.433 \cdot 10^{+10}$
$1/5 \cdot 2^{-2}$	$0.338 \cdot 10^{+01}$	1.00000	2.838	$0.166 \cdot 10^{+05}$
$1/5 \cdot 2^{-3}$	$0.472 \cdot 10^{+00}$	1.00000	2.332	$0.257 \cdot 10^{+04}$
$1/5 \cdot 2^{-4}$	$0.938 \cdot 10^{-01}$	1.00000	2.151	$0.116 \cdot 10^{+04}$
$1/5 \cdot 2^{-5}$	$0.211 \cdot 10^{-01}$	1.00000	2.072	$0.780 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.502 \cdot 10^{-02}$	1.00000	2.035	$0.630 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.122 \cdot 10^{-02}$	1.00000	2.018	$0.562 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.302 \cdot 10^{-03}$	1.00000	2.009	$0.527 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.751 \cdot 10^{-04}$	1.00000	2.004	$0.510 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.187 \cdot 10^{-04}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.149 \cdot 10^{+03}$	1.00000	8.227	$0.752 \cdot 10^{+13}$
$1/5 \cdot 2^{-3}$	$0.496 \cdot 10^{+00}$	1.00000	3.940	$0.102 \cdot 10^{+07}$
$1/5 \cdot 2^{-4}$	$0.323 \cdot 10^{-01}$	1.00000	3.380	$0.874 \cdot 10^{+05}$
$1/5 \cdot 2^{-5}$	$0.310 \cdot 10^{-02}$	1.00000	3.174	$0.308 \cdot 10^{+05}$
$1/5 \cdot 2^{-6}$	$0.344 \cdot 10^{-03}$	1.00000	3.084	$0.183 \cdot 10^{+05}$
$1/5 \cdot 2^{-7}$	$0.406 \cdot 10^{-04}$	1.00000	3.041	$0.139 \cdot 10^{+05}$
$1/5 \cdot 2^{-8}$	$0.493 \cdot 10^{-05}$	1.00000	3.020	$0.120 \cdot 10^{+05}$
$1/5 \cdot 2^{-9}$	$0.607 \cdot 10^{-06}$	1.00000	3.010	$0.110 \cdot 10^{+05}$
$1/5 \cdot 2^{-10}$	$0.754 \cdot 10^{-07}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.218 \cdot 10^{+02}$	1.00000	9.328	$0.192 \cdot 10^{+17}$
$1/5 \cdot 2^{-4}$	$0.339 \cdot 10^{-01}$	1.00000	4.988	$0.105 \cdot 10^{+09}$
$1/5 \cdot 2^{-5}$	$0.107 \cdot 10^{-02}$	1.00000	4.403	$0.542 \cdot 10^{+07}$
$1/5 \cdot 2^{-6}$	$0.505 \cdot 10^{-04}$	1.00000	4.186	$0.155 \cdot 10^{+07}$
$1/5 \cdot 2^{-7}$	$0.278 \cdot 10^{-05}$	1.00000	4.090	$0.831 \cdot 10^{+06}$
$1/5 \cdot 2^{-8}$	$0.163 \cdot 10^{-06}$	1.00000	4.044	$0.600 \cdot 10^{+06}$
$1/5 \cdot 2^{-9}$	$0.988 \cdot 10^{-08}$	1.00000	4.022	$0.504 \cdot 10^{+06}$
$1/5 \cdot 2^{-10}$	$0.608 \cdot 10^{-09}$	1.00000		

Table 4.16: Implicit Euler, basic solution and extrapolation steps $1 - 3$ for 3.2a

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-4}$	$0.149 \cdot 10^{+01}$	1.00000	10.376	$0.832 \cdot 10^{+20}$
$1/5 \cdot 2^{-5}$	$0.112 \cdot 10^{-02}$	1.00000	6.011	$0.199 \cdot 10^{+11}$
$1/5 \cdot 2^{-6}$	$0.174 \cdot 10^{-04}$	1.00000	5.415	$0.638 \cdot 10^{+09}$
$1/5 \cdot 2^{-7}$	$0.407 \cdot 10^{-06}$	1.00000	5.192	$0.151 \cdot 10^{+09}$
$1/5 \cdot 2^{-8}$	$0.111 \cdot 10^{-07}$	1.00000	5.092	$0.742 \cdot 10^{+08}$
$1/5 \cdot 2^{-9}$	$0.327 \cdot 10^{-09}$	1.00000	5.045	$0.513 \cdot 10^{+08}$
$1/5 \cdot 2^{-10}$	$0.989 \cdot 10^{-11}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-5}$	$0.492 \cdot 10^{-01}$	1.00000	11.400	$0.658 \cdot 10^{+24}$
$1/5 \cdot 2^{-6}$	$0.182 \cdot 10^{-04}$	1.00000	7.023	$0.713 \cdot 10^{+13}$
$1/5 \cdot 2^{-7}$	$0.140 \cdot 10^{-06}$	1.00000	6.421	$0.146 \cdot 10^{+12}$
$1/5 \cdot 2^{-8}$	$0.164 \cdot 10^{-08}$	1.00000	6.195	$0.289 \cdot 10^{+11}$
$1/5 \cdot 2^{-9}$	$0.223 \cdot 10^{-10}$	1.00000	6.094	$0.131 \cdot 10^{+11}$
$1/5 \cdot 2^{-10}$	$0.327 \cdot 10^{-12}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-6}$	$0.800 \cdot 10^{-03}$	1.00000	12.411	$0.988 \cdot 10^{+28}$
$1/5 \cdot 2^{-7}$	$0.147 \cdot 10^{-06}$	1.00000	8.028	$0.496 \cdot 10^{+16}$
$1/5 \cdot 2^{-8}$	$0.562 \cdot 10^{-09}$	1.00000	7.423	$0.655 \cdot 10^{+14}$
$1/5 \cdot 2^{-9}$	$0.328 \cdot 10^{-11}$	1.00000	7.196	$0.110 \cdot 10^{+14}$
$1/5 \cdot 2^{-10}$	$0.223 \cdot 10^{-13}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-7}$	$0.644 \cdot 10^{-05}$	1.00000	13.417	$0.288 \cdot 10^{+33}$
$1/5 \cdot 2^{-8}$	$0.589 \cdot 10^{-09}$	1.00000	9.031	$0.679 \cdot 10^{+19}$
$1/5 \cdot 2^{-9}$	$0.113 \cdot 10^{-11}$	1.00000	8.425	$0.583 \cdot 10^{+17}$
$1/5 \cdot 2^{-10}$	$0.328 \cdot 10^{-14}$	1.00000		

Table 4.17: Implicit Euler, extrapolation steps $4 - 7$ for 3.2a

In spite of the advantageous properties of the implicit Euler method, it seems natural to try and improve the efficiency of the IDeC method by using a higher order scheme like the box scheme or the trapezoidal rule as basic method, where for regular problems order sequences $O(h^2)$, $O(h^4), \dots$ are observed, or alternatively to use the (computationally cheaper) explicit Euler method. It is the aim of the next sections to show why neither of these is possible for singular initial value problems.

4.2   The Box Scheme

4.2.1   Convergence of the Basic Solution

A proof of convergence of the box scheme applied to (4.1) was given in [14]. It turns out that it retains its classical convergence order $O(h^2)$ except for the case where a multiple eigenvalue 0 of $M(0)$ occurs. Here, the convergence order is $O(|\ln(h)|^{n_0-1}h^2)$, where n_0 denotes the dimension of the largest Jordan block associated with the eigenvalue 0.

4.2.2   Iterated Defect Correction

From the fact that for problems 3.1a – 3.3a, order reductions potentially occur even for the basic solution, we cannot expect to have a satisfactory asymptotic error expansion for this class of problems. Therefore we expect IDeC to fail, as we found the error expansion to be the key factor in the convergence proof for the implicit Euler method in §4.1. This holds also for other numerical methods like the box scheme, see for example [8]. However, a special structure in the solution may permit some terms in the expansion (3.9) to exist. Nonetheless, IDeC will break down eventually. But also for the other singular problems where the classical convergence order can be observed, IDeC fails to improve the level of accuracy of the solution satisfactorily. In the sequel, we give numerical examples of the performance of IDeC where a polynomial degree $m = 10$ is used for the interpolation process defining the Zadunaisky polynomials $p^{[k]}(t)$. For regular problems in this case, the highest attainable convergence order is 10.

Consider example 1.1a first. The order of the basic solution is 2 according to theory, but IDeC does not improve this order when we consider the maximal error on $[0, 1]$, cf. Table 4.18. At a fixed point (we chose $t = 1/8$ and $t = 1$), however, a slight recovery can be observed, and an order sequence $O(h^2)$, $O(h^4)$, $O(h^5), \dots$ is obtained, see Tables 4.19 and 4.20. The reason for this phenomenon can clearly be attributed to the singularity, as $t^{\max} \to 0$ as $h \to 0$ in Table 4.18.

Now we take a look at example 1.2a. The picture is quite similar to that for 1.1a, again the convergence order does not exceed 2. At a fixed point t, however, the recovery of the order sequence only takes place up to order 3. For test results, refer to tables 4.21 – 4.23.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$6.3 \cdot 10^{-01}$	1.00000	1.989	$4.0 \cdot 10^{-02}$	0.10000	1.944
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-01}$	1.00000	1.997	$1.0 \cdot 10^{-02}$	0.05000	1.986
$1/5 \cdot 2^{-3}$	$4.0 \cdot 10^{-02}$	1.00000	1.999	$2.6 \cdot 10^{-03}$	0.02500	1.996
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-02}$	1.00000	1.999	$6.6 \cdot 10^{-04}$	0.01250	1.999
$1/5 \cdot 2^{-5}$	$2.5 \cdot 10^{-03}$	1.00000	1.999	$1.6 \cdot 10^{-04}$	0.00625	1.999
$1/5 \cdot 2^{-6}$	$6.2 \cdot 10^{-04}$	1.00000	1.999	$4.1 \cdot 10^{-05}$	0.00312	1.999
$1/5 \cdot 2^{-7}$	$1.5 \cdot 10^{-04}$	1.00000	1.999	$1.0 \cdot 10^{-05}$	0.00156	1.999
$1/5 \cdot 2^{-8}$	$3.9 \cdot 10^{-05}$	1.00000	1.999	$2.5 \cdot 10^{-06}$	0.00078	1.999
$1/5 \cdot 2^{-9}$	$9.7 \cdot 10^{-06}$	1.00000	1.999	$6.4 \cdot 10^{-07}$	0.00039	1.999
$1/5 \cdot 2^{-10}$	$2.4 \cdot 10^{-06}$	1.00000	1.999	$1.6 \cdot 10^{-07}$	0.00019	1.999
$1/5 \cdot 2^{-11}$	$6.1 \cdot 10^{-07}$	1.00000	1.999	$4.0 \cdot 10^{-08}$	0.00009	1.999
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-07}$	1.00000	2.000	$1.0 \cdot 10^{-08}$	0.00004	-29.262
$1/5 \cdot 2^{-13}$	$3.8 \cdot 10^{-08}$	0.99998	1.999	6.5	0.99998	33.258
$1/5 \cdot 2^{-14}$	$9.5 \cdot 10^{-09}$	1.00000		$6.3 \cdot 10^{-10}$	0.00001	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.8 \cdot 10^{-01}$	0.10000	1.959	$7.1 \cdot 10^{-01}$	0.10000	1.959
$1/5 \cdot 2^{-2}$	$4.6 \cdot 10^{-02}$	0.05000	1.989	$1.8 \cdot 10^{-01}$	0.05000	1.990
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-02}$	0.02500	1.997	$4.6 \cdot 10^{-02}$	0.02500	1.997
$1/5 \cdot 2^{-4}$	$2.9 \cdot 10^{-03}$	0.01250	1.999	$1.1 \cdot 10^{-02}$	0.01250	1.999
$1/5 \cdot 2^{-5}$	$7.3 \cdot 10^{-04}$	0.00625	1.999	$2.9 \cdot 10^{-03}$	0.00625	1.999
$1/5 \cdot 2^{-6}$	$1.8 \cdot 10^{-04}$	0.00312	1.999	$7.2 \cdot 10^{-04}$	0.00312	1.999
$1/5 \cdot 2^{-7}$	$4.6 \cdot 10^{-05}$	0.00156	1.999	$1.8 \cdot 10^{-04}$	0.00156	1.999
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-05}$	0.00078	1.999	$4.5 \cdot 10^{-05}$	0.00078	1.999
$1/5 \cdot 2^{-9}$	$2.8 \cdot 10^{-06}$	0.00039	1.999	$1.1 \cdot 10^{-05}$	0.00039	1.999
$1/5 \cdot 2^{-10}$	$7.1 \cdot 10^{-07}$	0.00019	1.999	$2.8 \cdot 10^{-06}$	0.00019	1.999
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-07}$	0.00009	1.999	$7.1 \cdot 10^{-07}$	0.00009	1.999
$1/5 \cdot 2^{-12}$	$4.4 \cdot 10^{-08}$	0.00004	-27.052	$1.7 \cdot 10^{-07}$	0.00004	-25.032
$1/5 \cdot 2^{-13}$	6.2	0.99998		6.1	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	2.6	0.10000	1.960	9.7	0.10000	1.960
$1/5 \cdot 2^{-2}$	$6.9 \cdot 10^{-01}$	0.05000	1.990	2.4	0.05000	1.990
$1/5 \cdot 2^{-3}$	$1.7 \cdot 10^{-01}$	0.02500	1.997	$6.2 \cdot 10^{-01}$	0.02500	1.997
$1/5 \cdot 2^{-4}$	$4.3 \cdot 10^{-02}$	0.01250	1.999	$1.5 \cdot 10^{-01}$	0.01250	1.999
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-02}$	0.00625	1.999	$3.9 \cdot 10^{-02}$	0.00625	1.999
$1/5 \cdot 2^{-6}$	$2.7 \cdot 10^{-03}$	0.00312	1.999	$9.8 \cdot 10^{-03}$	0.00312	1.999
$1/5 \cdot 2^{-7}$	$6.8 \cdot 10^{-04}$	0.00156	1.999	$2.4 \cdot 10^{-03}$	0.00156	1.999
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-04}$	0.00078	1.999	$6.1 \cdot 10^{-04}$	0.00078	1.999
$1/5 \cdot 2^{-9}$	$4.2 \cdot 10^{-05}$	0.00039	1.999	$1.5 \cdot 10^{-04}$	0.00039	1.999
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-05}$	0.00019	1.999	$3.8 \cdot 10^{-05}$	0.00019	1.999
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-06}$	0.00009	1.999	$9.6 \cdot 10^{-06}$	0.00009	1.999
$1/5 \cdot 2^{-12}$	$6.6 \cdot 10^{-07}$	0.00004		$2.4 \cdot 10^{-06}$	0.00004	

Table 4.18: Box scheme, IDeC method on $[0,1]$ for 1.1a.

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$2.1 \cdot 10^{-04}$	1.999	$1.1 \cdot 10^{-04}$	8.424
$1/5 \cdot 2^{-5}$	$5.3 \cdot 10^{-05}$	1.999	$3.2 \cdot 10^{-07}$	3.066
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-05}$	1.999	$3.8 \cdot 10^{-08}$	4.744
$1/5 \cdot 2^{-7}$	$3.3 \cdot 10^{-06}$	1.999	$1.4 \cdot 10^{-09}$	4.917
$1/5 \cdot 2^{-8}$	$8.3 \cdot 10^{-07}$	1.999	$4.7 \cdot 10^{-11}$	4.931
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-07}$	1.999	$1.5 \cdot 10^{-12}$	4.891
$1/5 \cdot 2^{-10}$	$5.1 \cdot 10^{-08}$	1.999	$5.2 \cdot 10^{-14}$	4.808
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-08}$	1.999	$1.8 \cdot 10^{-15}$	4.680
$1/5 \cdot 2^{-12}$	$3.2 \cdot 10^{-09}$	1.999	$7.3 \cdot 10^{-17}$	4.517
$1/5 \cdot 2^{-13}$	$8.1 \cdot 10^{-10}$	1.999	$3.1 \cdot 10^{-18}$	4.350
$1/5 \cdot 2^{-14}$	$2.0 \cdot 10^{-10}$	1.999	$1.5 \cdot 10^{-19}$	4.213
$1/5 \cdot 2^{-15}$	$5.0 \cdot 10^{-11}$	2.000	$8.4 \cdot 10^{-21}$	-60.031
$1/5 \cdot 2^{-16}$	$1.2 \cdot 10^{-11}$	1.999	$9.9 \cdot 10^{-03}$	68.208
$1/5 \cdot 2^{-17}$	$3.1 \cdot 10^{-12}$		$2.9 \cdot 10^{-23}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$4.8 \cdot 10^{-04}$	8.396	$1.8 \cdot 10^{-03}$	7.576
$1/5 \cdot 2^{-5}$	$1.4 \cdot 10^{-06}$	4.688	$9.7 \cdot 10^{-06}$	7.572
$1/5 \cdot 2^{-6}$	$5.5 \cdot 10^{-08}$	4.469	$5.1 \cdot 10^{-08}$	3.560
$1/5 \cdot 2^{-7}$	$2.5 \cdot 10^{-09}$	4.893	$4.3 \cdot 10^{-09}$	4.789
$1/5 \cdot 2^{-8}$	$8.4 \cdot 10^{-11}$	4.974	$1.5 \cdot 10^{-10}$	4.951
$1/5 \cdot 2^{-9}$	$2.6 \cdot 10^{-12}$	4.993	$5.0 \cdot 10^{-12}$	4.988
$1/5 \cdot 2^{-10}$	$8.4 \cdot 10^{-14}$	4.998	$1.5 \cdot 10^{-13}$	4.997
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-15}$	4.999	$4.9 \cdot 10^{-15}$	4.999
$1/5 \cdot 2^{-12}$	$8.2 \cdot 10^{-17}$	4.999	$1.5 \cdot 10^{-16}$	4.999
$1/5 \cdot 2^{-13}$	$2.5 \cdot 10^{-18}$	4.999	$4.8 \cdot 10^{-18}$	4.999
$1/5 \cdot 2^{-14}$	$8.0 \cdot 10^{-20}$	4.999	$1.5 \cdot 10^{-19}$	4.999
$1/5 \cdot 2^{-15}$	$2.5 \cdot 10^{-21}$	-61.721	$4.7 \cdot 10^{-21}$	-60.766
$1/5 \cdot 2^{-16}$	$9.5 \cdot 10^{-03}$		$9.3 \cdot 10^{-03}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$6.8 \cdot 10^{-03}$	7.307	$2.4 \cdot 10^{-02}$	7.189
$1/5 \cdot 2^{-5}$	$4.3 \cdot 10^{-05}$	8.824	$1.6 \cdot 10^{-04}$	7.625
$1/5 \cdot 2^{-6}$	$9.4 \cdot 10^{-08}$	3.834	$8.4 \cdot 10^{-07}$	6.871
$1/5 \cdot 2^{-7}$	$6.6 \cdot 10^{-09}$	4.556	$7.2 \cdot 10^{-09}$	3.888
$1/5 \cdot 2^{-8}$	$2.8 \cdot 10^{-10}$	4.907	$4.8 \cdot 10^{-10}$	4.818
$1/5 \cdot 2^{-9}$	$9.4 \cdot 10^{-12}$	4.977	$1.7 \cdot 10^{-11}$	4.957
$1/5 \cdot 2^{-10}$	$2.9 \cdot 10^{-13}$	4.994	$5.5 \cdot 10^{-13}$	4.989
$1/5 \cdot 2^{-11}$	$9.3 \cdot 10^{-15}$	4.998	$1.7 \cdot 10^{-14}$	4.997
$1/5 \cdot 2^{-12}$	$2.9 \cdot 10^{-16}$	4.999	$5.5 \cdot 10^{-16}$	4.999
$1/5 \cdot 2^{-13}$	$9.1 \cdot 10^{-18}$	4.999	$1.7 \cdot 10^{-17}$	4.999
$1/5 \cdot 2^{-14}$	$2.8 \cdot 10^{-19}$	4.999	$5.3 \cdot 10^{-19}$	4.999
$1/5 \cdot 2^{-15}$	$8.9 \cdot 10^{-21}$		$1.6 \cdot 10^{-20}$	

Table 4.19: Box scheme, IDeC method at $t = 1/8$ for 1.1a.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$6.3 \cdot 10^{-01}$	1.989	$8.9 \cdot 10^{-04}$	0.804
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-01}$	1.997	$5.1 \cdot 10^{-04}$	3.944
$1/5 \cdot 2^{-3}$	$4.0 \cdot 10^{-02}$	1.999	$3.3 \cdot 10^{-05}$	4.042
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-02}$	1.999	$2.0 \cdot 10^{-06}$	4.031
$1/5 \cdot 2^{-5}$	$2.5 \cdot 10^{-03}$	1.999	$1.2 \cdot 10^{-07}$	4.016
$1/5 \cdot 2^{-6}$	$6.2 \cdot 10^{-04}$	1.999	$7.6 \cdot 10^{-09}$	4.008
$1/5 \cdot 2^{-7}$	$1.5 \cdot 10^{-04}$	1.999	$4.7 \cdot 10^{-10}$	4.004
$1/5 \cdot 2^{-8}$	$3.9 \cdot 10^{-05}$	1.999	$2.9 \cdot 10^{-11}$	4.002
$1/5 \cdot 2^{-9}$	$9.7 \cdot 10^{-06}$	1.999	$1.8 \cdot 10^{-12}$	4.001
$1/5 \cdot 2^{-10}$	$2.4 \cdot 10^{-06}$	1.999	$1.1 \cdot 10^{-13}$	4.000
$1/5 \cdot 2^{-11}$	$6.1 \cdot 10^{-07}$	1.999	$7.2 \cdot 10^{-15}$	4.000
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-07}$	2.000	$4.4 \cdot 10^{-16}$	-53.685
$1/5 \cdot 2^{-13}$	$3.8 \cdot 10^{-08}$	1.999	6.5	61.679
$1/5 \cdot 2^{-14}$	$9.5 \cdot 10^{-09}$		$1.7 \cdot 10^{-18}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$2.9 \cdot 10^{-02}$	8.343	$1.1 \cdot 10^{-01}$	7.537
$1/5 \cdot 2^{-2}$	$9.1 \cdot 10^{-05}$	4.701	$6.1 \cdot 10^{-04}$	7.563
$1/5 \cdot 2^{-3}$	$3.5 \cdot 10^{-06}$	4.462	$3.2 \cdot 10^{-06}$	3.558
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-07}$	4.891	$2.7 \cdot 10^{-07}$	4.788
$1/5 \cdot 2^{-5}$	$5.4 \cdot 10^{-09}$	4.973	$1.0 \cdot 10^{-08}$	4.951
$1/5 \cdot 2^{-6}$	$1.7 \cdot 10^{-10}$	4.993	$3.2 \cdot 10^{-10}$	4.988
$1/5 \cdot 2^{-7}$	$5.3 \cdot 10^{-12}$	4.998	$1.0 \cdot 10^{-11}$	4.997
$1/5 \cdot 2^{-8}$	$1.6 \cdot 10^{-13}$	4.999	$3.1 \cdot 10^{-13}$	4.999
$1/5 \cdot 2^{-9}$	$5.2 \cdot 10^{-15}$	4.999	$9.9 \cdot 10^{-15}$	4.999
$1/5 \cdot 2^{-10}$	$1.6 \cdot 10^{-16}$	4.999	$3.1 \cdot 10^{-16}$	4.999
$1/5 \cdot 2^{-11}$	$5.1 \cdot 10^{-18}$	4.999	$9.7 \cdot 10^{-18}$	4.999
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-19}$	-65.077	$3.0 \cdot 10^{-19}$	-64.121
$1/5 \cdot 2^{-13}$	6.2		6.1	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$4.2 \cdot 10^{-01}$	7.268	1.5	7.150
$1/5 \cdot 2^{-2}$	$2.7 \cdot 10^{-03}$	8.814	$1.0 \cdot 10^{-02}$	7.615
$1/5 \cdot 2^{-3}$	$6.0 \cdot 10^{-06}$	3.832	$5.4 \cdot 10^{-05}$	6.869
$1/5 \cdot 2^{-4}$	$4.2 \cdot 10^{-07}$	4.555	$4.6 \cdot 10^{-07}$	3.887
$1/5 \cdot 2^{-5}$	$1.8 \cdot 10^{-08}$	4.907	$3.1 \cdot 10^{-08}$	4.818
$1/5 \cdot 2^{-6}$	$6.0 \cdot 10^{-10}$	4.977	$1.1 \cdot 10^{-09}$	4.957
$1/5 \cdot 2^{-7}$	$1.9 \cdot 10^{-11}$	4.994	$3.5 \cdot 10^{-11}$	4.989
$1/5 \cdot 2^{-8}$	$5.9 \cdot 10^{-13}$	4.998	$1.1 \cdot 10^{-12}$	4.997
$1/5 \cdot 2^{-9}$	$1.8 \cdot 10^{-14}$	4.999	$3.5 \cdot 10^{-14}$	4.999
$1/5 \cdot 2^{-10}$	$5.8 \cdot 10^{-16}$	4.999	$1.1 \cdot 10^{-15}$	4.999
$1/5 \cdot 2^{-11}$	$1.8 \cdot 10^{-17}$	4.999	$3.4 \cdot 10^{-17}$	4.999
$1/5 \cdot 2^{-12}$	$5.7 \cdot 10^{-19}$		$1.0 \cdot 10^{-18}$	

Table 4.20: Box scheme, IDeC method at $t = 1$ for 1.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.2 \cdot 10^{-02}$	0.10000	1.860	$2.0 \cdot 10^{-02}$	0.10000	1.964
$1/5 \cdot 2^{-2}$	$6.1 \cdot 10^{-03}$	0.05000	1.932	$5.2 \cdot 10^{-03}$	0.05000	1.984
$1/5 \cdot 2^{-3}$	$1.6 \cdot 10^{-03}$	0.02500	1.966	$1.3 \cdot 10^{-03}$	0.02500	1.992
$1/5 \cdot 2^{-4}$	$4.1 \cdot 10^{-04}$	0.01250	1.983	$3.3 \cdot 10^{-04}$	0.01250	1.996
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-04}$	0.00625	1.991	$8.3 \cdot 10^{-05}$	0.00625	1.998
$1/5 \cdot 2^{-6}$	$2.6 \cdot 10^{-05}$	0.00312	1.995	$2.0 \cdot 10^{-05}$	0.00312	1.999
$1/5 \cdot 2^{-7}$	$6.6 \cdot 10^{-06}$	0.00156	1.997	$5.2 \cdot 10^{-06}$	0.00156	1.999
$1/5 \cdot 2^{-8}$	$1.6 \cdot 10^{-06}$	0.00078	1.998	$1.3 \cdot 10^{-06}$	0.00078	1.999
$1/5 \cdot 2^{-9}$	$4.1 \cdot 10^{-07}$	0.00039	1.999	$3.2 \cdot 10^{-07}$	0.00039	1.999
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-07}$	0.00019	1.999	$8.1 \cdot 10^{-08}$	0.00019	1.999
$1/5 \cdot 2^{-11}$	$2.5 \cdot 10^{-08}$	0.00009	1.999	$2.0 \cdot 10^{-08}$	0.00009	1.999
$1/5 \cdot 2^{-12}$	$6.4 \cdot 10^{-09}$	0.00004	1.999	$5.0 \cdot 10^{-09}$	0.00004	-25.370
$1/5 \cdot 2^{-13}$	$1.6 \cdot 10^{-09}$	0.00002	1.999	$2.2 \cdot 10^{-01}$	0.99998	29.367
$1/5 \cdot 2^{-14}$	$4.0 \cdot 10^{-10}$	0.00001		$3.1 \cdot 10^{-10}$	0.00001	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.8 \cdot 10^{-02}$	0.10000	1.764	$1.2 \cdot 10^{-02}$	0.10000	1.294
$1/5 \cdot 2^{-2}$	$5.4 \cdot 10^{-03}$	0.05000	1.892	$5.0 \cdot 10^{-03}$	0.05000	1.725
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-03}$	0.02500	1.948	$1.5 \cdot 10^{-03}$	0.02500	1.875
$1/5 \cdot 2^{-4}$	$3.7 \cdot 10^{-04}$	0.01250	1.974	$4.1 \cdot 10^{-04}$	0.01250	1.940
$1/5 \cdot 2^{-5}$	$9.6 \cdot 10^{-05}$	0.00625	1.987	$1.0 \cdot 10^{-04}$	0.00625	1.970
$1/5 \cdot 2^{-6}$	$2.4 \cdot 10^{-05}$	0.00312	1.993	$2.7 \cdot 10^{-05}$	0.00312	1.985
$1/5 \cdot 2^{-7}$	$6.1 \cdot 10^{-06}$	0.00156	1.996	$7.0 \cdot 10^{-06}$	0.00156	1.992
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-06}$	0.00078	1.998	$1.7 \cdot 10^{-06}$	0.00078	1.996
$1/5 \cdot 2^{-9}$	$3.8 \cdot 10^{-07}$	0.00039	1.999	$4.4 \cdot 10^{-07}$	0.00039	1.998
$1/5 \cdot 2^{-10}$	$9.5 \cdot 10^{-08}$	0.00019	1.999	$1.1 \cdot 10^{-07}$	0.00019	1.999
$1/5 \cdot 2^{-11}$	$2.3 \cdot 10^{-08}$	0.00009	1.999	$2.7 \cdot 10^{-08}$	0.00009	1.999
$1/5 \cdot 2^{-12}$	$5.9 \cdot 10^{-09}$	0.00004	-25.080	$6.9 \cdot 10^{-09}$	0.00004	-24.833
$1/5 \cdot 2^{-13}$	$2.1 \cdot 10^{-01}$	0.99998		$2.0 \cdot 10^{-01}$	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-03}$	0.10000	0.209	$1.8 \cdot 10^{-03}$	0.10000	-0.824
$1/5 \cdot 2^{-2}$	$4.2 \cdot 10^{-03}$	0.05000	1.478	$3.3 \cdot 10^{-03}$	0.05000	1.146
$1/5 \cdot 2^{-3}$	$1.5 \cdot 10^{-03}$	0.02500	1.779	$1.5 \cdot 10^{-03}$	0.02500	1.666
$1/5 \cdot 2^{-4}$	$4.4 \cdot 10^{-04}$	0.01250	1.897	$4.7 \cdot 10^{-04}$	0.01250	1.848
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-04}$	0.00625	1.950	$1.3 \cdot 10^{-04}$	0.00625	1.927
$1/5 \cdot 2^{-6}$	$3.1 \cdot 10^{-05}$	0.00312	1.975	$3.4 \cdot 10^{-05}$	0.00312	1.964
$1/5 \cdot 2^{-7}$	$7.9 \cdot 10^{-06}$	0.00156	1.987	$8.8 \cdot 10^{-06}$	0.00156	1.982
$1/5 \cdot 2^{-8}$	$1.9 \cdot 10^{-06}$	0.00078	1.993	$2.2 \cdot 10^{-06}$	0.00078	1.991
$1/5 \cdot 2^{-9}$	$5.0 \cdot 10^{-07}$	0.00039	1.996	$5.6 \cdot 10^{-07}$	0.00039	1.995
$1/5 \cdot 2^{-10}$	$1.2 \cdot 10^{-07}$	0.00019	1.998	$1.4 \cdot 10^{-07}$	0.00019	1.997
$1/5 \cdot 2^{-11}$	$3.1 \cdot 10^{-08}$	0.00009	1.999	$3.5 \cdot 10^{-08}$	0.00009	1.998
$1/5 \cdot 2^{-12}$	$7.8 \cdot 10^{-09}$	0.00004		$8.8 \cdot 10^{-09}$	0.00004	

Table 4.21: Box scheme, IDeC method on $[0, 1]$ for 1.2a.

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-04}$	2.003	$2.0 \cdot 10^{-05}$	5.077
$1/5 \cdot 2^{-5}$	$2.7 \cdot 10^{-05}$	1.999	$6.0 \cdot 10^{-07}$	2.122
$1/5 \cdot 2^{-6}$	$6.7 \cdot 10^{-06}$	1.999	$1.3 \cdot 10^{-07}$	2.531
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-06}$	1.999	$2.3 \cdot 10^{-08}$	2.798
$1/5 \cdot 2^{-8}$	$4.2 \cdot 10^{-07}$	1.999	$3.4 \cdot 10^{-09}$	2.945
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-07}$	1.999	$4.4 \cdot 10^{-10}$	2.992
$1/5 \cdot 2^{-10}$	$2.6 \cdot 10^{-08}$	1.999	$5.5 \cdot 10^{-11}$	3.005
$1/5 \cdot 2^{-11}$	$6.6 \cdot 10^{-09}$	1.999	$6.9 \cdot 10^{-12}$	3.007
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-09}$	1.999	$8.6 \cdot 10^{-13}$	3.006
$1/5 \cdot 2^{-13}$	$4.1 \cdot 10^{-10}$	1.999	$1.0 \cdot 10^{-13}$	3.004
$1/5 \cdot 2^{-14}$	$1.0 \cdot 10^{-10}$	1.999	$1.3 \cdot 10^{-14}$	3.002
$1/5 \cdot 2^{-15}$	$2.5 \cdot 10^{-11}$	1.999	$1.6 \cdot 10^{-15}$	-43.072
$1/5 \cdot 2^{-16}$	$6.4 \cdot 10^{-12}$	2.000	$1.5 \cdot 10^{-02}$	49.070
$1/5 \cdot 2^{-17}$	$1.6 \cdot 10^{-12}$		$2.6 \cdot 10^{-17}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$3.2 \cdot 10^{-05}$	3.996	$3.8 \cdot 10^{-05}$	3.817
$1/5 \cdot 2^{-5}$	$2.0 \cdot 10^{-06}$	4.882	$2.7 \cdot 10^{-06}$	4.337
$1/5 \cdot 2^{-6}$	$6.8 \cdot 10^{-08}$	3.433	$1.3 \cdot 10^{-07}$	5.120
$1/5 \cdot 2^{-7}$	$6.3 \cdot 10^{-09}$	2.806	$3.8 \cdot 10^{-09}$	3.065
$1/5 \cdot 2^{-8}$	$9.0 \cdot 10^{-10}$	2.720	$4.6 \cdot 10^{-10}$	3.068
$1/5 \cdot 2^{-9}$	$1.3 \cdot 10^{-10}$	2.939	$5.5 \cdot 10^{-11}$	2.871
$1/5 \cdot 2^{-10}$	$1.7 \cdot 10^{-11}$	3.002	$7.5 \cdot 10^{-12}$	2.992
$1/5 \cdot 2^{-11}$	$2.2 \cdot 10^{-12}$	3.016	$9.5 \cdot 10^{-13}$	3.038
$1/5 \cdot 2^{-12}$	$2.7 \cdot 10^{-13}$	3.016	$1.1 \cdot 10^{-13}$	3.041
$1/5 \cdot 2^{-13}$	$3.4 \cdot 10^{-14}$	3.012	$1.4 \cdot 10^{-14}$	3.032
$1/5 \cdot 2^{-14}$	$4.2 \cdot 10^{-15}$	3.008	$1.7 \cdot 10^{-15}$	3.022
$1/5 \cdot 2^{-15}$	$5.2 \cdot 10^{-16}$	-44.684	$2.1 \cdot 10^{-16}$	-45.965
$1/5 \cdot 2^{-16}$	$1.4 \cdot 10^{-02}$		$1.4 \cdot 10^{-02}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$4.2 \cdot 10^{-05}$	3.730	$4.5 \cdot 10^{-05}$	3.666
$1/5 \cdot 2^{-5}$	$3.1 \cdot 10^{-06}$	4.202	$3.5 \cdot 10^{-06}$	4.142
$1/5 \cdot 2^{-6}$	$1.7 \cdot 10^{-07}$	4.616	$2.0 \cdot 10^{-07}$	4.485
$1/5 \cdot 2^{-7}$	$7.0 \cdot 10^{-09}$	4.482	$9.0 \cdot 10^{-09}$	5.053
$1/5 \cdot 2^{-8}$	$3.1 \cdot 10^{-10}$	3.086	$2.7 \cdot 10^{-10}$	3.096
$1/5 \cdot 2^{-9}$	$3.7 \cdot 10^{-11}$	3.283	$3.1 \cdot 10^{-11}$	3.376
$1/5 \cdot 2^{-10}$	$3.8 \cdot 10^{-12}$	3.071	$3.0 \cdot 10^{-12}$	3.456
$1/5 \cdot 2^{-11}$	$4.5 \cdot 10^{-13}$	3.086	$2.7 \cdot 10^{-13}$	3.241
$1/5 \cdot 2^{-12}$	$5.3 \cdot 10^{-14}$	3.100	$2.9 \cdot 10^{-14}$	3.209
$1/5 \cdot 2^{-13}$	$6.2 \cdot 10^{-15}$	3.082	$3.1 \cdot 10^{-15}$	3.185
$1/5 \cdot 2^{-14}$	$7.3 \cdot 10^{-16}$	3.058	$3.4 \cdot 10^{-16}$	3.140
$1/5 \cdot 2^{-15}$	$8.8 \cdot 10^{-17}$		$3.9 \cdot 10^{-17}$	

Table 4.22: Box scheme, IDeC method at $t = 1/8$ for 1.2a.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$8.7 \cdot 10^{-03}$	2.073	$7.3 \cdot 10^{-04}$	3.341
$1/5 \cdot 2^{-2}$	$2.0 \cdot 10^{-03}$	2.014	$7.2 \cdot 10^{-05}$	3.105
$1/5 \cdot 2^{-3}$	$5.1 \cdot 10^{-04}$	2.002	$8.3 \cdot 10^{-06}$	3.057
$1/5 \cdot 2^{-4}$	$1.2 \cdot 10^{-04}$	2.000	$1.0 \cdot 10^{-06}$	4.408
$1/5 \cdot 2^{-5}$	$3.2 \cdot 10^{-05}$	2.000	$4.7 \cdot 10^{-08}$	3.379
$1/5 \cdot 2^{-6}$	$8.0 \cdot 10^{-06}$	2.000	$4.5 \cdot 10^{-09}$	2.902
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-06}$	2.000	$6.0 \cdot 10^{-10}$	2.583
$1/5 \cdot 2^{-8}$	$5.0 \cdot 10^{-07}$	1.999	$1.0 \cdot 10^{-10}$	2.862
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-07}$	1.999	$1.3 \cdot 10^{-11}$	2.954
$1/5 \cdot 2^{-10}$	$3.1 \cdot 10^{-08}$	1.999	$1.8 \cdot 10^{-12}$	2.987
$1/5 \cdot 2^{-11}$	$7.8 \cdot 10^{-09}$	1.999	$2.2 \cdot 10^{-13}$	2.998
$1/5 \cdot 2^{-12}$	$1.9 \cdot 10^{-09}$	1.999	$2.8 \cdot 10^{-14}$	-42.821
$1/5 \cdot 2^{-13}$	$4.8 \cdot 10^{-10}$	2.000	$2.2 \cdot 10^{-01}$	48.820
$1/5 \cdot 2^{-14}$	$1.2 \cdot 10^{-10}$		$4.4 \cdot 10^{-16}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$8.7 \cdot 10^{-04}$	2.476	$6.0 \cdot 10^{-04}$	1.845
$1/5 \cdot 2^{-2}$	$1.5 \cdot 10^{-04}$	4.633	$1.6 \cdot 10^{-04}$	4.770
$1/5 \cdot 2^{-3}$	$6.3 \cdot 10^{-06}$	2.276	$6.1 \cdot 10^{-06}$	2.026
$1/5 \cdot 2^{-4}$	$1.3 \cdot 10^{-06}$	3.742	$1.5 \cdot 10^{-06}$	3.610
$1/5 \cdot 2^{-5}$	$9.7 \cdot 10^{-08}$	4.342	$1.2 \cdot 10^{-07}$	4.083
$1/5 \cdot 2^{-6}$	$4.8 \cdot 10^{-09}$	4.382	$7.2 \cdot 10^{-09}$	4.461
$1/5 \cdot 2^{-7}$	$2.3 \cdot 10^{-10}$	3.074	$3.3 \cdot 10^{-10}$	4.284
$1/5 \cdot 2^{-8}$	$2.7 \cdot 10^{-11}$	2.856	$1.6 \cdot 10^{-11}$	3.261
$1/5 \cdot 2^{-9}$	$3.7 \cdot 10^{-12}$	2.795	$1.7 \cdot 10^{-12}$	3.182
$1/5 \cdot 2^{-10}$	$5.4 \cdot 10^{-13}$	2.938	$1.9 \cdot 10^{-13}$	2.801
$1/5 \cdot 2^{-11}$	$7.0 \cdot 10^{-14}$	2.986	$2.7 \cdot 10^{-14}$	2.954
$1/5 \cdot 2^{-12}$	$8.9 \cdot 10^{-15}$	-44.430	$3.6 \cdot 10^{-15}$	-45.705
$1/5 \cdot 2^{-13}$	$2.1 \cdot 10^{-01}$		$2.0 \cdot 10^{-01}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$2.7 \cdot 10^{-04}$	0.822	$2.0 \cdot 10^{-04}$	0.698
$1/5 \cdot 2^{-2}$	$1.5 \cdot 10^{-04}$	4.649	$1.2 \cdot 10^{-04}$	4.373
$1/5 \cdot 2^{-3}$	$6.0 \cdot 10^{-06}$	1.896	$6.0 \cdot 10^{-06}$	1.805
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-06}$	3.535	$1.7 \cdot 10^{-06}$	3.474
$1/5 \cdot 2^{-5}$	$1.4 \cdot 10^{-07}$	3.997	$1.5 \cdot 10^{-07}$	3.953
$1/5 \cdot 2^{-6}$	$8.8 \cdot 10^{-09}$	4.267	$1.0 \cdot 10^{-08}$	4.201
$1/5 \cdot 2^{-7}$	$4.5 \cdot 10^{-10}$	4.594	$5.4 \cdot 10^{-10}$	4.418
$1/5 \cdot 2^{-8}$	$1.9 \cdot 10^{-11}$	3.842	$2.5 \cdot 10^{-11}$	4.396
$1/5 \cdot 2^{-9}$	$1.3 \cdot 10^{-12}$	3.430	$1.2 \cdot 10^{-12}$	3.536
$1/5 \cdot 2^{-10}$	$1.2 \cdot 10^{-13}$	3.410	$1.0 \cdot 10^{-13}$	3.561
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-14}$	2.921	$8.8 \cdot 10^{-15}$	3.528
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-15}$		$7.6 \cdot 10^{-16}$	

Table 4.23: Box scheme, IDeC method at $t = 1$ for 1.2a.

Although the spectral properties of $M(0)$ are different from the previous examples, IDeC shows a similar behaviour for 2.1a, see Table 4.24. Again, the order sequence at a fixed point is $O(h^2)$, $O(h^3)$,... (these tables are omitted to avoid repetitions).

For a variable coefficient matrix as in example 2.2a or the nonlinear problem 2.3a, the same convergence properties of IDeC are observed, cf. Table 4.25 and Table 4.26, respectively.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.2 \cdot 10^{-02}$	0.10000	1.995	$4.8 \cdot 10^{-03}$	0.10000	2.073
$1/5 \cdot 2^{-2}$	$5.6 \cdot 10^{-03}$	0.05000	1.998	$1.1 \cdot 10^{-03}$	0.05000	2.019
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-03}$	0.02500	1.999	$2.8 \cdot 10^{-04}$	0.02500	2.004
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-04}$	0.01250	1.999	$7.0 \cdot 10^{-05}$	0.01250	2.001
$1/5 \cdot 2^{-5}$	$8.7 \cdot 10^{-05}$	0.00625	1.999	$1.7 \cdot 10^{-05}$	0.00625	2.000
$1/5 \cdot 2^{-6}$	$2.1 \cdot 10^{-05}$	0.00312	1.999	$4.3 \cdot 10^{-06}$	0.00312	2.000
$1/5 \cdot 2^{-7}$	$5.4 \cdot 10^{-06}$	0.00156	1.999	$1.0 \cdot 10^{-06}$	0.00156	2.000
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-06}$	0.00078	1.999	$2.7 \cdot 10^{-07}$	0.00078	2.000
$1/5 \cdot 2^{-9}$	$3.4 \cdot 10^{-07}$	0.00039	1.999	$6.8 \cdot 10^{-08}$	0.00039	2.000
$1/5 \cdot 2^{-10}$	$8.5 \cdot 10^{-08}$	0.00019	1.999	$1.7 \cdot 10^{-08}$	0.00019	2.000
$1/5 \cdot 2^{-11}$	$2.1 \cdot 10^{-08}$	0.00009	1.999	$4.2 \cdot 10^{-09}$	0.00009	2.000
$1/5 \cdot 2^{-12}$	$5.3 \cdot 10^{-09}$	0.00004	1.999	$1.0 \cdot 10^{-09}$	0.00004	-26.377
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-09}$	0.00002	1.999	$9.3 \cdot 10^{-02}$	0.99998	30.374
$1/5 \cdot 2^{-14}$	$3.3 \cdot 10^{-10}$	0.00001		$6.6 \cdot 10^{-11}$	0.00001	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.6 \cdot 10^{-03}$	0.10000	2.048	$6.5 \cdot 10^{-04}$	0.10000	2.044
$1/5 \cdot 2^{-2}$	$4.0 \cdot 10^{-04}$	0.05000	2.011	$1.5 \cdot 10^{-04}$	0.05000	2.011
$1/5 \cdot 2^{-3}$	$1.0 \cdot 10^{-04}$	0.02500	2.002	$3.9 \cdot 10^{-05}$	0.02500	2.002
$1/5 \cdot 2^{-4}$	$2.5 \cdot 10^{-05}$	0.01250	2.000	$9.8 \cdot 10^{-06}$	0.01250	2.000
$1/5 \cdot 2^{-5}$	$6.2 \cdot 10^{-06}$	0.00625	2.000	$2.4 \cdot 10^{-06}$	0.00625	2.000
$1/5 \cdot 2^{-6}$	$1.5 \cdot 10^{-06}$	0.00312	2.000	$6.1 \cdot 10^{-07}$	0.00312	2.000
$1/5 \cdot 2^{-7}$	$3.9 \cdot 10^{-07}$	0.00156	2.000	$1.5 \cdot 10^{-07}$	0.00156	2.000
$1/5 \cdot 2^{-8}$	$9.8 \cdot 10^{-08}$	0.00078	2.000	$3.8 \cdot 10^{-08}$	0.00078	2.000
$1/5 \cdot 2^{-9}$	$2.4 \cdot 10^{-08}$	0.00039	2.000	$9.6 \cdot 10^{-09}$	0.00039	2.000
$1/5 \cdot 2^{-10}$	$6.1 \cdot 10^{-09}$	0.00019	2.000	$2.4 \cdot 10^{-09}$	0.00019	2.000
$1/5 \cdot 2^{-11}$	$1.5 \cdot 10^{-09}$	0.00009	2.000	$6.0 \cdot 10^{-10}$	0.00009	2.000
$1/5 \cdot 2^{-12}$	$3.8 \cdot 10^{-10}$	0.00004	-27.801	$1.5 \cdot 10^{-10}$	0.00004	-29.119
$1/5 \cdot 2^{-13}$	$8.9 \cdot 10^{-02}$	0.99998		$8.7 \cdot 10^{-02}$	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.6 \cdot 10^{-04}$	0.10000	2.046	$1.0 \cdot 10^{-04}$	0.10000	2.043
$1/5 \cdot 2^{-2}$	$6.3 \cdot 10^{-05}$	0.05000	2.010	$2.5 \cdot 10^{-05}$	0.05000	2.010
$1/5 \cdot 2^{-3}$	$1.5 \cdot 10^{-05}$	0.02500	2.002	$6.2 \cdot 10^{-06}$	0.02500	2.002
$1/5 \cdot 2^{-4}$	$3.9 \cdot 10^{-06}$	0.01250	2.000	$1.5 \cdot 10^{-06}$	0.01250	2.000
$1/5 \cdot 2^{-5}$	$9.8 \cdot 10^{-07}$	0.00625	2.000	$3.9 \cdot 10^{-07}$	0.00625	2.000
$1/5 \cdot 2^{-6}$	$2.4 \cdot 10^{-07}$	0.00312	2.000	$9.7 \cdot 10^{-08}$	0.00312	2.000
$1/5 \cdot 2^{-7}$	$6.1 \cdot 10^{-08}$	0.00156	2.000	$2.4 \cdot 10^{-08}$	0.00156	2.000
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-08}$	0.00078	2.000	$6.1 \cdot 10^{-09}$	0.00078	2.000
$1/5 \cdot 2^{-9}$	$3.8 \cdot 10^{-09}$	0.00039	2.000	$1.5 \cdot 10^{-09}$	0.00039	2.000
$1/5 \cdot 2^{-10}$	$9.5 \cdot 10^{-10}$	0.00019	2.000	$3.8 \cdot 10^{-10}$	0.00019	2.000
$1/5 \cdot 2^{-11}$	$2.3 \cdot 10^{-10}$	0.00009	2.000	$9.5 \cdot 10^{-11}$	0.00009	2.000
$1/5 \cdot 2^{-12}$	$5.9 \cdot 10^{-11}$	0.00004		$2.3 \cdot 10^{-11}$	0.00004	

Table 4.24: Box scheme, IDeC method on $[0, 1]$ for 2.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.6 \cdot 10^{-02}$	1.00000	1.998	$1.8 \cdot 10^{-03}$	0.10000	2.032
$1/5 \cdot 2^{-2}$	$6.5 \cdot 10^{-03}$	1.00000	1.999	$4.6 \cdot 10^{-04}$	0.05000	2.008
$1/5 \cdot 2^{-3}$	$1.6 \cdot 10^{-03}$	1.00000	1.999	$1.1 \cdot 10^{-04}$	0.02500	2.002
$1/5 \cdot 2^{-4}$	$4.0 \cdot 10^{-04}$	1.00000	1.999	$2.8 \cdot 10^{-05}$	0.01250	2.000
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-04}$	1.00000	1.999	$7.1 \cdot 10^{-06}$	0.00625	2.000
$1/5 \cdot 2^{-6}$	$2.5 \cdot 10^{-05}$	1.00000	1.999	$1.7 \cdot 10^{-06}$	0.00312	2.000
$1/5 \cdot 2^{-7}$	$6.4 \cdot 10^{-06}$	1.00000	1.999	$4.4 \cdot 10^{-07}$	0.00156	2.000
$1/5 \cdot 2^{-8}$	$1.6 \cdot 10^{-06}$	1.00000	1.999	$1.1 \cdot 10^{-07}$	0.00078	2.000
$1/5 \cdot 2^{-9}$	$4.0 \cdot 10^{-07}$	1.00000	1.999	$2.8 \cdot 10^{-08}$	0.00039	2.000
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-07}$	1.00000	1.999	$7.0 \cdot 10^{-09}$	0.00019	2.000
$1/5 \cdot 2^{-11}$	$2.5 \cdot 10^{-08}$	1.00000	1.999	$1.7 \cdot 10^{-09}$	0.00009	2.000
$1/5 \cdot 2^{-12}$	$6.2 \cdot 10^{-09}$	1.00000	2.000	$4.3 \cdot 10^{-10}$	0.00004	-31.369
$1/5 \cdot 2^{-13}$	$1.5 \cdot 10^{-09}$	0.99998	1.999	1.2	0.99998	35.366
$1/5 \cdot 2^{-14}$	$3.9 \cdot 10^{-10}$	1.00000		$2.7 \cdot 10^{-11}$	0.00001	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$6.9 \cdot 10^{-04}$	0.10000	2.064	$2.8 \cdot 10^{-04}$	0.10000	2.115
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-04}$	0.05000	2.016	$6.6 \cdot 10^{-05}$	0.05000	2.029
$1/5 \cdot 2^{-3}$	$4.1 \cdot 10^{-05}$	0.02500	2.004	$1.6 \cdot 10^{-05}$	0.02500	2.007
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-05}$	0.01250	2.001	$4.0 \cdot 10^{-06}$	0.01250	2.001
$1/5 \cdot 2^{-5}$	$2.5 \cdot 10^{-06}$	0.00625	2.000	$1.0 \cdot 10^{-06}$	0.00625	2.000
$1/5 \cdot 2^{-6}$	$6.4 \cdot 10^{-07}$	0.00312	2.000	$2.5 \cdot 10^{-07}$	0.00312	2.000
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-07}$	0.00156	2.000	$6.2 \cdot 10^{-08}$	0.00156	2.000
$1/5 \cdot 2^{-8}$	$4.0 \cdot 10^{-08}$	0.00078	2.000	$1.5 \cdot 10^{-08}$	0.00078	2.000
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-08}$	0.00039	2.000	$3.9 \cdot 10^{-09}$	0.00039	2.000
$1/5 \cdot 2^{-10}$	$2.5 \cdot 10^{-09}$	0.00019	2.000	$9.8 \cdot 10^{-10}$	0.00019	2.000
$1/5 \cdot 2^{-11}$	$6.2 \cdot 10^{-10}$	0.00009	2.000	$2.4 \cdot 10^{-10}$	0.00009	2.000
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-10}$	0.00004	-32.793	$6.1 \cdot 10^{-11}$	0.00004	-34.111
$1/5 \cdot 2^{-13}$	1.1	0.99998		1.1	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.2 \cdot 10^{-04}$	0.10000	2.170	$5.0 \cdot 10^{-05}$	0.10000	2.225
$1/5 \cdot 2^{-2}$	$2.6 \cdot 10^{-05}$	0.05000	2.044	$1.0 \cdot 10^{-05}$	0.05000	2.058
$1/5 \cdot 2^{-3}$	$6.4 \cdot 10^{-06}$	0.02500	2.011	$2.5 \cdot 10^{-06}$	0.02500	2.014
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-06}$	0.01250	2.002	$6.4 \cdot 10^{-07}$	0.01250	2.003
$1/5 \cdot 2^{-5}$	$4.0 \cdot 10^{-07}$	0.00625	2.000	$1.5 \cdot 10^{-07}$	0.00625	2.000
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{-07}$	0.00312	2.000	$3.9 \cdot 10^{-08}$	0.00312	2.000
$1/5 \cdot 2^{-7}$	$2.5 \cdot 10^{-08}$	0.00156	2.000	$9.9 \cdot 10^{-09}$	0.00156	2.000
$1/5 \cdot 2^{-8}$	$6.2 \cdot 10^{-09}$	0.00078	2.000	$2.4 \cdot 10^{-09}$	0.00078	2.000
$1/5 \cdot 2^{-9}$	$1.5 \cdot 10^{-09}$	0.00039	2.000	$6.2 \cdot 10^{-10}$	0.00039	2.000
$1/5 \cdot 2^{-10}$	$3.9 \cdot 10^{-10}$	0.00019	2.000	$1.5 \cdot 10^{-10}$	0.00019	2.000
$1/5 \cdot 2^{-11}$	$9.7 \cdot 10^{-11}$	0.00009	2.000	$3.9 \cdot 10^{-11}$	0.00009	2.000
$1/5 \cdot 2^{-12}$	$2.4 \cdot 10^{-11}$	0.00004		$9.7 \cdot 10^{-12}$	0.00004	

Table 4.25: Box scheme, IDeC method on $[0, 1]$ for 2.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$9.2 \cdot 10^{-04}$	0.60000	1.999	$1.6 \cdot 10^{-04}$	0.10000	1.972
$1/5 \cdot 2^{-2}$	$2.3 \cdot 10^{-04}$	0.55000	2.000	$4.1 \cdot 10^{-05}$	0.05000	1.993
$1/5 \cdot 2^{-3}$	$5.7 \cdot 10^{-05}$	0.55000	2.000	$1.0 \cdot 10^{-05}$	0.02500	1.998
$1/5 \cdot 2^{-4}$	$1.4 \cdot 10^{-05}$	0.56250	2.000	$2.5 \cdot 10^{-06}$	0.01250	1.999
$1/5 \cdot 2^{-5}$	$3.6 \cdot 10^{-06}$	0.55625	2.000	$6.4 \cdot 10^{-07}$	0.00625	1.999
$1/5 \cdot 2^{-6}$	$9.0 \cdot 10^{-07}$	0.55625	2.000	$1.6 \cdot 10^{-07}$	0.00312	1.999
$1/5 \cdot 2^{-7}$	$2.2 \cdot 10^{-07}$	0.55781	2.000	$4.0 \cdot 10^{-08}$	0.00156	1.999
$1/5 \cdot 2^{-8}$	$5.6 \cdot 10^{-08}$	0.55703	2.000	$1.0 \cdot 10^{-08}$	0.00078	1.999
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-08}$	0.55703	2.000	$2.5 \cdot 10^{-09}$	0.00039	1.999
$1/5 \cdot 2^{-10}$	$3.5 \cdot 10^{-09}$	0.55722	2.000	$6.3 \cdot 10^{-10}$	0.00019	1.999
$1/5 \cdot 2^{-11}$	$8.8 \cdot 10^{-10}$	0.55712	2.000	$1.5 \cdot 10^{-10}$	0.00009	1.999
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-10}$	0.55717	2.000	$3.9 \cdot 10^{-11}$	0.00004	-32.165
$1/5 \cdot 2^{-13}$	$5.5 \cdot 10^{-11}$	0.55717	2.000	$1.9 \cdot 10^{-01}$	0.99998	36.162
$1/5 \cdot 2^{-14}$	$1.3 \cdot 10^{-11}$	0.55716		$2.4 \cdot 10^{-12}$	0.00001	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$5.5 \cdot 10^{-05}$	0.10000	1.920	$2.0 \cdot 10^{-05}$	0.10000	1.855
$1/5 \cdot 2^{-2}$	$1.4 \cdot 10^{-05}$	0.05000	1.980	$5.6 \cdot 10^{-06}$	0.05000	1.964
$1/5 \cdot 2^{-3}$	$3.7 \cdot 10^{-06}$	0.02500	1.995	$1.4 \cdot 10^{-06}$	0.02500	1.991
$1/5 \cdot 2^{-4}$	$9.3 \cdot 10^{-07}$	0.01250	1.998	$3.6 \cdot 10^{-07}$	0.01250	1.997
$1/5 \cdot 2^{-5}$	$2.3 \cdot 10^{-07}$	0.00625	1.999	$9.1 \cdot 10^{-08}$	0.00625	1.999
$1/5 \cdot 2^{-6}$	$5.8 \cdot 10^{-08}$	0.00312	1.999	$2.2 \cdot 10^{-08}$	0.00312	1.999
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-08}$	0.00156	1.999	$5.6 \cdot 10^{-09}$	0.00156	1.999
$1/5 \cdot 2^{-8}$	$3.6 \cdot 10^{-09}$	0.00078	1.999	$1.4 \cdot 10^{-09}$	0.00078	1.999
$1/5 \cdot 2^{-9}$	$9.0 \cdot 10^{-10}$	0.00039	1.999	$3.5 \cdot 10^{-10}$	0.00039	1.999
$1/5 \cdot 2^{-10}$	$2.2 \cdot 10^{-10}$	0.00019	1.999	$8.9 \cdot 10^{-11}$	0.00019	1.999
$1/5 \cdot 2^{-11}$	$5.6 \cdot 10^{-11}$	0.00009	1.999	$2.2 \cdot 10^{-11}$	0.00009	1.999
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-11}$	0.00004	-33.590	$5.5 \cdot 10^{-12}$	0.00004	-34.907
$1/5 \cdot 2^{-13}$	$1.8 \cdot 10^{-01}$	0.99998		$1.7 \cdot 10^{-01}$	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$7.6 \cdot 10^{-06}$	0.10000	1.787	$2.8 \cdot 10^{-06}$	0.10000	1.712
$1/5 \cdot 2^{-2}$	$2.2 \cdot 10^{-06}$	0.05000	1.948	$8.6 \cdot 10^{-07}$	0.05000	1.930
$1/5 \cdot 2^{-3}$	$5.7 \cdot 10^{-07}$	0.02500	1.987	$2.2 \cdot 10^{-07}$	0.02500	1.982
$1/5 \cdot 2^{-4}$	$1.4 \cdot 10^{-07}$	0.01250	1.996	$5.7 \cdot 10^{-08}$	0.01250	1.995
$1/5 \cdot 2^{-5}$	$3.6 \cdot 10^{-08}$	0.00625	1.999	$1.4 \cdot 10^{-08}$	0.00625	1.998
$1/5 \cdot 2^{-6}$	$9.0 \cdot 10^{-09}$	0.00312	1.999	$3.6 \cdot 10^{-09}$	0.00312	1.999
$1/5 \cdot 2^{-7}$	$2.2 \cdot 10^{-09}$	0.00156	1.999	$9.0 \cdot 10^{-10}$	0.00156	1.999
$1/5 \cdot 2^{-8}$	$5.6 \cdot 10^{-10}$	0.00078	1.999	$2.2 \cdot 10^{-10}$	0.00078	1.999
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-10}$	0.00039	1.999	$5.6 \cdot 10^{-11}$	0.00039	1.999
$1/5 \cdot 2^{-10}$	$3.5 \cdot 10^{-11}$	0.00019	1.999	$1.4 \cdot 10^{-11}$	0.00019	1.999
$1/5 \cdot 2^{-11}$	$8.8 \cdot 10^{-12}$	0.00009	1.999	$3.5 \cdot 10^{-12}$	0.00009	1.999
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-12}$	0.00004		$8.8 \cdot 10^{-13}$	0.00004	

Table 4.26: Box scheme, IDeC method on $[0,1]$ for 2.3a.

The observed phenomena are different for the case where $M(0)$ has a double eigenvalue 0. Theory predicts that it is well possible that an order reduction down to $O(|\ln(h)|h^2)$ occurs for the two-dimensional testproblems. Of course special properties of the test problems can nevertheless allow for a full convergence order 2 of the basic solution. Naturally, no error expansion (3.9) can exist in the first case. Nonetheless, the IDeC iteration may restore the order $O(h^2)$. In the second case, the error expansion is likely to break down eventually and this fact should be reflected in the behavior of IDeC.

In order to see if logarithmic terms are present in the convergence orders, we make both the ansatz $\delta \approx ch^p$ and $\delta \approx c|\ln(h)|h^p$ for the errors of the basic and improved approximations, and compare the results. We accept the guess at the error structure as correct if p stabilizes for $h \to 0$. However, the results are not unambiguous in all cases, because of the slow growth of $|\ln(h)|$ which makes such terms difficult to detect numerically.

Tables 4.27 and 4.28 demonstrate this effect clearly for example 3.1a. While the basic solution does not seem to stabilize at $p = 2$ for the polynomial ansatz, the further IDeC iterates clearly show a convergence rate $O(h^2)$ in Table 4.27. Another interesting phenomenon is first encountered here: The 5th and last iteration suddenly improves the convergence order to 4, after the previous steps yielded the same convergence order. The logarithmic ansatz in Table 4.28 improves the stabilization for the basic solution markedly, naturally the other terms are quite blurred now. This leads to the conclusion that for test problem 3.1a, the convergence order of the basic solution is $O(|\ln(h)|h^2)$, whereas for the iterates $1, \ldots, 4$ the order is 2, and the last step we computed yields an $O(h^4)$ approximation.

The asymptotic behavior of IDeC for this example as well as for all the other test problems with the same spectral properties is the same at a fixed point t as for the error on $[0, 1]$, so we only display the one example in Table 4.29.

The results for example 3.1b are very similar to those for 3.1a, see Tables 4.30 and 4.31. However, it is important to note the difference to 3.1c (Tables 4.32 and 4.33), which has a similar structure but has a different solution. Here, a higher order of convergence can be obtained with the IDeC iteration. The numerical results suggest an order sequence $O(h^2)$, $O(h^4)$, $O(|\ln(h)|h^6)$, $O(h^6)$, \ldots. This is apparently due to the fact that the first four derivatives of the solution vanish at $t = 0$. We shall consider this point further in §4.2.3.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$6.9 \cdot 10^{-02}$	0.60000	1.709	$4.8 \cdot 10^{-03}$	0.90000	1.922
$1/5 \cdot 2^{-2}$	$2.1 \cdot 10^{-02}$	0.55000	1.757	$1.2 \cdot 10^{-03}$	0.45000	1.970
$1/5 \cdot 2^{-3}$	$6.2 \cdot 10^{-03}$	0.55000	1.792	$3.2 \cdot 10^{-04}$	0.22500	1.991
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-03}$	0.55000	1.818	$8.2 \cdot 10^{-05}$	0.11250	1.997
$1/5 \cdot 2^{-5}$	$5.1 \cdot 10^{-04}$	0.55000	1.838	$2.0 \cdot 10^{-05}$	0.05625	1.999
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-04}$	0.55312	1.854	$5.1 \cdot 10^{-06}$	0.02812	1.999
$1/5 \cdot 2^{-7}$	$3.9 \cdot 10^{-05}$	0.55312	1.868	$1.2 \cdot 10^{-06}$	0.01406	1.999
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-05}$	0.55234	1.879	$3.2 \cdot 10^{-07}$	0.00703	1.999
$1/5 \cdot 2^{-9}$	$2.9 \cdot 10^{-06}$	0.55273	1.888	$8.0 \cdot 10^{-08}$	0.00351	1.999
$1/5 \cdot 2^{-10}$	$8.0 \cdot 10^{-07}$	0.55273	1.896	$2.0 \cdot 10^{-08}$	0.00175	1.999
$1/5 \cdot 2^{-11}$	$2.1 \cdot 10^{-07}$	0.55263	1.903	$5.0 \cdot 10^{-09}$	0.00087	1.999
$1/5 \cdot 2^{-12}$	$5.7 \cdot 10^{-08}$	0.55268	1.909	$1.2 \cdot 10^{-09}$	0.00043	-26.142
$1/5 \cdot 2^{-13}$	$1.5 \cdot 10^{-08}$	0.55265	1.914	$9.3 \cdot 10^{-02}$	0.99998	30.139
$1/5 \cdot 2^{-14}$	$4.0 \cdot 10^{-09}$	0.55266		$7.8 \cdot 10^{-11}$	0.00010	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$5.1 \cdot 10^{-04}$	0.80000	1.933	$2.8 \cdot 10^{-05}$	0.70000	1.973
$1/5 \cdot 2^{-2}$	$1.3 \cdot 10^{-04}$	0.40000	1.981	$7.2 \cdot 10^{-06}$	0.35000	1.964
$1/5 \cdot 2^{-3}$	$3.3 \cdot 10^{-05}$	0.20000	1.995	$1.8 \cdot 10^{-06}$	0.17500	1.991
$1/5 \cdot 2^{-4}$	$8.4 \cdot 10^{-06}$	0.10000	1.998	$4.6 \cdot 10^{-07}$	0.08750	1.997
$1/5 \cdot 2^{-5}$	$2.1 \cdot 10^{-06}$	0.05000	1.999	$1.1 \cdot 10^{-07}$	0.04375	1.999
$1/5 \cdot 2^{-6}$	$5.3 \cdot 10^{-07}$	0.02500	1.999	$2.9 \cdot 10^{-08}$	0.02187	1.999
$1/5 \cdot 2^{-7}$	$1.3 \cdot 10^{-07}$	0.01250	1.999	$7.2 \cdot 10^{-09}$	0.01093	1.999
$1/5 \cdot 2^{-8}$	$3.3 \cdot 10^{-08}$	0.00625	1.999	$1.8 \cdot 10^{-09}$	0.00546	1.999
$1/5 \cdot 2^{-9}$	$8.3 \cdot 10^{-09}$	0.00312	1.999	$4.5 \cdot 10^{-10}$	0.00273	1.999
$1/5 \cdot 2^{-10}$	$2.0 \cdot 10^{-09}$	0.00156	1.999	$1.1 \cdot 10^{-10}$	0.00136	1.999
$1/5 \cdot 2^{-11}$	$5.1 \cdot 10^{-10}$	0.00078	1.999	$2.8 \cdot 10^{-11}$	0.00068	1.999
$1/5 \cdot 2^{-12}$	$1.2 \cdot 10^{-10}$	0.00039	-29.366	$7.1 \cdot 10^{-12}$	0.00034	-33.519
$1/5 \cdot 2^{-13}$	$8.9 \cdot 10^{-02}$	0.99998		$8.7 \cdot 10^{-02}$	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.6 \cdot 10^{-06}$	0.70000	4.295	$2.4 \cdot 10^{-06}$	1.00000	11.079
$1/5 \cdot 2^{-2}$	$1.3 \cdot 10^{-07}$	0.30000	1.904	$1.1 \cdot 10^{-09}$	1.00000	6.083
$1/5 \cdot 2^{-3}$	$3.6 \cdot 10^{-08}$	0.15000	1.974	$1.6 \cdot 10^{-11}$	0.15000	3.964
$1/5 \cdot 2^{-4}$	$9.1 \cdot 10^{-09}$	0.07500	1.993	$1.0 \cdot 10^{-12}$	0.07500	3.995
$1/5 \cdot 2^{-5}$	$2.3 \cdot 10^{-09}$	0.03750	1.998	$6.7 \cdot 10^{-14}$	0.03750	3.998
$1/5 \cdot 2^{-6}$	$5.7 \cdot 10^{-10}$	0.01875	1.999	$4.2 \cdot 10^{-15}$	0.01875	3.999
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-10}$	0.00937	1.999	$2.6 \cdot 10^{-16}$	0.00937	3.999
$1/5 \cdot 2^{-8}$	$3.6 \cdot 10^{-11}$	0.00468	1.999	$1.6 \cdot 10^{-17}$	0.00468	3.999
$1/5 \cdot 2^{-9}$	$9.0 \cdot 10^{-12}$	0.00234	1.999	$1.0 \cdot 10^{-18}$	0.00234	3.999
$1/5 \cdot 2^{-10}$	$2.2 \cdot 10^{-12}$	0.00117	1.999	$6.4 \cdot 10^{-20}$	0.00117	3.999
$1/5 \cdot 2^{-11}$	$5.6 \cdot 10^{-13}$	0.00058	1.999	$4.0 \cdot 10^{-21}$	0.00058	3.999
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-13}$	0.00029		$2.5 \cdot 10^{-22}$	0.00029	

Table 4.27: Box scheme, IDeC method on $[0, 1]$ for 3.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5\cdot2^{-1}$	$6.9\cdot10^{-02}$	0.60000	2.088	$4.8\cdot10^{-03}$	0.90000	2.301
$1/5\cdot2^{-2}$	$2.1\cdot10^{-02}$	0.55000	2.058	$1.2\cdot10^{-03}$	0.45000	2.270
$1/5\cdot2^{-3}$	$6.2\cdot10^{-03}$	0.55000	2.040	$3.2\cdot10^{-04}$	0.22500	2.240
$1/5\cdot2^{-4}$	$1.8\cdot10^{-03}$	0.55000	2.030	$8.2\cdot10^{-05}$	0.11250	2.209
$1/5\cdot2^{-5}$	$5.1\cdot10^{-04}$	0.55000	2.023	$2.0\cdot10^{-05}$	0.05625	2.184
$1/5\cdot2^{-6}$	$1.4\cdot10^{-04}$	0.55312	2.018	$5.1\cdot10^{-06}$	0.02812	2.163
$1/5\cdot2^{-7}$	$3.9\cdot10^{-05}$	0.55312	2.015	$1.2\cdot10^{-06}$	0.01406	2.146
$1/5\cdot2^{-8}$	$1.0\cdot10^{-05}$	0.55234	2.012	$3.2\cdot10^{-07}$	0.00703	2.133
$1/5\cdot2^{-9}$	$2.9\cdot10^{-06}$	0.55273	2.010	$8.0\cdot10^{-08}$	0.00351	2.122
$1/5\cdot2^{-10}$	$8.0\cdot10^{-07}$	0.55273	2.009	$2.0\cdot10^{-08}$	0.00175	2.112
$1/5\cdot2^{-11}$	$2.1\cdot10^{-07}$	0.55263	2.007	$5.0\cdot10^{-09}$	0.00087	2.104
$1/5\cdot2^{-12}$	$5.7\cdot10^{-08}$	0.55268	2.006	$1.2\cdot10^{-09}$	0.00043	-26.045
$1/5\cdot2^{-13}$	$1.5\cdot10^{-08}$	0.55265	2.006	$9.3\cdot10^{-02}$	0.99998	30.230
$1/5\cdot2^{-14}$	$4.0\cdot10^{-09}$	0.55266		$7.8\cdot10^{-11}$	0.00010	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5\cdot2^{-1}$	$5.1\cdot10^{-04}$	0.80000	2.313	$2.8\cdot10^{-05}$	0.70000	2.353
$1/5\cdot2^{-2}$	$1.3\cdot10^{-04}$	0.40000	2.281	$7.2\cdot10^{-06}$	0.35000	2.264
$1/5\cdot2^{-3}$	$3.3\cdot10^{-05}$	0.20000	2.243	$1.8\cdot10^{-06}$	0.17500	2.239
$1/5\cdot2^{-4}$	$8.4\cdot10^{-06}$	0.10000	2.210	$4.6\cdot10^{-07}$	0.08750	2.209
$1/5\cdot2^{-5}$	$2.1\cdot10^{-06}$	0.05000	2.184	$1.1\cdot10^{-07}$	0.04375	2.184
$1/5\cdot2^{-6}$	$5.3\cdot10^{-07}$	0.02500	2.163	$2.9\cdot10^{-08}$	0.02187	2.163
$1/5\cdot2^{-7}$	$1.3\cdot10^{-07}$	0.01250	2.146	$7.2\cdot10^{-09}$	0.01093	2.146
$1/5\cdot2^{-8}$	$3.3\cdot10^{-08}$	0.00625	2.133	$1.8\cdot10^{-09}$	0.00546	2.133
$1/5\cdot2^{-9}$	$8.3\cdot10^{-09}$	0.00312	2.122	$4.5\cdot10^{-10}$	0.00273	2.122
$1/5\cdot2^{-10}$	$2.0\cdot10^{-09}$	0.00156	2.112	$1.1\cdot10^{-10}$	0.00136	2.112
$1/5\cdot2^{-11}$	$5.1\cdot10^{-10}$	0.00078	2.104	$2.8\cdot10^{-11}$	0.00068	2.104
$1/5\cdot2^{-12}$	$1.2\cdot10^{-10}$	0.00039	-29.269	$7.1\cdot10^{-12}$	0.00034	-33.421
$1/5\cdot2^{-13}$	$8.9\cdot10^{-02}$	0.99998		$8.7\cdot10^{-02}$	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5\cdot2^{-1}$	$2.6\cdot10^{-06}$	0.70000	4.675	$2.4\cdot10^{-06}$	1.00000	11.459
$1/5\cdot2^{-2}$	$1.3\cdot10^{-07}$	0.30000	2.204	$1.1\cdot10^{-09}$	1.00000	6.383
$1/5\cdot2^{-3}$	$3.6\cdot10^{-08}$	0.15000	2.223	$1.6\cdot10^{-11}$	0.15000	4.212
$1/5\cdot2^{-4}$	$9.1\cdot10^{-09}$	0.07500	2.205	$1.0\cdot10^{-12}$	0.07500	4.207
$1/5\cdot2^{-5}$	$2.3\cdot10^{-09}$	0.03750	2.183	$6.7\cdot10^{-14}$	0.03750	4.183
$1/5\cdot2^{-6}$	$5.7\cdot10^{-10}$	0.01875	2.163	$4.2\cdot10^{-15}$	0.01875	4.163
$1/5\cdot2^{-7}$	$1.4\cdot10^{-10}$	0.00937	2.146	$2.6\cdot10^{-16}$	0.00937	4.146
$1/5\cdot2^{-8}$	$3.6\cdot10^{-11}$	0.00468	2.133	$1.6\cdot10^{-17}$	0.00468	4.133
$1/5\cdot2^{-9}$	$9.0\cdot10^{-12}$	0.00234	2.122	$1.0\cdot10^{-18}$	0.00234	4.122
$1/5\cdot2^{-10}$	$2.2\cdot10^{-12}$	0.00117	2.112	$6.4\cdot10^{-20}$	0.00117	4.112
$1/5\cdot2^{-11}$	$5.6\cdot10^{-13}$	0.00058	2.104	$4.0\cdot10^{-21}$	0.00058	4.104
$1/5\cdot2^{-12}$	$1.4\cdot10^{-13}$	0.00029		$2.5\cdot10^{-22}$	0.00029	

Table 4.28: Box scheme, IDeC method on $[0,1]$ for 3.1a, logarithmic.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$6.5 \cdot 10^{-02}$	1.693	$4.6 \cdot 10^{-03}$	1.915
$1/5 \cdot 2^{-2}$	$2.0 \cdot 10^{-02}$	1.746	$1.2 \cdot 10^{-03}$	1.973
$1/5 \cdot 2^{-3}$	$6.0 \cdot 10^{-03}$	1.783	$3.1 \cdot 10^{-04}$	1.991
$1/5 \cdot 2^{-4}$	$1.7 \cdot 10^{-03}$	1.811	$7.8 \cdot 10^{-05}$	1.997
$1/5 \cdot 2^{-5}$	$4.9 \cdot 10^{-04}$	1.833	$1.9 \cdot 10^{-05}$	1.999
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-04}$	1.850	$4.9 \cdot 10^{-06}$	1.999
$1/5 \cdot 2^{-7}$	$3.8 \cdot 10^{-05}$	1.864	$1.2 \cdot 10^{-06}$	1.999
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-05}$	1.876	$3.0 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-9}$	$2.8 \cdot 10^{-06}$	1.886	$7.7 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-10}$	$7.8 \cdot 10^{-07}$	1.894	$1.9 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-11}$	$2.1 \cdot 10^{-07}$	1.901	$4.8 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-12}$	$5.6 \cdot 10^{-08}$	1.907	$1.2 \cdot 10^{-09}$	-26.207
$1/5 \cdot 2^{-13}$	$1.5 \cdot 10^{-08}$	1.913	$9.3 \cdot 10^{-02}$	30.204
$1/5 \cdot 2^{-14}$	$3.9 \cdot 10^{-09}$		$7.5 \cdot 10^{-11}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$4.2 \cdot 10^{-04}$	1.923	$1.8 \cdot 10^{-05}$	2.007
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-04}$	1.978	$4.6 \cdot 10^{-06}$	1.952
$1/5 \cdot 2^{-3}$	$2.8 \cdot 10^{-05}$	1.994	$1.1 \cdot 10^{-06}$	1.988
$1/5 \cdot 2^{-4}$	$7.1 \cdot 10^{-06}$	1.998	$2.9 \cdot 10^{-07}$	1.997
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-06}$	1.999	$7.5 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-6}$	$4.4 \cdot 10^{-07}$	1.999	$1.8 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-7}$	$1.1 \cdot 10^{-07}$	1.999	$4.6 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-8}$	$2.7 \cdot 10^{-08}$	1.999	$1.1 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-9}$	$6.9 \cdot 10^{-09}$	1.999	$2.9 \cdot 10^{-10}$	1.999
$1/5 \cdot 2^{-10}$	$1.7 \cdot 10^{-09}$	1.999	$7.3 \cdot 10^{-11}$	1.999
$1/5 \cdot 2^{-11}$	$4.3 \cdot 10^{-10}$	1.999	$1.8 \cdot 10^{-11}$	1.999
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-10}$	-29.621	$4.5 \cdot 10^{-12}$	-34.154
$1/5 \cdot 2^{-13}$	$8.9 \cdot 10^{-02}$		$8.7 \cdot 10^{-02}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$2.5 \cdot 10^{-06}$	5.793	$2.4 \cdot 10^{-06}$	11.079
$1/5 \cdot 2^{-2}$	$4.6 \cdot 10^{-08}$	1.844	$1.1 \cdot 10^{-09}$	7.738
$1/5 \cdot 2^{-3}$	$1.2 \cdot 10^{-08}$	1.953	$5.3 \cdot 10^{-12}$	3.780
$1/5 \cdot 2^{-4}$	$3.3 \cdot 10^{-09}$	1.988	$3.8 \cdot 10^{-13}$	3.989
$1/5 \cdot 2^{-5}$	$8.3 \cdot 10^{-10}$	1.997	$2.4 \cdot 10^{-14}$	3.997
$1/5 \cdot 2^{-6}$	$2.0 \cdot 10^{-10}$	1.999	$1.5 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-7}$	$5.2 \cdot 10^{-11}$	1.999	$9.5 \cdot 10^{-17}$	3.999
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-11}$	1.999	$5.9 \cdot 10^{-18}$	3.999
$1/5 \cdot 2^{-9}$	$3.2 \cdot 10^{-12}$	1.999	$3.7 \cdot 10^{-19}$	3.999
$1/5 \cdot 2^{-10}$	$8.1 \cdot 10^{-13}$	1.999	$2.3 \cdot 10^{-20}$	3.999
$1/5 \cdot 2^{-11}$	$2.0 \cdot 10^{-13}$	1.999	$1.4 \cdot 10^{-21}$	3.999
$1/5 \cdot 2^{-12}$	$5.0 \cdot 10^{-14}$		$9.1 \cdot 10^{-23}$	

Table 4.29: Box scheme, IDeC method at $t = 1$ for 3.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.5 \cdot 10^{-02}$	1.00000	1.892	$1.3 \cdot 10^{-03}$	0.90000	2.150
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-02}$	1.00000	1.901	$3.0 \cdot 10^{-04}$	0.45000	2.029
$1/5 \cdot 2^{-3}$	$3.3 \cdot 10^{-03}$	1.00000	1.908	$7.3 \cdot 10^{-05}$	0.22500	2.006
$1/5 \cdot 2^{-4}$	$8.8 \cdot 10^{-04}$	1.00000	1.913	$1.8 \cdot 10^{-05}$	0.11250	2.001
$1/5 \cdot 2^{-5}$	$2.3 \cdot 10^{-04}$	1.00000	1.918	$4.5 \cdot 10^{-06}$	0.05625	2.000
$1/5 \cdot 2^{-6}$	$6.1 \cdot 10^{-05}$	1.00000	1.923	$1.1 \cdot 10^{-06}$	0.02812	2.000
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-05}$	1.00000	1.926	$2.8 \cdot 10^{-07}$	0.01406	2.000
$1/5 \cdot 2^{-8}$	$4.2 \cdot 10^{-06}$	1.00000	1.930	$7.1 \cdot 10^{-08}$	0.00703	2.000
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-06}$	1.00000	1.933	$1.7 \cdot 10^{-08}$	0.00351	2.000
$1/5 \cdot 2^{-10}$	$2.9 \cdot 10^{-07}$	1.00000	1.936	$4.4 \cdot 10^{-09}$	0.00175	2.000
$1/5 \cdot 2^{-11}$	$7.6 \cdot 10^{-08}$	1.00000	1.939	$1.1 \cdot 10^{-09}$	0.00087	2.000
$1/5 \cdot 2^{-12}$	$2.0 \cdot 10^{-08}$	1.00000	1.941	$2.8 \cdot 10^{-10}$	0.00043	-32.581
$1/5 \cdot 2^{-13}$	$5.2 \cdot 10^{-09}$	0.99998	1.943	1.7	0.99998	36.577
$1/5 \cdot 2^{-14}$	$1.3 \cdot 10^{-09}$	1.00000		$1.7 \cdot 10^{-11}$	0.00010	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.2 \cdot 10^{-04}$	0.80000	2.050	$7.2 \cdot 10^{-06}$	0.70000	2.091
$1/5 \cdot 2^{-2}$	$3.0 \cdot 10^{-05}$	0.40000	2.012	$1.6 \cdot 10^{-06}$	0.35000	2.023
$1/5 \cdot 2^{-3}$	$7.5 \cdot 10^{-06}$	0.20000	2.003	$4.1 \cdot 10^{-07}$	0.17500	2.005
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-06}$	0.10000	2.000	$1.0 \cdot 10^{-07}$	0.08750	2.001
$1/5 \cdot 2^{-5}$	$4.7 \cdot 10^{-07}$	0.05000	2.000	$2.5 \cdot 10^{-08}$	0.04375	2.000
$1/5 \cdot 2^{-6}$	$1.1 \cdot 10^{-07}$	0.02500	2.000	$6.4 \cdot 10^{-09}$	0.02187	2.000
$1/5 \cdot 2^{-7}$	$2.9 \cdot 10^{-08}$	0.01250	2.000	$1.6 \cdot 10^{-09}$	0.01093	2.000
$1/5 \cdot 2^{-8}$	$7.3 \cdot 10^{-09}$	0.00625	2.000	$4.0 \cdot 10^{-10}$	0.00546	2.000
$1/5 \cdot 2^{-9}$	$1.8 \cdot 10^{-09}$	0.00312	2.000	$1.0 \cdot 10^{-10}$	0.00273	2.000
$1/5 \cdot 2^{-10}$	$4.6 \cdot 10^{-10}$	0.00156	2.000	$2.5 \cdot 10^{-11}$	0.00136	2.000
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-10}$	0.00078	2.000	$6.3 \cdot 10^{-12}$	0.00068	2.000
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-11}$	0.00039	-35.805	$1.5 \cdot 10^{-12}$	0.00034	-39.957
$1/5 \cdot 2^{-13}$	1.7	0.99998		1.6	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.6 \cdot 10^{-07}$	0.60000	2.217	$2.6 \cdot 10^{-09}$	1.00000	6.004
$1/5 \cdot 2^{-2}$	$3.4 \cdot 10^{-08}$	0.30000	2.065	$4.0 \cdot 10^{-11}$	0.30000	3.987
$1/5 \cdot 2^{-3}$	$8.3 \cdot 10^{-09}$	0.15000	2.016	$2.5 \cdot 10^{-12}$	0.15000	4.004
$1/5 \cdot 2^{-4}$	$2.0 \cdot 10^{-09}$	0.07500	2.004	$1.6 \cdot 10^{-13}$	0.07500	4.001
$1/5 \cdot 2^{-5}$	$5.1 \cdot 10^{-10}$	0.03750	2.001	$1.0 \cdot 10^{-14}$	0.03750	4.000
$1/5 \cdot 2^{-6}$	$1.2 \cdot 10^{-10}$	0.01875	2.000	$6.2 \cdot 10^{-16}$	0.01875	4.000
$1/5 \cdot 2^{-7}$	$3.2 \cdot 10^{-11}$	0.00937	2.000	$3.9 \cdot 10^{-17}$	0.00937	4.000
$1/5 \cdot 2^{-8}$	$8.0 \cdot 10^{-12}$	0.00468	2.000	$2.4 \cdot 10^{-18}$	0.00468	4.000
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-12}$	0.00234	2.000	$1.5 \cdot 10^{-19}$	0.00234	4.000
$1/5 \cdot 2^{-10}$	$5.0 \cdot 10^{-13}$	0.00117	2.000	$9.5 \cdot 10^{-21}$	0.00117	4.000
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-13}$	0.00058	2.000	$5.9 \cdot 10^{-22}$	0.00058	4.000
$1/5 \cdot 2^{-12}$	$3.1 \cdot 10^{-14}$	0.00029		$3.7 \cdot 10^{-23}$	0.00029	

Table 4.30: Box scheme, IDeC method on $[0, 1]$ for 3.1b.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.5 \cdot 10^{-02}$	1.00000	2.272	$1.3 \cdot 10^{-03}$	0.90000	2.529
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-02}$	1.00000	2.201	$3.0 \cdot 10^{-04}$	0.45000	2.329
$1/5 \cdot 2^{-3}$	$3.3 \cdot 10^{-03}$	1.00000	2.156	$7.3 \cdot 10^{-05}$	0.22500	2.254
$1/5 \cdot 2^{-4}$	$8.8 \cdot 10^{-04}$	1.00000	2.125	$1.8 \cdot 10^{-05}$	0.11250	2.213
$1/5 \cdot 2^{-5}$	$2.3 \cdot 10^{-04}$	1.00000	2.103	$4.5 \cdot 10^{-06}$	0.05625	2.185
$1/5 \cdot 2^{-6}$	$6.1 \cdot 10^{-05}$	1.00000	2.086	$1.1 \cdot 10^{-06}$	0.02812	2.163
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-05}$	1.00000	2.073	$2.8 \cdot 10^{-07}$	0.01406	2.147
$1/5 \cdot 2^{-8}$	$4.2 \cdot 10^{-06}$	1.00000	2.063	$7.1 \cdot 10^{-08}$	0.00703	2.133
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-06}$	1.00000	2.055	$1.7 \cdot 10^{-08}$	0.00351	2.122
$1/5 \cdot 2^{-10}$	$2.9 \cdot 10^{-07}$	1.00000	2.049	$4.4 \cdot 10^{-09}$	0.00175	2.112
$1/5 \cdot 2^{-11}$	$7.6 \cdot 10^{-08}$	1.00000	2.043	$1.1 \cdot 10^{-09}$	0.00087	2.104
$1/5 \cdot 2^{-12}$	$2.0 \cdot 10^{-08}$	1.00000	2.039	$2.8 \cdot 10^{-10}$	0.00043	-32.483
$1/5 \cdot 2^{-13}$	$5.2 \cdot 10^{-09}$	0.99998	2.035	1.7	0.99998	36.668
$1/5 \cdot 2^{-14}$	$1.3 \cdot 10^{-09}$	1.00000		$1.7 \cdot 10^{-11}$	0.00010	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.2 \cdot 10^{-04}$	0.80000	2.430	$7.2 \cdot 10^{-06}$	0.70000	2.471
$1/5 \cdot 2^{-2}$	$3.0 \cdot 10^{-05}$	0.40000	2.312	$1.6 \cdot 10^{-06}$	0.35000	2.323
$1/5 \cdot 2^{-3}$	$7.5 \cdot 10^{-06}$	0.20000	2.251	$4.1 \cdot 10^{-07}$	0.17500	2.254
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-06}$	0.10000	2.212	$1.0 \cdot 10^{-07}$	0.08750	2.213
$1/5 \cdot 2^{-5}$	$4.7 \cdot 10^{-07}$	0.05000	2.184	$2.5 \cdot 10^{-08}$	0.04375	2.185
$1/5 \cdot 2^{-6}$	$1.1 \cdot 10^{-07}$	0.02500	2.163	$6.4 \cdot 10^{-09}$	0.02187	2.163
$1/5 \cdot 2^{-7}$	$2.9 \cdot 10^{-08}$	0.01250	2.147	$1.6 \cdot 10^{-09}$	0.01093	2.147
$1/5 \cdot 2^{-8}$	$7.3 \cdot 10^{-09}$	0.00625	2.133	$4.0 \cdot 10^{-10}$	0.00546	2.133
$1/5 \cdot 2^{-9}$	$1.8 \cdot 10^{-09}$	0.00312	2.122	$1.0 \cdot 10^{-10}$	0.00273	2.122
$1/5 \cdot 2^{-10}$	$4.6 \cdot 10^{-10}$	0.00156	2.112	$2.5 \cdot 10^{-11}$	0.00136	2.112
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-10}$	0.00078	2.104	$6.3 \cdot 10^{-12}$	0.00068	2.104
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-11}$	0.00039	-35.707	$1.5 \cdot 10^{-12}$	0.00034	-39.859
$1/5 \cdot 2^{-13}$	1.7	0.99998		1.6	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.6 \cdot 10^{-07}$	0.60000	2.596	$2.6 \cdot 10^{-09}$	1.00000	6.384
$1/5 \cdot 2^{-2}$	$3.4 \cdot 10^{-08}$	0.30000	2.365	$4.0 \cdot 10^{-11}$	0.30000	4.287
$1/5 \cdot 2^{-3}$	$8.3 \cdot 10^{-09}$	0.15000	2.265	$2.5 \cdot 10^{-12}$	0.15000	4.253
$1/5 \cdot 2^{-4}$	$2.0 \cdot 10^{-09}$	0.07500	2.216	$1.6 \cdot 10^{-13}$	0.07500	4.213
$1/5 \cdot 2^{-5}$	$5.1 \cdot 10^{-10}$	0.03750	2.185	$1.0 \cdot 10^{-14}$	0.03750	4.185
$1/5 \cdot 2^{-6}$	$1.2 \cdot 10^{-10}$	0.01875	2.163	$6.2 \cdot 10^{-16}$	0.01875	4.163
$1/5 \cdot 2^{-7}$	$3.2 \cdot 10^{-11}$	0.00937	2.147	$3.9 \cdot 10^{-17}$	0.00937	4.147
$1/5 \cdot 2^{-8}$	$8.0 \cdot 10^{-12}$	0.00468	2.133	$2.4 \cdot 10^{-18}$	0.00468	4.133
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-12}$	0.00234	2.122	$1.5 \cdot 10^{-19}$	0.00234	4.122
$1/5 \cdot 2^{-10}$	$5.0 \cdot 10^{-13}$	0.00117	2.112	$9.5 \cdot 10^{-21}$	0.00117	4.112
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-13}$	0.00058	2.104	$5.9 \cdot 10^{-22}$	0.00058	4.104
$1/5 \cdot 2^{-12}$	$3.1 \cdot 10^{-14}$	0.00029		$3.7 \cdot 10^{-23}$	0.00029	

Table 4.31: Box scheme, IDeC method on $[0,1]$ for 3.1b, logarithmic.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.2 \cdot 10^{-01}$	1.00000	1.994	$1.6 \cdot 10^{-03}$	1.00000	3.994
$1/5 \cdot 2^{-2}$	$5.5 \cdot 10^{-02}$	1.00000	1.998	$1.0 \cdot 10^{-04}$	1.00000	3.998
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-02}$	1.00000	1.999	$6.5 \cdot 10^{-06}$	1.00000	3.999
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-03}$	1.00000	1.999	$4.0 \cdot 10^{-07}$	1.00000	3.999
$1/5 \cdot 2^{-5}$	$8.7 \cdot 10^{-04}$	1.00000	1.999	$2.5 \cdot 10^{-08}$	1.00000	3.999
$1/5 \cdot 2^{-6}$	$2.1 \cdot 10^{-04}$	1.00000	1.999	$1.5 \cdot 10^{-09}$	1.00000	3.999
$1/5 \cdot 2^{-7}$	$5.4 \cdot 10^{-05}$	1.00000	1.999	$9.9 \cdot 10^{-11}$	1.00000	3.999
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-05}$	1.00000	1.999	$6.2 \cdot 10^{-12}$	1.00000	3.999
$1/5 \cdot 2^{-9}$	$3.4 \cdot 10^{-06}$	1.00000	1.999	$3.9 \cdot 10^{-13}$	1.00000	3.999
$1/5 \cdot 2^{-10}$	$8.5 \cdot 10^{-07}$	1.00000	1.999	$2.4 \cdot 10^{-14}$	1.00000	3.999
$1/5 \cdot 2^{-11}$	$2.1 \cdot 10^{-07}$	1.00000	1.999	$1.5 \cdot 10^{-15}$	1.00000	3.999
$1/5 \cdot 2^{-12}$	$5.3 \cdot 10^{-08}$	1.00000	2.000	$9.5 \cdot 10^{-17}$	1.00000	-55.070
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-08}$	0.99998	1.999	3.5	0.99998	63.064
$1/5 \cdot 2^{-14}$	$3.3 \cdot 10^{-09}$	1.00000		$3.7 \cdot 10^{-19}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.2 \cdot 10^{-05}$	1.00000	5.749	$1.6 \cdot 10^{-06}$	1.00000	9.290
$1/5 \cdot 2^{-2}$	$2.2 \cdot 10^{-07}$	1.00000	5.930	$2.5 \cdot 10^{-09}$	0.20000	5.757
$1/5 \cdot 2^{-3}$	$3.7 \cdot 10^{-09}$	1.00000	5.939	$4.7 \cdot 10^{-11}$	0.22500	6.001
$1/5 \cdot 2^{-4}$	$6.1 \cdot 10^{-11}$	1.00000	5.942	$7.4 \cdot 10^{-13}$	0.11250	6.001
$1/5 \cdot 2^{-5}$	$9.9 \cdot 10^{-13}$	1.00000	5.945	$1.1 \cdot 10^{-14}$	0.05625	6.000
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-14}$	1.00000	5.947	$1.8 \cdot 10^{-16}$	0.02812	6.000
$1/5 \cdot 2^{-7}$	$2.6 \cdot 10^{-16}$	1.00000	5.948	$2.8 \cdot 10^{-18}$	0.01406	6.000
$1/5 \cdot 2^{-8}$	$4.2 \cdot 10^{-18}$	1.00000	5.950	$4.4 \cdot 10^{-20}$	0.00703	6.000
$1/5 \cdot 2^{-9}$	$6.8 \cdot 10^{-20}$	1.00000	5.952	$6.8 \cdot 10^{-22}$	0.00351	6.000
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-21}$	1.00000	5.953	$1.0 \cdot 10^{-23}$	0.00175	6.000
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-23}$	1.00000	5.956	$1.6 \cdot 10^{-25}$	0.00087	6.000
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-25}$	1.00000	-83.314	$2.6 \cdot 10^{-27}$	0.00043	-90.058
$1/5 \cdot 2^{-13}$	3.4	0.99998		3.3	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.8 \cdot 10^{-06}$	1.00000	11.149	$1.8 \cdot 10^{-06}$	1.00000	10.876
$1/5 \cdot 2^{-2}$	$8.1 \cdot 10^{-10}$	1.00000	7.345	$9.9 \cdot 10^{-10}$	1.00000	10.259
$1/5 \cdot 2^{-3}$	$4.9 \cdot 10^{-12}$	0.07500	5.946	$8.0 \cdot 10^{-13}$	1.00000	7.469
$1/5 \cdot 2^{-4}$	$8.0 \cdot 10^{-14}$	0.10000	5.999	$4.5 \cdot 10^{-15}$	0.05000	5.973
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-15}$	0.05000	6.000	$7.2 \cdot 10^{-17}$	0.04375	5.999
$1/5 \cdot 2^{-6}$	$1.9 \cdot 10^{-17}$	0.02500	6.000	$1.1 \cdot 10^{-18}$	0.02187	6.000
$1/5 \cdot 2^{-7}$	$3.0 \cdot 10^{-19}$	0.01250	6.000	$1.7 \cdot 10^{-20}$	0.01093	6.000
$1/5 \cdot 2^{-8}$	$4.8 \cdot 10^{-21}$	0.00625	6.000	$2.7 \cdot 10^{-22}$	0.00546	6.000
$1/5 \cdot 2^{-9}$	$7.5 \cdot 10^{-23}$	0.00312	6.000	$4.3 \cdot 10^{-24}$	0.00273	6.000
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-24}$	0.00156	6.000	$6.7 \cdot 10^{-26}$	0.00136	6.000
$1/5 \cdot 2^{-11}$	$1.8 \cdot 10^{-26}$	0.00078	3.520	$1.0 \cdot 10^{-27}$	0.00068	-0.622
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-27}$	0.99848		$1.6 \cdot 10^{-27}$	0.99848	

Table 4.32: Box scheme, IDeC method on $[0, 1]$ for 3.1c.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.2 \cdot 10^{-01}$	1.00000	2.373	$1.6 \cdot 10^{-03}$	1.00000	4.374
$1/5 \cdot 2^{-2}$	$5.5 \cdot 10^{-02}$	1.00000	2.298	$1.0 \cdot 10^{-04}$	1.00000	4.298
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-02}$	1.00000	2.248	$6.5 \cdot 10^{-06}$	1.00000	4.248
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-03}$	1.00000	2.211	$4.0 \cdot 10^{-07}$	1.00000	4.211
$1/5 \cdot 2^{-5}$	$8.7 \cdot 10^{-04}$	1.00000	2.184	$2.5 \cdot 10^{-08}$	1.00000	4.184
$1/5 \cdot 2^{-6}$	$2.1 \cdot 10^{-04}$	1.00000	2.163	$1.5 \cdot 10^{-09}$	1.00000	4.163
$1/5 \cdot 2^{-7}$	$5.4 \cdot 10^{-05}$	1.00000	2.147	$9.9 \cdot 10^{-11}$	1.00000	4.147
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-05}$	1.00000	2.133	$6.2 \cdot 10^{-12}$	1.00000	4.133
$1/5 \cdot 2^{-9}$	$3.4 \cdot 10^{-06}$	1.00000	2.122	$3.9 \cdot 10^{-13}$	1.00000	4.122
$1/5 \cdot 2^{-10}$	$8.5 \cdot 10^{-07}$	1.00000	2.112	$2.4 \cdot 10^{-14}$	1.00000	4.112
$1/5 \cdot 2^{-11}$	$2.1 \cdot 10^{-07}$	1.00000	2.104	$1.5 \cdot 10^{-15}$	1.00000	4.104
$1/5 \cdot 2^{-12}$	$5.3 \cdot 10^{-08}$	1.00000	2.097	$9.5 \cdot 10^{-17}$	1.00000	-54.973
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-08}$	0.99998	2.091	3.5	0.99998	63.155
$1/5 \cdot 2^{-14}$	$3.3 \cdot 10^{-09}$	1.00000		$3.7 \cdot 10^{-19}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.2 \cdot 10^{-05}$	1.00000	6.128	$1.6 \cdot 10^{-06}$	1.00000	9.669
$1/5 \cdot 2^{-2}$	$2.2 \cdot 10^{-07}$	1.00000	6.230	$2.5 \cdot 10^{-09}$	0.20000	6.057
$1/5 \cdot 2^{-3}$	$3.7 \cdot 10^{-09}$	1.00000	6.188	$4.7 \cdot 10^{-11}$	0.22500	6.249
$1/5 \cdot 2^{-4}$	$6.1 \cdot 10^{-11}$	1.00000	6.154	$7.4 \cdot 10^{-13}$	0.11250	6.213
$1/5 \cdot 2^{-5}$	$9.9 \cdot 10^{-13}$	1.00000	6.129	$1.1 \cdot 10^{-14}$	0.05625	6.185
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-14}$	1.00000	6.110	$1.8 \cdot 10^{-16}$	0.02812	6.163
$1/5 \cdot 2^{-7}$	$2.6 \cdot 10^{-16}$	1.00000	6.095	$2.8 \cdot 10^{-18}$	0.01406	6.147
$1/5 \cdot 2^{-8}$	$4.2 \cdot 10^{-18}$	1.00000	6.084	$4.4 \cdot 10^{-20}$	0.00703	6.133
$1/5 \cdot 2^{-9}$	$6.8 \cdot 10^{-20}$	1.00000	6.074	$6.8 \cdot 10^{-22}$	0.00351	6.122
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-21}$	1.00000	6.066	$1.0 \cdot 10^{-23}$	0.00175	6.112
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-23}$	1.00000	6.060	$1.6 \cdot 10^{-25}$	0.00087	6.104
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-25}$	1.00000	-83.217	$2.6 \cdot 10^{-27}$	0.00043	-89.961
$1/5 \cdot 2^{-13}$	3.4	0.99998		3.3	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.8 \cdot 10^{-06}$	1.00000	11.529	$1.8 \cdot 10^{-06}$	1.00000	11.255
$1/5 \cdot 2^{-2}$	$8.1 \cdot 10^{-10}$	1.00000	7.645	$9.9 \cdot 10^{-10}$	1.00000	10.559
$1/5 \cdot 2^{-3}$	$4.9 \cdot 10^{-12}$	0.07500	6.195	$8.0 \cdot 10^{-13}$	1.00000	7.718
$1/5 \cdot 2^{-4}$	$8.0 \cdot 10^{-14}$	0.10000	6.210	$4.5 \cdot 10^{-15}$	0.05000	6.185
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-15}$	0.05000	6.184	$7.2 \cdot 10^{-17}$	0.04375	6.184
$1/5 \cdot 2^{-6}$	$1.9 \cdot 10^{-17}$	0.02500	6.163	$1.1 \cdot 10^{-18}$	0.02187	6.163
$1/5 \cdot 2^{-7}$	$3.0 \cdot 10^{-19}$	0.01250	6.147	$1.7 \cdot 10^{-20}$	0.01093	6.147
$1/5 \cdot 2^{-8}$	$4.8 \cdot 10^{-21}$	0.00625	6.133	$2.7 \cdot 10^{-22}$	0.00546	6.133
$1/5 \cdot 2^{-9}$	$7.5 \cdot 10^{-23}$	0.00312	6.122	$4.3 \cdot 10^{-24}$	0.00273	6.122
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-24}$	0.00156	6.112	$6.7 \cdot 10^{-26}$	0.00136	6.112
$1/5 \cdot 2^{-11}$	$1.8 \cdot 10^{-26}$	0.00078	3.624	$1.0 \cdot 10^{-27}$	0.00068	-0.517
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-27}$	0.99848		$1.6 \cdot 10^{-27}$	0.99848	

Table 4.33: Box scheme, IDeC method on $[0, 1]$ for 3.1c, logarithmic.

The behavior of IDeC for example 3.2a does not exactly fit into the picture we established so far. This can be seen in Tables 4.34 and 4.35. While the order sequence not unexpectedly starts with $O(|\ln(h)|h^2)$, $O(h^2)$, the level of accuracy seems to be increasing gradually until in the fifth iteration it is almost 3. Note that the error constant does not stabilize well for this problem. This irregular behavior occurs even though $t^{\max} \approx 1$. Accordingly, the same effect is observed at a fixed point t, see Tables 4.36 and 4.37. From Table 4.35 it is clear that the observed behavior cannot be attributed to the presence of logarithmic terms in the convergence orders except for the basic solution. At the moment we cannot offer an explanation for these test results.

For the nonlinear example 3.3a, the order sequence $O(|\ln(h)|h^2)$, $O(h^2), \ldots$ is observed, where the level of accuracy surprisingly jumps to $O(h^4)$ in the last step of the iteration, a phenomenon which is not encountered for the first time. See Tables 4.38 and 4.39 for these results.

4.2.3   Asymptotic Error Expansions

For some of the examples with constant coefficient matrix, it is possible to explicitly compute the asymptotic error expansions of the numerical solution obtained with the box scheme using the computer algebra system MAPLE. It turns out that the behavior of the IDeC observed in §4.2.2 can be explained by comparing the length of this error expansion with the break-down of the IDeC iteration.

From the sketch of the convergence proof for the implicit Euler method given in §4.1, it is clear that the length of the error expansions in (4.5) is crucial for the convergence properties of IDeC. No improvement of the convergence order of the numerical solution can be expected when the order of the remainder term r_h is reached. The following results were computed using the computer algebra system MAPLE and support the above line of reasoning.

First, we formally derive the variational equations for the box scheme applied to (4.1). They are obtained by substituting the ansatz (3.8) into the defining equation for the box scheme (3.7), using Taylor expansion of the involved quantities at $t = t_j + \frac{h}{2}$ and equating coefficients of the same powers of h. For our purpose it is sufficient to look at $e_1, \ldots, e_6$.

Note, however, that when a coefficient e_j does not satisfy the smoothness requirements, then all further equations become meaningless.

$$
\begin{aligned}
e_1'(t) - \frac{M(t)}{t}e_1(t) &= D_2 f(t, z(t))e_1(t), \\
e_1(0) &= 0, \\
e_2'(t) - \frac{M(t)}{t}e_2(t) &= D_2 f(t, z(t))e_2(t) + \frac{1}{8}D_2 f(t, z(t))z''(t) \\
&\quad + \frac{1}{2}D_2^2 f(t, z(t))e_1^2(t) - \frac{1}{24}z^{(3)}(t) + \frac{1}{8}\frac{M(t)}{t}z''(t), \\
e_2(0) &= 0,
\end{aligned}
$$

h	δ	$t^{\max}$	p	c	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-2}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-3}$	$5.0 \cdot 10^{-02}$	1.00	1.808	$-3.9 \cdot 10^{+01}$	$1.9 \cdot 10^{-03}$	1.00	1.719	$-1.1 \cdot 10^{+00}$
$1/5 \cdot 2^{-4}$	$1.4 \cdot 10^{-02}$	1.00	1.820	$-4.2 \cdot 10^{+01}$	$5.9 \cdot 10^{-04}$	1.00	1.900	$-2.4 \cdot 10^{+00}$
$1/5 \cdot 2^{-5}$	$4.0 \cdot 10^{-03}$	1.00	1.835	$-4.5 \cdot 10^{+01}$	$1.5 \cdot 10^{-04}$	1.00	1.963	$-3.3 \cdot 10^{+00}$
$1/5 \cdot 2^{-6}$	$1.1 \cdot 10^{-03}$	1.00	1.850	$-4.9 \cdot 10^{+01}$	$4.0 \cdot 10^{-05}$	1.00	1.987	$-3.8 \cdot 10^{+00}$
$1/5 \cdot 2^{-7}$	$3.1 \cdot 10^{-04}$	1.00	1.863	$-5.3 \cdot 10^{+01}$	$1.0 \cdot 10^{-05}$	1.00	1.997	$-4.1 \cdot 10^{+00}$
$1/5 \cdot 2^{-8}$	$8.7 \cdot 10^{-05}$	1.00	1.875	$-5.8 \cdot 10^{+01}$	$2.5 \cdot 10^{-06}$	1.00	2.000	$-4.2 \cdot 10^{+00}$
$1/5 \cdot 2^{-9}$	$2.3 \cdot 10^{-05}$	1.00	1.884	$-6.3 \cdot 10^{+01}$	$6.4 \cdot 10^{-07}$	1.00	2.000	$-4.2 \cdot 10^{+00}$
$1/5 \cdot 2^{-10}$	$6.4 \cdot 10^{-06}$	1.00	1.893	$-6.7 \cdot 10^{+01}$	$1.6 \cdot 10^{-07}$	1.00	2.000	$-4.2 \cdot 10^{+00}$
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-06}$	1.00	1.900	$-7.2 \cdot 10^{+01}$	$4.0 \cdot 10^{-08}$	1.00	2.000	$-4.2 \cdot 10^{+00}$

h	δ	$t^{\max}$	p	c	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-2}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-3}$	$3.0 \cdot 10^{-04}$	1.00	2.011	$-5.0 \cdot 10^{-01}$	$3.5 \cdot 10^{-05}$	1.00	2.257	$-1.4 \cdot 10^{-01}$
$1/5 \cdot 2^{-4}$	$7.5 \cdot 10^{-05}$	1.00	2.068	$-6.4 \cdot 10^{-01}$	$7.4 \cdot 10^{-06}$	1.00	2.351	$-2.2 \cdot 10^{-01}$
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-05}$	1.00	2.078	$-6.8 \cdot 10^{-01}$	$1.4 \cdot 10^{-06}$	1.00	2.345	$-2.1 \cdot 10^{-01}$
$1/5 \cdot 2^{-6}$	$4.2 \cdot 10^{-06}$	1.00	2.066	$-6.3 \cdot 10^{-01}$	$2.8 \cdot 10^{-07}$	1.00	2.289	$-1.5 \cdot 10^{-01}$
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-06}$	1.00	2.049	$-5.6 \cdot 10^{-01}$	$5.8 \cdot 10^{-08}$	1.00	2.216	$-9.7 \cdot 10^{-02}$
$1/5 \cdot 2^{-8}$	$2.4 \cdot 10^{-07}$	1.00	2.033	$-5.0 \cdot 10^{-01}$	$1.2 \cdot 10^{-08}$	1.00	2.147	$-5.9 \cdot 10^{-02}$
$1/5 \cdot 2^{-9}$	$5.9 \cdot 10^{-08}$	1.00	2.020	$-4.6 \cdot 10^{-01}$	$2.8 \cdot 10^{-09}$	1.00	2.093	$-3.8 \cdot 10^{-02}$
$1/5 \cdot 2^{-10}$	$1.4 \cdot 10^{-08}$	1.00	2.012	$-4.3 \cdot 10^{-01}$	$6.6 \cdot 10^{-10}$	1.00	2.056	$-2.8 \cdot 10^{-02}$
$1/5 \cdot 2^{-11}$	$3.6 \cdot 10^{-09}$	1.00	2.007	$-4.1 \cdot 10^{-01}$	$1.6 \cdot 10^{-10}$	1.00	2.032	$-2.2 \cdot 10^{-02}$

h	δ	$t^{\max}$	p	c	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-2}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-3}$	$3.9 \cdot 10^{-06}$	1.00	2.460	$-3.4 \cdot 10^{-02}$	$5.2 \cdot 10^{-07}$	1.00	2.706	$-1.1 \cdot 10^{-02}$
$1/5 \cdot 2^{-4}$	$7.2 \cdot 10^{-07}$	1.00	2.665	$-8.5 \cdot 10^{-02}$	$8.0 \cdot 10^{-08}$	1.00	2.878	$-2.4 \cdot 10^{-02}$
$1/5 \cdot 2^{-5}$	$1.1 \cdot 10^{-07}$	1.00	2.737	$-1.2 \cdot 10^{-01}$	$1.0 \cdot 10^{-08}$	1.00	2.957	$-3.6 \cdot 10^{-02}$
$1/5 \cdot 2^{-6}$	$1.7 \cdot 10^{-08}$	1.00	2.739	$-1.2 \cdot 10^{-01}$	$1.4 \cdot 10^{-09}$	1.00	2.987	$-4.2 \cdot 10^{-02}$
$1/5 \cdot 2^{-7}$	$2.5 \cdot 10^{-09}$	1.00	2.688	$-8.9 \cdot 10^{-02}$	$1.7 \cdot 10^{-10}$	1.00	2.995	$-4.5 \cdot 10^{-02}$
$1/5 \cdot 2^{-8}$	$3.9 \cdot 10^{-10}$	1.00	2.591	$-4.5 \cdot 10^{-02}$	$2.2 \cdot 10^{-11}$	1.00	2.994	$-4.4 \cdot 10^{-02}$
$1/5 \cdot 2^{-9}$	$6.6 \cdot 10^{-11}$	1.00	2.464	$-1.6 \cdot 10^{-02}$	$2.7 \cdot 10^{-12}$	1.00	2.992	$-4.4 \cdot 10^{-02}$
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-11}$	1.00	2.328	$-5.2 \cdot 10^{-03}$	$3.5 \cdot 10^{-13}$	1.00	2.989	$-4.3 \cdot 10^{-02}$
$1/5 \cdot 2^{-11}$	$2.3 \cdot 10^{-12}$	1.00	2.212	$-1.7 \cdot 10^{-03}$	$4.4 \cdot 10^{-14}$	1.00	2.987	$-4.2 \cdot 10^{-02}$

Table 4.34: Box scheme, IDeC method on $[0, 1]$ for 3.2a.

h	δ	$t^{\max}$	p	c	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-2}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-3}$	$5.0 \cdot 10^{-02}$	1.00	2.056	$2.6 \cdot 10^{+01}$	$1.9 \cdot 10^{-03}$	1.00	1.967	$7.4 \cdot 10^{-01}$
$1/5 \cdot 2^{-4}$	$1.4 \cdot 10^{-02}$	1.00	2.032	$2.4 \cdot 10^{+01}$	$5.9 \cdot 10^{-04}$	1.00	2.111	$1.4 \cdot 10^{+00}$
$1/5 \cdot 2^{-5}$	$4.0 \cdot 10^{-03}$	1.00	2.020	$2.2 \cdot 10^{+01}$	$1.5 \cdot 10^{-04}$	1.00	2.148	$1.6 \cdot 10^{+00}$
$1/5 \cdot 2^{-6}$	$1.1 \cdot 10^{-03}$	1.00	2.014	$2.2 \cdot 10^{+01}$	$4.0 \cdot 10^{-05}$	1.00	2.151	$1.7 \cdot 10^{+00}$
$1/5 \cdot 2^{-7}$	$3.1 \cdot 10^{-04}$	1.00	2.010	$2.1 \cdot 10^{+01}$	$1.0 \cdot 10^{-05}$	1.00	2.144	$1.6 \cdot 10^{+00}$
$1/5 \cdot 2^{-8}$	$8.7 \cdot 10^{-05}$	1.00	2.008	$2.1 \cdot 10^{+01}$	$2.5 \cdot 10^{-06}$	1.00	2.133	$1.5 \cdot 10^{+00}$
$1/5 \cdot 2^{-9}$	$2.3 \cdot 10^{-05}$	1.00	2.007	$2.0 \cdot 10^{+01}$	$6.4 \cdot 10^{-07}$	1.00	2.123	$1.4 \cdot 10^{+00}$
$1/5 \cdot 2^{-10}$	$6.4 \cdot 10^{-06}$	1.00	2.005	$2.0 \cdot 10^{+01}$	$1.6 \cdot 10^{-07}$	1.00	2.113	$1.2 \cdot 10^{+00}$
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-06}$	1.00	2.005	$2.0 \cdot 10^{+01}$	$4.0 \cdot 10^{-08}$	1.00	2.105	$1.2 \cdot 10^{+00}$

h	δ	$t^{\max}$	p	c	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-2}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-3}$	$3.0 \cdot 10^{-04}$	1.00	2.260	$3.4 \cdot 10^{-01}$	$3.5 \cdot 10^{-05}$	1.00	2.505	$9.9 \cdot 10^{-02}$
$1/5 \cdot 2^{-4}$	$7.5 \cdot 10^{-05}$	1.00	2.279	$3.7 \cdot 10^{-01}$	$7.4 \cdot 10^{-06}$	1.00	2.563	$1.2 \cdot 10^{-01}$
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-05}$	1.00	2.263	$3.4 \cdot 10^{-01}$	$1.4 \cdot 10^{-06}$	1.00	2.529	$1.0 \cdot 10^{-01}$
$1/5 \cdot 2^{-6}$	$4.2 \cdot 10^{-06}$	1.00	2.230	$2.8 \cdot 10^{-01}$	$2.8 \cdot 10^{-07}$	1.00	2.453	$6.9 \cdot 10^{-02}$
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-06}$	1.00	2.196	$2.2 \cdot 10^{-01}$	$5.8 \cdot 10^{-08}$	1.00	2.363	$3.8 \cdot 10^{-02}$
$1/5 \cdot 2^{-8}$	$2.4 \cdot 10^{-07}$	1.00	2.166	$1.8 \cdot 10^{-01}$	$1.2 \cdot 10^{-08}$	1.00	2.281	$2.1 \cdot 10^{-02}$
$1/5 \cdot 2^{-9}$	$5.9 \cdot 10^{-08}$	1.00	2.142	$1.5 \cdot 10^{-01}$	$2.8 \cdot 10^{-09}$	1.00	2.215	$1.2 \cdot 10^{-02}$
$1/5 \cdot 2^{-10}$	$1.4 \cdot 10^{-08}$	1.00	2.125	$1.3 \cdot 10^{-01}$	$6.6 \cdot 10^{-10}$	1.00	2.169	$8.6 \cdot 10^{-03}$
$1/5 \cdot 2^{-11}$	$3.6 \cdot 10^{-09}$	1.00	2.111	$1.1 \cdot 10^{-01}$	$1.6 \cdot 10^{-10}$	1.00	2.137	$6.4 \cdot 10^{-03}$

h	δ	$t^{\max}$	p	c	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-2}$	0.0	0.00			0.0	0.00		
$1/5 \cdot 2^{-3}$	$3.9 \cdot 10^{-06}$	1.00	2.709	$2.3 \cdot 10^{-02}$	$5.2 \cdot 10^{-07}$	1.00	2.954	$7.6 \cdot 10^{-03}$
$1/5 \cdot 2^{-4}$	$7.2 \cdot 10^{-07}$	1.00	2.877	$4.9 \cdot 10^{-02}$	$8.0 \cdot 10^{-08}$	1.00	3.090	$1.3 \cdot 10^{-02}$
$1/5 \cdot 2^{-5}$	$1.1 \cdot 10^{-07}$	1.00	2.922	$6.2 \cdot 10^{-02}$	$1.0 \cdot 10^{-08}$	1.00	3.142	$1.8 \cdot 10^{-02}$
$1/5 \cdot 2^{-6}$	$1.7 \cdot 10^{-08}$	1.00	2.902	$5.5 \cdot 10^{-02}$	$1.4 \cdot 10^{-09}$	1.00	3.150	$1.9 \cdot 10^{-02}$
$1/5 \cdot 2^{-7}$	$2.5 \cdot 10^{-09}$	1.00	2.835	$3.5 \cdot 10^{-02}$	$1.7 \cdot 10^{-10}$	1.00	3.142	$1.8 \cdot 10^{-02}$
$1/5 \cdot 2^{-8}$	$3.9 \cdot 10^{-10}$	1.00	2.725	$1.6 \cdot 10^{-02}$	$2.2 \cdot 10^{-11}$	1.00	3.128	$1.6 \cdot 10^{-02}$
$1/5 \cdot 2^{-9}$	$6.6 \cdot 10^{-11}$	1.00	2.586	$5.4 \cdot 10^{-03}$	$2.7 \cdot 10^{-12}$	1.00	3.114	$1.4 \cdot 10^{-02}$
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-11}$	1.00	2.441	$1.5 \cdot 10^{-03}$	$3.5 \cdot 10^{-13}$	1.00	3.102	$1.3 \cdot 10^{-02}$
$1/5 \cdot 2^{-11}$	$2.3 \cdot 10^{-12}$	1.00	2.316	$5.0 \cdot 10^{-04}$	$4.4 \cdot 10^{-14}$	1.00	3.092	$1.2 \cdot 10^{-02}$

Table 4.35: Box scheme, IDeC method on $[0, 1]$ for 3.2a, logarithmic.

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$5.2 \cdot 10^{-04}$	1.775	$2.7 \cdot 10^{-05}$	1.893
$1/5 \cdot 2^{-5}$	$1.5 \cdot 10^{-04}$	1.800	$7.3 \cdot 10^{-06}$	1.959
$1/5 \cdot 2^{-6}$	$4.4 \cdot 10^{-05}$	1.822	$1.8 \cdot 10^{-06}$	1.985
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-05}$	1.840	$4.7 \cdot 10^{-07}$	1.996
$1/5 \cdot 2^{-8}$	$3.4 \cdot 10^{-06}$	1.856	$1.1 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-9}$	$9.6 \cdot 10^{-07}$	1.869	$2.9 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-10}$	$2.6 \cdot 10^{-07}$	1.879	$7.4 \cdot 10^{-09}$	2.000
$1/5 \cdot 2^{-11}$	$7.1 \cdot 10^{-08}$	1.889	$1.8 \cdot 10^{-09}$	2.000
$1/5 \cdot 2^{-12}$	$1.9 \cdot 10^{-08}$	1.896	$4.6 \cdot 10^{-10}$	2.000
$1/5 \cdot 2^{-13}$	$5.1 \cdot 10^{-09}$	1.903	$1.1 \cdot 10^{-10}$	2.000
$1/5 \cdot 2^{-14}$	$1.3 \cdot 10^{-09}$	1.909	$2.9 \cdot 10^{-11}$	2.000
$1/5 \cdot 2^{-15}$	$3.6 \cdot 10^{-10}$	1.915	$7.2 \cdot 10^{-12}$	-30.090
$1/5 \cdot 2^{-16}$	$9.7 \cdot 10^{-11}$	1.919	$8.2 \cdot 10^{-03}$	34.087
$1/5 \cdot 2^{-17}$	$2.5 \cdot 10^{-11}$		$4.5 \cdot 10^{-13}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$3.3 \cdot 10^{-06}$	2.028	$3.2 \cdot 10^{-07}$	2.315
$1/5 \cdot 2^{-5}$	$8.1 \cdot 10^{-07}$	2.059	$6.4 \cdot 10^{-08}$	2.321
$1/5 \cdot 2^{-6}$	$1.9 \cdot 10^{-07}$	2.057	$1.2 \cdot 10^{-08}$	2.273
$1/5 \cdot 2^{-7}$	$4.6 \cdot 10^{-08}$	2.044	$2.6 \cdot 10^{-09}$	2.206
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-08}$	2.030	$5.7 \cdot 10^{-10}$	2.141
$1/5 \cdot 2^{-9}$	$2.7 \cdot 10^{-09}$	2.019	$1.3 \cdot 10^{-10}$	2.090
$1/5 \cdot 2^{-10}$	$6.8 \cdot 10^{-10}$	2.011	$3.0 \cdot 10^{-11}$	2.054
$1/5 \cdot 2^{-11}$	$1.6 \cdot 10^{-10}$	2.007	$7.4 \cdot 10^{-12}$	2.031
$1/5 \cdot 2^{-12}$	$4.2 \cdot 10^{-11}$	2.004	$1.8 \cdot 10^{-12}$	2.017
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-11}$	2.002	$4.4 \cdot 10^{-13}$	2.010
$1/5 \cdot 2^{-14}$	$2.6 \cdot 10^{-12}$	2.001	$1.1 \cdot 10^{-13}$	2.005
$1/5 \cdot 2^{-15}$	$6.5 \cdot 10^{-13}$	-33.503	$2.7 \cdot 10^{-14}$	-38.031
$1/5 \cdot 2^{-16}$	$7.9 \cdot 10^{-03}$		$7.7 \cdot 10^{-03}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$3.2 \cdot 10^{-08}$	2.652	$3.5 \cdot 10^{-09}$	2.842
$1/5 \cdot 2^{-5}$	$5.1 \cdot 10^{-09}$	2.730	$4.9 \cdot 10^{-10}$	2.942
$1/5 \cdot 2^{-6}$	$7.7 \cdot 10^{-10}$	2.733	$6.4 \cdot 10^{-11}$	2.980
$1/5 \cdot 2^{-7}$	$1.1 \cdot 10^{-10}$	2.682	$8.1 \cdot 10^{-12}$	2.991
$1/5 \cdot 2^{-8}$	$1.8 \cdot 10^{-11}$	2.586	$1.0 \cdot 10^{-12}$	2.993
$1/5 \cdot 2^{-9}$	$3.0 \cdot 10^{-12}$	2.458	$1.2 \cdot 10^{-13}$	2.991
$1/5 \cdot 2^{-10}$	$5.5 \cdot 10^{-13}$	2.324	$1.6 \cdot 10^{-14}$	2.989
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-13}$	2.208	$2.0 \cdot 10^{-15}$	2.987
$1/5 \cdot 2^{-12}$	$2.3 \cdot 10^{-14}$	2.124	$2.5 \cdot 10^{-16}$	2.986
$1/5 \cdot 2^{-13}$	$5.4 \cdot 10^{-15}$	2.070	$3.2 \cdot 10^{-17}$	2.985
$1/5 \cdot 2^{-14}$	$1.3 \cdot 10^{-15}$	2.038	$4.1 \cdot 10^{-18}$	2.985
$1/5 \cdot 2^{-15}$	$3.1 \cdot 10^{-16}$		$5.1 \cdot 10^{-19}$	

Table 4.36: Box scheme, IDeC method at $t = 1/8$ for 3.2a.

h	δ	p	δ	p
$1/5\cdot 2^{-1}$	0.0		0.0	
$1/5\cdot 2^{-2}$	0.0		0.0	
$1/5\cdot 2^{-3}$	$5.0\cdot 10^{-02}$	1.808	$1.9\cdot 10^{-03}$	1.719
$1/5\cdot 2^{-4}$	$1.4\cdot 10^{-02}$	1.820	$5.9\cdot 10^{-04}$	1.900
$1/5\cdot 2^{-5}$	$4.0\cdot 10^{-03}$	1.835	$1.5\cdot 10^{-04}$	1.963
$1/5\cdot 2^{-6}$	$1.1\cdot 10^{-03}$	1.850	$4.0\cdot 10^{-05}$	1.987
$1/5\cdot 2^{-7}$	$3.1\cdot 10^{-04}$	1.863	$1.0\cdot 10^{-05}$	1.997
$1/5\cdot 2^{-8}$	$8.7\cdot 10^{-05}$	1.875	$2.5\cdot 10^{-06}$	2.000
$1/5\cdot 2^{-9}$	$2.3\cdot 10^{-05}$	1.884	$6.4\cdot 10^{-07}$	2.000
$1/5\cdot 2^{-10}$	$6.4\cdot 10^{-06}$	1.893	$1.6\cdot 10^{-07}$	2.000
$1/5\cdot 2^{-11}$	$1.7\cdot 10^{-06}$	1.900	$4.0\cdot 10^{-08}$	2.000
$1/5\cdot 2^{-12}$	$4.6\cdot 10^{-07}$	1.907	$1.0\cdot 10^{-08}$	-27.421
$1/5\cdot 2^{-13}$	$1.2\cdot 10^{-07}$	1.912	1.7	31.419
$1/5\cdot 2^{-14}$	$3.2\cdot 10^{-08}$		$6.2\cdot 10^{-10}$	

h	δ	p	δ	p
$1/5\cdot 2^{-1}$	0.0		0.0	
$1/5\cdot 2^{-2}$	0.0		0.0	
$1/5\cdot 2^{-3}$	$3.0\cdot 10^{-04}$	2.011	$3.5\cdot 10^{-05}$	2.257
$1/5\cdot 2^{-4}$	$7.5\cdot 10^{-05}$	2.068	$7.4\cdot 10^{-06}$	2.351
$1/5\cdot 2^{-5}$	$1.7\cdot 10^{-05}$	2.078	$1.4\cdot 10^{-06}$	2.345
$1/5\cdot 2^{-6}$	$4.2\cdot 10^{-06}$	2.066	$2.8\cdot 10^{-07}$	2.289
$1/5\cdot 2^{-7}$	$1.0\cdot 10^{-06}$	2.049	$5.8\cdot 10^{-08}$	2.216
$1/5\cdot 2^{-8}$	$2.4\cdot 10^{-07}$	2.033	$1.2\cdot 10^{-08}$	2.147
$1/5\cdot 2^{-9}$	$5.9\cdot 10^{-08}$	2.020	$2.8\cdot 10^{-09}$	2.093
$1/5\cdot 2^{-10}$	$1.4\cdot 10^{-08}$	2.012	$6.6\cdot 10^{-10}$	2.056
$1/5\cdot 2^{-11}$	$3.6\cdot 10^{-09}$	2.007	$1.6\cdot 10^{-10}$	2.032
$1/5\cdot 2^{-12}$	$9.0\cdot 10^{-10}$	-30.827	$3.9\cdot 10^{-11}$	-35.328
$1/5\cdot 2^{-13}$	1.7		1.6	

h	δ	p	δ	p
$1/5\cdot 2^{-1}$	0.0		0.0	
$1/5\cdot 2^{-2}$	0.0		0.0	
$1/5\cdot 2^{-3}$	$3.9\cdot 10^{-06}$	2.460	$5.2\cdot 10^{-07}$	2.706
$1/5\cdot 2^{-4}$	$7.2\cdot 10^{-07}$	2.665	$8.0\cdot 10^{-08}$	2.878
$1/5\cdot 2^{-5}$	$1.1\cdot 10^{-07}$	2.737	$1.0\cdot 10^{-08}$	2.957
$1/5\cdot 2^{-6}$	$1.7\cdot 10^{-08}$	2.739	$1.4\cdot 10^{-09}$	2.987
$1/5\cdot 2^{-7}$	$2.5\cdot 10^{-09}$	2.688	$1.7\cdot 10^{-10}$	2.995
$1/5\cdot 2^{-8}$	$3.9\cdot 10^{-10}$	2.591	$2.2\cdot 10^{-11}$	2.994
$1/5\cdot 2^{-9}$	$6.6\cdot 10^{-11}$	2.464	$2.7\cdot 10^{-12}$	2.992
$1/5\cdot 2^{-10}$	$1.1\cdot 10^{-11}$	2.328	$3.5\cdot 10^{-13}$	2.989
$1/5\cdot 2^{-11}$	$2.3\cdot 10^{-12}$	2.212	$4.4\cdot 10^{-14}$	2.987
$1/5\cdot 2^{-12}$	$5.1\cdot 10^{-13}$		$5.5\cdot 10^{-15}$	

Table 4.37: Box scheme, IDeC method at $t = 1$ for 3.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$7.8 \cdot 10^{-03}$	0.80000	1.685	$8.2 \cdot 10^{-04}$	0.70000	2.317
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-03}$	0.80000	1.750	$1.6 \cdot 10^{-04}$	0.70000	2.135
$1/5 \cdot 2^{-3}$	$7.2 \cdot 10^{-04}$	0.80000	1.789	$3.7 \cdot 10^{-05}$	0.75000	2.026
$1/5 \cdot 2^{-4}$	$2.0 \cdot 10^{-04}$	0.81250	1.816	$9.2 \cdot 10^{-06}$	0.77500	2.003
$1/5 \cdot 2^{-5}$	$5.9 \cdot 10^{-05}$	0.80625	1.837	$2.2 \cdot 10^{-06}$	0.78125	2.000
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-05}$	0.80312	1.854	$5.7 \cdot 10^{-07}$	0.78437	1.999
$1/5 \cdot 2^{-7}$	$4.6 \cdot 10^{-06}$	0.80156	1.867	$1.4 \cdot 10^{-07}$	0.78593	1.999
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-06}$	0.80000	1.878	$3.5 \cdot 10^{-08}$	0.78593	1.999
$1/5 \cdot 2^{-9}$	$3.4 \cdot 10^{-07}$	0.79843	1.888	$8.9 \cdot 10^{-09}$	0.78593	1.999
$1/5 \cdot 2^{-10}$	$9.2 \cdot 10^{-08}$	0.79765	1.896	$2.2 \cdot 10^{-09}$	0.78574	1.999
$1/5 \cdot 2^{-11}$	$2.4 \cdot 10^{-08}$	0.79687	1.903	$5.6 \cdot 10^{-10}$	0.78574	1.999
$1/5 \cdot 2^{-12}$	$6.6 \cdot 10^{-09}$	0.79614	1.909	$1.4 \cdot 10^{-10}$	0.78579	-30.052
$1/5 \cdot 2^{-13}$	$1.7 \cdot 10^{-09}$	0.79553	1.914	$1.5 \cdot 10^{-01}$	0.99998	34.049
$1/5 \cdot 2^{-14}$	$4.6 \cdot 10^{-10}$	0.79498		$8.7 \cdot 10^{-12}$	0.78579	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$7.5 \cdot 10^{-05}$	0.60000	2.196	$9.5 \cdot 10^{-06}$	0.90000	2.473
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-05}$	0.65000	2.116	$1.7 \cdot 10^{-06}$	0.65000	2.769
$1/5 \cdot 2^{-3}$	$3.8 \cdot 10^{-06}$	0.07500	2.012	$2.5 \cdot 10^{-07}$	0.10000	2.202
$1/5 \cdot 2^{-4}$	$9.4 \cdot 10^{-07}$	0.03750	2.003	$5.4 \cdot 10^{-08}$	0.05000	2.058
$1/5 \cdot 2^{-5}$	$2.3 \cdot 10^{-07}$	0.01875	1.996	$1.3 \cdot 10^{-08}$	0.02500	2.014
$1/5 \cdot 2^{-6}$	$5.8 \cdot 10^{-08}$	0.02500	1.998	$3.2 \cdot 10^{-09}$	0.02187	2.003
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-08}$	0.01250	1.999	$8.1 \cdot 10^{-10}$	0.01093	2.000
$1/5 \cdot 2^{-8}$	$3.6 \cdot 10^{-09}$	0.00625	1.999	$2.0 \cdot 10^{-10}$	0.00546	2.000
$1/5 \cdot 2^{-9}$	$9.2 \cdot 10^{-10}$	0.00312	1.999	$5.0 \cdot 10^{-11}$	0.00273	2.000
$1/5 \cdot 2^{-10}$	$2.3 \cdot 10^{-10}$	0.00156	1.999	$1.2 \cdot 10^{-11}$	0.00136	2.000
$1/5 \cdot 2^{-11}$	$5.7 \cdot 10^{-11}$	0.00078	1.999	$3.1 \cdot 10^{-12}$	0.00068	2.000
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-11}$	0.00039	-33.277	$7.9 \cdot 10^{-13}$	0.00034	-37.429
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-01}$	0.99998		$1.4 \cdot 10^{-01}$	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.1 \cdot 10^{-05}$	0.90000	6.365	$2.3 \cdot 10^{-05}$	0.90000	9.045
$1/5 \cdot 2^{-2}$	$2.5 \cdot 10^{-07}$	0.75000	3.944	$4.5 \cdot 10^{-08}$	0.80000	4.684
$1/5 \cdot 2^{-3}$	$1.6 \cdot 10^{-08}$	0.12500	3.167	$1.7 \cdot 10^{-09}$	0.75000	4.011
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-09}$	0.07500	2.582	$1.0 \cdot 10^{-10}$	0.06250	3.968
$1/5 \cdot 2^{-5}$	$3.0 \cdot 10^{-10}$	0.03750	2.195	$6.9 \cdot 10^{-12}$	0.05625	3.990
$1/5 \cdot 2^{-6}$	$6.7 \cdot 10^{-11}$	0.01875	2.053	$4.3 \cdot 10^{-13}$	0.02812	3.997
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-11}$	0.00937	2.013	$2.7 \cdot 10^{-14}$	0.01406	3.999
$1/5 \cdot 2^{-8}$	$4.0 \cdot 10^{-12}$	0.00468	2.003	$1.7 \cdot 10^{-15}$	0.00703	3.999
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-12}$	0.00234	2.000	$1.0 \cdot 10^{-16}$	0.00351	3.999
$1/5 \cdot 2^{-10}$	$2.5 \cdot 10^{-13}$	0.00117	2.000	$6.7 \cdot 10^{-18}$	0.00175	3.999
$1/5 \cdot 2^{-11}$	$6.2 \cdot 10^{-14}$	0.00058	2.000	$4.1 \cdot 10^{-19}$	0.00087	3.999
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-14}$	0.00029		$2.6 \cdot 10^{-20}$	0.00043	

Table 4.38: Box scheme, IDeC method on $[0, 1]$ for 3.3a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$7.8 \cdot 10^{-03}$	0.80000	2.065	$8.2 \cdot 10^{-04}$	0.70000	2.696
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-03}$	0.80000	2.050	$1.6 \cdot 10^{-04}$	0.70000	2.435
$1/5 \cdot 2^{-3}$	$7.2 \cdot 10^{-04}$	0.80000	2.037	$3.7 \cdot 10^{-05}$	0.75000	2.274
$1/5 \cdot 2^{-4}$	$2.0 \cdot 10^{-04}$	0.81250	2.028	$9.2 \cdot 10^{-06}$	0.77500	2.215
$1/5 \cdot 2^{-5}$	$5.9 \cdot 10^{-05}$	0.80625	2.022	$2.2 \cdot 10^{-06}$	0.78125	2.184
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-05}$	0.80312	2.017	$5.7 \cdot 10^{-07}$	0.78437	2.163
$1/5 \cdot 2^{-7}$	$4.6 \cdot 10^{-06}$	0.80156	2.014	$1.4 \cdot 10^{-07}$	0.78593	2.146
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-06}$	0.80000	2.012	$3.5 \cdot 10^{-08}$	0.78593	2.133
$1/5 \cdot 2^{-9}$	$3.4 \cdot 10^{-07}$	0.79843	2.010	$8.9 \cdot 10^{-09}$	0.78593	2.122
$1/5 \cdot 2^{-10}$	$9.2 \cdot 10^{-08}$	0.79765	2.008	$2.2 \cdot 10^{-09}$	0.78574	2.112
$1/5 \cdot 2^{-11}$	$2.4 \cdot 10^{-08}$	0.79687	2.007	$5.6 \cdot 10^{-10}$	0.78574	2.104
$1/5 \cdot 2^{-12}$	$6.6 \cdot 10^{-09}$	0.79614	2.006	$1.4 \cdot 10^{-10}$	0.78579	-29.955
$1/5 \cdot 2^{-13}$	$1.7 \cdot 10^{-09}$	0.79553	2.005	$1.5 \cdot 10^{-01}$	0.99998	34.140
$1/5 \cdot 2^{-14}$	$4.6 \cdot 10^{-10}$	0.79498		$8.7 \cdot 10^{-12}$	0.78579	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$7.5 \cdot 10^{-05}$	0.60000	2.575	$9.5 \cdot 10^{-06}$	0.90000	2.852
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-05}$	0.65000	2.416	$1.7 \cdot 10^{-06}$	0.65000	3.069
$1/5 \cdot 2^{-3}$	$3.8 \cdot 10^{-06}$	0.07500	2.260	$2.5 \cdot 10^{-07}$	0.10000	2.451
$1/5 \cdot 2^{-4}$	$9.4 \cdot 10^{-07}$	0.03750	2.215	$5.4 \cdot 10^{-08}$	0.05000	2.270
$1/5 \cdot 2^{-5}$	$2.3 \cdot 10^{-07}$	0.01875	2.181	$1.3 \cdot 10^{-08}$	0.02500	2.199
$1/5 \cdot 2^{-6}$	$5.8 \cdot 10^{-08}$	0.02500	2.162	$3.2 \cdot 10^{-09}$	0.02187	2.166
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-08}$	0.01250	2.146	$8.1 \cdot 10^{-10}$	0.01093	2.147
$1/5 \cdot 2^{-8}$	$3.6 \cdot 10^{-09}$	0.00625	2.133	$2.0 \cdot 10^{-10}$	0.00546	2.133
$1/5 \cdot 2^{-9}$	$9.2 \cdot 10^{-10}$	0.00312	2.122	$5.0 \cdot 10^{-11}$	0.00273	2.122
$1/5 \cdot 2^{-10}$	$2.3 \cdot 10^{-10}$	0.00156	2.112	$1.2 \cdot 10^{-11}$	0.00136	2.112
$1/5 \cdot 2^{-11}$	$5.7 \cdot 10^{-11}$	0.00078	2.104	$3.1 \cdot 10^{-12}$	0.00068	2.104
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-11}$	0.00039	-33.179	$7.9 \cdot 10^{-13}$	0.00034	-37.331
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-01}$	0.99998		$1.4 \cdot 10^{-01}$	0.99998	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.1 \cdot 10^{-05}$	0.90000	6.745	$2.3 \cdot 10^{-05}$	0.90000	9.425
$1/5 \cdot 2^{-2}$	$2.5 \cdot 10^{-07}$	0.75000	4.245	$4.5 \cdot 10^{-08}$	0.80000	4.985
$1/5 \cdot 2^{-3}$	$1.6 \cdot 10^{-08}$	0.12500	3.415	$1.7 \cdot 10^{-09}$	0.75000	4.259
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-09}$	0.07500	2.794	$1.0 \cdot 10^{-10}$	0.06250	4.180
$1/5 \cdot 2^{-5}$	$3.0 \cdot 10^{-10}$	0.03750	2.380	$6.9 \cdot 10^{-12}$	0.05625	4.174
$1/5 \cdot 2^{-6}$	$6.7 \cdot 10^{-11}$	0.01875	2.217	$4.3 \cdot 10^{-13}$	0.02812	4.161
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-11}$	0.00937	2.160	$2.7 \cdot 10^{-14}$	0.01406	4.146
$1/5 \cdot 2^{-8}$	$4.0 \cdot 10^{-12}$	0.00468	2.136	$1.7 \cdot 10^{-15}$	0.00703	4.133
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-12}$	0.00234	2.122	$1.0 \cdot 10^{-16}$	0.00351	4.122
$1/5 \cdot 2^{-10}$	$2.5 \cdot 10^{-13}$	0.00117	2.112	$6.7 \cdot 10^{-18}$	0.00175	4.112
$1/5 \cdot 2^{-11}$	$6.2 \cdot 10^{-14}$	0.00058	2.104	$4.1 \cdot 10^{-19}$	0.00087	4.104
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-14}$	0.00029		$2.6 \cdot 10^{-20}$	0.00043	

Table 4.39: Box scheme, IDeC method on $[0, 1]$ for 3.3a, logarithmic.

$$e_3'(t) - \frac{M(t)}{t}e_3(t) = D_2f(t,z(t))e_3(t) + \frac{1}{8}D_2f(t,z(t))e_1''(t)$$

$$+ D_2^2f(t,z(t))(e_1(t)e_2(t) + \frac{1}{8}e_1(t)z''(t))$$

$$+ \frac{1}{6}D_2^3f(t,z(t))e_1^3(t) - \frac{1}{24}e_1^{(3)}(t) + \frac{1}{8}\frac{M(t)}{t}e_1''(t),$$

$$e_3(0) = 0,$$

$$e_4'(t) - \frac{M(t)}{t}e_4(t) = D_2f(t,z(t))e_4(t) + D_2f(t,z(t))\left(\frac{1}{8}e_2''(t) + \frac{1}{384}z^{(4)}(t)\right)$$

$$+ D_2^2f(t,z(t))\left(\frac{1}{128}(z'')^2(t) + \frac{1}{8}e_2(t)z''(t)\right.$$

$$\left. + \frac{1}{2}e_2^2(t) + \frac{1}{8}e_1(t)e_1''(t) + e_1(t)e_3(t)\right)$$

$$+ \frac{1}{2}D_2^3f(t,z(t))\left(e_1^2(t)e_2(t) + \frac{1}{8}e_1^2(t)z''(t)\right)$$

$$+ \frac{1}{24}D_2^4f(t,z(t))e_1^4(t) - \frac{1}{24}e_2^{(3)}(t) - \frac{1}{1920}z^{(5)}(t)$$

$$+ \frac{M(t)}{t}\left(\frac{1}{8}e_2''(t) + \frac{1}{384}z^{(4)}(t)\right),$$

$$e_4(0) = 0,$$

$$e_5'(t) - \frac{M(t)}{t}e_5(t) = D_2f(t,z(t))e_5(t) + D_2f(t,z(t))\left(\frac{1}{8}e_3''(t) + \frac{1}{384}e_1^{(4)}(t)\right)$$

$$+ D_2^2f(t,z(t))\left(e_2(t)e_3(t) + \frac{1}{8}(e_2(t)e_1''(t) + e_3(t)z''(t))\right.$$

$$\left. + \frac{1}{64}e_1''(t)z''(t) + e_1(t)e_4(t) + \frac{1}{8}e_1(t)e_2''(t) + \frac{1}{384}e_1(t)z^{(4)}(t)\right)$$

$$+ \frac{1}{6}D_2^3f(t,z(t))\left(\frac{3}{8}e_1^2(t)e_1''(t) + 3e_1^2(t)e_3(t)\right.$$

$$\left. + \frac{3}{4}e_1(t)e_2(t)z''(t) + \frac{3}{64}e_1(t)(z'')^2(t)\right)$$

$$+ \frac{1}{6}D_2^4f(t,z(t))\left(e_1^3(t)e_2(t) + \frac{1}{8}e_1^3(t)z''(t)\right)$$

$$+ \frac{1}{120}D_2^5f(t,z(t))e_1^5(t) - \frac{1}{24}e_3^{(3)}(t)$$

$$- \frac{1}{1920}e_1^{(5)}(t) + \frac{M(t)}{t}\left(\frac{1}{8}e_3''(t) + \frac{1}{384}e_1^{(4)}(t)\right),$$

$$e_5(0) = 0,$$

$$e_6'(t) - \frac{M(t)}{t}e_6(t) = D_2f(t,z(t))e_6(t) + D_2f(t,z(t))\left(\frac{1}{8}e_4''(t) + \frac{1}{384}e_2^{(4)}(t)\right.$$

$$+ \frac{1}{46080} z^{(6)}(t) \Bigg) + \frac{1}{2} D_2^2 f(t, z(t)) \left(\frac{1}{4} e_1(t) e_3''(t) \right.$$

$$+ \frac{1}{192} e_1(t) e_1^{(4)}(t) + 2 e_1(t) e_5(t) + 2 e_2(t) e_4(t)$$

$$+ \frac{1}{4} e_4(t) z''(t) + \frac{1}{4} e_2(t) e_2''(t) + \frac{1}{32} e_2''(t) z''(t)$$

$$+ \frac{1}{192} e_2(t) z^{(4)}(t) + \frac{1}{1536} z''(t) z^{(4)}(t) + e_3^2(t)$$

$$+ \frac{1}{4} e_1(t) e_3(t) + \frac{1}{64} (e_1'')^2(t) \Bigg) + \frac{1}{6} D_2^3 f(t, z(t)) \Bigg($$

$$6 e_1(t) e_2(t) e_3(t) + \frac{3}{4} e_1(t) e_3(t) z''(t) + \frac{3}{4} e_1(t) e_2(t) e_1''(t)$$

$$+ \frac{3}{32} e_1(t) e_1''(t) z''(t) + 3 e_1^2(t) e_4(t) + \frac{3}{8} e_1^2(t) e_2''(t)$$

$$+ \frac{3}{384} e_1^2(t) z^{(4)}(t) + e_2^3(t) + \frac{3}{8} e_2^2(t) z''(t) + \frac{3}{64} e_2(t)$$

$$\left(z'' \right)^2 (t) + \frac{1}{256} \left(z'' \right)^3 (t) \Bigg)$$

$$+ \frac{1}{24} D_2^4 f(t, z(t)) \left(4 e_1^3(t) e_3(t) + \frac{1}{2} e_1^3(t) e_1''(t) \right.$$

$$+ 6 e_1^2(t) e_2^2(t) + \frac{3}{2} e_1^2(t) e_2(t) z''(t) + \frac{3}{32} e_1^2(t) \left(z'' \right)^2 (t) \Bigg)$$

$$+ \frac{1}{24} D_2^5 f(t, z(t)) \left(e_1^4(t) e_2(t) + \frac{1}{8} e_1^4(t) z''(t) \right)$$

$$+ \frac{1}{720} D_2^6 f(t, z(t)) e_1^6(t) - \frac{1}{24} e_4^{(3)}(t) - \frac{1}{1920} e_2^{(5)}(t)$$

$$- \frac{1}{322560} z^{(7)}(t) + \frac{M(t)}{t} \left(\frac{1}{8} e_4''(t) + \frac{1}{384} e_2^{(4)}(t) \right.$$

$$+ \frac{1}{46080} z^{(6)}(t) \Bigg),$$

$$e_6(0) = 0.$$

In the above equations, $D_2^p f(t, z) e_1 \cdots e_p$ denotes the p-th Fréchet derivative with respect to the second argument, which is applied to $e_1, \ldots, e_p$ successively.

The unique solution of the first variational equation is $e_1(t) = 0$. Thus, it turns out that the equations for e_3 and e_5 also reduce to a homogeneous initial value problem, which has only the trivial solution (when the equation is meaningful). Thus, if e_j are sufficiently smooth, we expect an asymptotic error expansion in even powers of h, cf. (3.9). The following computations contradict this expectation, however, which also explains the failure of IDeC observed in §4.2.2.

First, we look at example 1.1a. Since this equation has a very simple struc-

ture (the coefficient matrix is constant), it is easy to compute the variational equations explicitly and solve them analytically. The solutions become quite complicated quickly, and therefore only the leading terms of the series expansions are given. They read

$$
\begin{aligned}
e_1(t) &= (0,0), \\
e_2(t) &= \left(-\frac{c_1}{5}t^{-5} - \left(\frac{c_2}{3}+1\right)t^{-3} - 1 - 4t + O(t^2), \dots \right. \\
&\qquad \left. c_1 t^{-5} + (c_2+3)t^{-3} - 2 - 20t + O(t^2) \right),
\end{aligned}
$$

and there is no choice of the constants in the general solution for e_2 which would yield a continuous solution satisfying the homogeneous initial conditions. Thus, the asymptotic error expansion breaks down and IDeC does not improve the convergence orders of the iterates, cf. Table 4.18. The basic convergence order $O(h^2)$ is preserved nonetheless. These facts do not contradict each other, however. The result only means that the error does not have the desired structure but only the expected asymptotic properties. Note that the error expansion explains only the behavior of the maximal error on $[0,1]$, but not the results displayed in Tables 4.19 and 4.20.

For problem 2.1a, the situation is similar. The general solutions of the variational equations satisfy

$$
\begin{aligned}
e_1(t) &= (0,0), \\
e_2(t) &= \left(c_2 t^{-1} + \left(-\frac{9}{8}+c_1\right) + \frac{45}{16}t^2 + O(t^4), \dots \right. \\
&\qquad \left. -c_2 t^{-1} + \frac{9}{4} - \frac{45}{4}t^2 + O(t^4) \right).
\end{aligned}
$$

Again, the error expansion breaks down and an order sequence $O(h^2)$, $O(h^2), \dots$ is observed, cf. Table 4.24.

For example 3.1a, an order reduction for the basic solution down to $O(|\ln(h)|h^2)$ was observed in Table 4.27. Thus, we cannot expect an asymptotic error expansion to exist. Indeed[10],

$$
\begin{aligned}
e_1(t) &= (0,0), \\
e_2(t) &= \left(-\frac{3}{2}\ln(t) + c_2\ln(t) - \frac{3}{2}\gamma - \frac{3}{2}\ln(3) + c_1 - \frac{3}{8} + \frac{81}{16}t^2 + O(t^4), \dots \right. \\
&\qquad \left. \left(\frac{3}{4}+c_2\right) - \frac{27}{4}t^2 + O(t^4) \right).
\end{aligned}
$$

The last examples where the asymptotic error expansion was computed explicitly are 3.1b and 3.1c.

---

[10]γ denotes *Euler's constant*, $\gamma = \lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{1}{i} - \ln(n)\right)$.

For 3.1b, again an order reduction of the basic solution is observed (Table 4.30, and, as expected, no solution of the variational equation for e_2 exists:

$$
\begin{aligned}
e_1(t) &= (0,0), \\
e_2(t) &= \left(\frac{1}{3}\gamma + \frac{1}{3}\ln(t) + c_2 \ln(t) + \frac{7}{12} + c_1 + \frac{5}{4}t + O(t^2), \ldots \right. \\
&\qquad \left. -\frac{1}{6} + c_2 - \frac{3}{4}t + O(t^2) \right)
\end{aligned}
$$

The situation is different for example 3.1c, however. The order sequence that was observed in Table 4.32 is $O(h^2)$, $O(h^4)$, $O(|\ln(t)|h^6)$, $O(h^6)$, …. The corresponding solutions of the variational equations exist up to e_5, accordingly. The apparent reason for this fact is the special structure of the solution, $z(t) = t^5 e^t$, which satisfies $z(0) = z'(0) = \cdots = z^{(4)}(0) = 0$. Thus,

$$
\begin{aligned}
e_1(t) &= (0,0), \\
e_2(t) &= \left(\frac{35}{18}t^3 + O(t^4), -\frac{25}{6}t^3 + O(t^4) \right), \\
e_3(t) &= (0,0), \\
e_4(t) &= \left(-\frac{199}{144}t + O(t^2), \frac{35}{48}t + O(t^2) \right), \\
e_5(t) &= (0,0), \\
e_6(t) &= \left(\frac{1}{7}\gamma + \frac{1}{7}\ln(t) + c_2 \ln(t) + \frac{743}{1680} + c_1 + \frac{6913}{5760}t + O(t^2), \ldots \right. \\
&\qquad \left. -\frac{31}{224} + c_2 - \frac{217}{384}t + O(t^2) \right).
\end{aligned}
$$

4.3   The Trapezoidal Rule

When analyzing the trapezoidal rule, we are facing a different situation. The basic convergence order $O(h^2)$ could be shown for all well-posed problems (1.1) in [14][11]. In this case, however, even the existence of smooth coefficient functions e_j in (4.3) does not guarantee the existence of a smooth error expansion. This paradoxical situation is obviously caused by the fact that the remainder term in (3.9) cannot be estimated at the necessary level of accuracy. We confirm this fact by comparing the performance of IDeC with that of extrapolation. For regular problems, we expect to observe order sequences $O(h^2)$, $O(h^4)$, … for both methods. For singular problems, the results which in general diverge from this expectation are given in the following.

---

[11]Note that an evaluation of the right-hand side at $t = 0$ can be replaced by $z'(0)$, which is known in the case of (1.1), cf. [15].

4.3.1   Iterated Defect Correction

For all the examples discussed in this section, a polynomial degree $m = 10$ is used in the definition of the IDeC method. Thus, for regular problems the maximal attainable convergence order is 10.

Example 1.1a demonstrates very clearly the possible failure of IDeC to improve the convergence order of a numerical solution. No higher level of accuracy is obtained after the first step of the iteration which yields $O(h^3)$, see Table 4.40. For a fixed point t, further improvement is possible nonetheless and we observe an order sequence $O(h^2)$, $O(h^4)$, $O(h^6)$, $O(h^6)$, ..., cf. Tables 4.41 and 4.42. This phenomenon may be explained by noting that $t^{\max} \to 0$ as $h \to 0$.

The situation is similar for example 1.2a. At the points $t = 1/8$ and $t = 1$ the order sequence is $O(h^2)$, $O(h^4)$, $O(h^4)$, ... For these results, see Tables 4.43 – 4.45.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5\cdot 2^{-1}$	$8.2\cdot 10^{-02}$	1.00000	1.996	$2.7\cdot 10^{-03}$	0.20000	2.995
$1/5\cdot 2^{-2}$	$2.0\cdot 10^{-02}$	1.00000	1.999	$3.5\cdot 10^{-04}$	0.10000	2.998
$1/5\cdot 2^{-3}$	$5.1\cdot 10^{-03}$	1.00000	1.999	$4.3\cdot 10^{-05}$	0.05000	2.999
$1/5\cdot 2^{-4}$	$1.2\cdot 10^{-03}$	1.00000	1.999	$5.4\cdot 10^{-06}$	0.02500	2.999
$1/5\cdot 2^{-5}$	$3.2\cdot 10^{-04}$	1.00000	1.999	$6.8\cdot 10^{-07}$	0.01250	2.999
$1/5\cdot 2^{-6}$	$8.0\cdot 10^{-05}$	1.00000	1.999	$8.5\cdot 10^{-08}$	0.00625	2.999
$1/5\cdot 2^{-7}$	$2.0\cdot 10^{-05}$	1.00000	1.999	$1.0\cdot 10^{-08}$	0.00312	2.999
$1/5\cdot 2^{-8}$	$5.0\cdot 10^{-06}$	1.00000	1.999	$1.3\cdot 10^{-09}$	0.00156	2.999
$1/5\cdot 2^{-9}$	$1.2\cdot 10^{-06}$	1.00000	1.999	$1.6\cdot 10^{-10}$	0.00078	2.999
$1/5\cdot 2^{-10}$	$3.1\cdot 10^{-07}$	1.00000	1.999	$2.0\cdot 10^{-11}$	0.00039	2.999
$1/5\cdot 2^{-11}$	$7.8\cdot 10^{-08}$	1.00000	1.999	$2.6\cdot 10^{-12}$	0.00019	2.999
$1/5\cdot 2^{-12}$	$1.9\cdot 10^{-08}$	1.00000	2.000	$3.2\cdot 10^{-13}$	0.00009	3.000
$1/5\cdot 2^{-13}$	$4.9\cdot 10^{-09}$	1.00000	1.999	$4.0\cdot 10^{-14}$	0.00004	2.999
$1/5\cdot 2^{-14}$	$1.2\cdot 10^{-09}$	1.00000		$5.1\cdot 10^{-15}$	0.00002	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5\cdot 2^{-1}$	$9.4\cdot 10^{-03}$	0.20000	2.987	$3.2\cdot 10^{-02}$	0.10000	2.986
$1/5\cdot 2^{-2}$	$1.1\cdot 10^{-03}$	0.10000	2.996	$4.1\cdot 10^{-03}$	0.05000	2.996
$1/5\cdot 2^{-3}$	$1.4\cdot 10^{-04}$	0.05000	2.999	$5.1\cdot 10^{-04}$	0.02500	2.999
$1/5\cdot 2^{-4}$	$1.8\cdot 10^{-05}$	0.02500	2.999	$6.4\cdot 10^{-05}$	0.01250	2.999
$1/5\cdot 2^{-5}$	$2.3\cdot 10^{-06}$	0.01250	2.999	$8.0\cdot 10^{-06}$	0.00625	2.999
$1/5\cdot 2^{-6}$	$2.9\cdot 10^{-07}$	0.00625	2.999	$1.0\cdot 10^{-06}$	0.00312	2.999
$1/5\cdot 2^{-7}$	$3.6\cdot 10^{-08}$	0.00312	2.999	$1.2\cdot 10^{-07}$	0.00156	2.999
$1/5\cdot 2^{-8}$	$4.5\cdot 10^{-09}$	0.00156	2.999	$1.5\cdot 10^{-08}$	0.00078	2.999
$1/5\cdot 2^{-9}$	$5.6\cdot 10^{-10}$	0.00078	2.999	$1.9\cdot 10^{-09}$	0.00039	2.999
$1/5\cdot 2^{-10}$	$7.1\cdot 10^{-11}$	0.00039	2.999	$2.4\cdot 10^{-10}$	0.00019	2.999
$1/5\cdot 2^{-11}$	$8.8\cdot 10^{-12}$	0.00019	2.999	$3.0\cdot 10^{-11}$	0.00009	2.999
$1/5\cdot 2^{-12}$	$1.1\cdot 10^{-12}$	0.00009	3.000	$3.8\cdot 10^{-12}$	0.00004	3.000
$1/5\cdot 2^{-13}$	$1.3\cdot 10^{-13}$	0.00004		$4.7\cdot 10^{-13}$	0.00002	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5\cdot 2^{-1}$	$1.0\cdot 10^{-01}$	0.10000	2.987	$3.4\cdot 10^{-01}$	0.10000	2.988
$1/5\cdot 2^{-2}$	$1.3\cdot 10^{-02}$	0.05000	2.996	$4.2\cdot 10^{-02}$	0.05000	2.997
$1/5\cdot 2^{-3}$	$1.7\cdot 10^{-03}$	0.02500	2.999	$5.3\cdot 10^{-03}$	0.02500	2.999
$1/5\cdot 2^{-4}$	$2.1\cdot 10^{-04}$	0.01250	2.999	$6.7\cdot 10^{-04}$	0.01250	2.999
$1/5\cdot 2^{-5}$	$2.6\cdot 10^{-05}$	0.00625	2.999	$8.4\cdot 10^{-05}$	0.00625	2.999
$1/5\cdot 2^{-6}$	$3.3\cdot 10^{-06}$	0.00312	2.999	$1.0\cdot 10^{-05}$	0.00312	2.999
$1/5\cdot 2^{-7}$	$4.1\cdot 10^{-07}$	0.00156	2.999	$1.3\cdot 10^{-06}$	0.00156	2.999
$1/5\cdot 2^{-8}$	$5.2\cdot 10^{-08}$	0.00078	2.999	$1.6\cdot 10^{-07}$	0.00078	2.999
$1/5\cdot 2^{-9}$	$6.5\cdot 10^{-09}$	0.00039	2.999	$2.0\cdot 10^{-08}$	0.00039	2.999
$1/5\cdot 2^{-10}$	$8.1\cdot 10^{-10}$	0.00019	2.999	$2.5\cdot 10^{-09}$	0.00019	2.999
$1/5\cdot 2^{-11}$	$1.0\cdot 10^{-10}$	0.00009	2.999	$3.2\cdot 10^{-10}$	0.00009	2.999
$1/5\cdot 2^{-12}$	$1.2\cdot 10^{-11}$	0.00004		$4.0\cdot 10^{-11}$	0.00004	

Table 4.40: Trapezoidal rule, IDeC method on $[0,1]$ for 1.1a.

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$1.4 \cdot 10^{-05}$	1.999	$4.3 \cdot 10^{-06}$	7.972
$1/5 \cdot 2^{-5}$	$3.6 \cdot 10^{-06}$	1.999	$1.7 \cdot 10^{-08}$	4.927
$1/5 \cdot 2^{-6}$	$9.0 \cdot 10^{-07}$	1.999	$5.7 \cdot 10^{-10}$	5.895
$1/5 \cdot 2^{-7}$	$2.2 \cdot 10^{-07}$	1.999	$9.6 \cdot 10^{-12}$	6.287
$1/5 \cdot 2^{-8}$	$5.6 \cdot 10^{-08}$	1.999	$1.2 \cdot 10^{-13}$	7.211
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-08}$	1.999	$8.3 \cdot 10^{-16}$	2.739
$1/5 \cdot 2^{-10}$	$3.5 \cdot 10^{-09}$	1.999	$1.2 \cdot 10^{-16}$	3.683
$1/5 \cdot 2^{-11}$	$8.8 \cdot 10^{-10}$	1.999	$9.6 \cdot 10^{-18}$	3.930
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-10}$	1.999	$6.3 \cdot 10^{-19}$	3.983
$1/5 \cdot 2^{-13}$	$5.5 \cdot 10^{-11}$	1.999	$4.0 \cdot 10^{-20}$	3.995
$1/5 \cdot 2^{-14}$	$1.3 \cdot 10^{-11}$	1.999	$2.5 \cdot 10^{-21}$	3.998
$1/5 \cdot 2^{-15}$	$3.4 \cdot 10^{-12}$	2.000	$1.5 \cdot 10^{-22}$	3.999
$1/5 \cdot 2^{-16}$	$8.6 \cdot 10^{-13}$	1.999	$9.8 \cdot 10^{-24}$	3.999
$1/5 \cdot 2^{-17}$	$2.1 \cdot 10^{-13}$		$6.1 \cdot 10^{-25}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$1.7 \cdot 10^{-05}$	10.047	$5.8 \cdot 10^{-05}$	9.850
$1/5 \cdot 2^{-5}$	$1.6 \cdot 10^{-08}$	3.732	$6.3 \cdot 10^{-08}$	5.006
$1/5 \cdot 2^{-6}$	$1.2 \cdot 10^{-09}$	5.740	$1.9 \cdot 10^{-09}$	5.550
$1/5 \cdot 2^{-7}$	$2.3 \cdot 10^{-11}$	5.942	$4.2 \cdot 10^{-11}$	5.907
$1/5 \cdot 2^{-8}$	$3.8 \cdot 10^{-13}$	5.985	$7.0 \cdot 10^{-13}$	5.977
$1/5 \cdot 2^{-9}$	$6.0 \cdot 10^{-15}$	5.996	$1.1 \cdot 10^{-14}$	5.994
$1/5 \cdot 2^{-10}$	$9.4 \cdot 10^{-17}$	5.999	$1.7 \cdot 10^{-16}$	5.998
$1/5 \cdot 2^{-11}$	$1.4 \cdot 10^{-18}$	5.999	$2.7 \cdot 10^{-18}$	5.999
$1/5 \cdot 2^{-12}$	$2.3 \cdot 10^{-20}$	5.999	$4.2 \cdot 10^{-20}$	5.999
$1/5 \cdot 2^{-13}$	$3.6 \cdot 10^{-22}$	5.999	$6.6 \cdot 10^{-22}$	5.999
$1/5 \cdot 2^{-14}$	$5.6 \cdot 10^{-24}$	5.999	$1.0 \cdot 10^{-23}$	6.000
$1/5 \cdot 2^{-15}$	$8.7 \cdot 10^{-26}$	5.995	$1.6 \cdot 10^{-25}$	6.003
$1/5 \cdot 2^{-16}$	$1.3 \cdot 10^{-27}$		$2.5 \cdot 10^{-27}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-04}$	8.891	$5.6 \cdot 10^{-04}$	8.544
$1/5 \cdot 2^{-5}$	$3.8 \cdot 10^{-07}$	7.213	$1.5 \cdot 10^{-06}$	9.694
$1/5 \cdot 2^{-6}$	$2.6 \cdot 10^{-09}$	5.146	$1.8 \cdot 10^{-09}$	3.865
$1/5 \cdot 2^{-7}$	$7.3 \cdot 10^{-11}$	5.848	$1.2 \cdot 10^{-10}$	5.746
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-12}$	5.964	$2.3 \cdot 10^{-12}$	5.943
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-14}$	5.991	$3.7 \cdot 10^{-14}$	5.986
$1/5 \cdot 2^{-10}$	$3.2 \cdot 10^{-16}$	5.997	$5.9 \cdot 10^{-16}$	5.996
$1/5 \cdot 2^{-11}$	$5.0 \cdot 10^{-18}$	5.999	$9.3 \cdot 10^{-18}$	5.999
$1/5 \cdot 2^{-12}$	$7.8 \cdot 10^{-20}$	5.999	$1.4 \cdot 10^{-19}$	5.999
$1/5 \cdot 2^{-13}$	$1.2 \cdot 10^{-21}$	5.999	$2.2 \cdot 10^{-21}$	5.999
$1/5 \cdot 2^{-14}$	$1.9 \cdot 10^{-23}$	5.999	$3.5 \cdot 10^{-23}$	5.999
$1/5 \cdot 2^{-15}$	$3.0 \cdot 10^{-25}$		$5.5 \cdot 10^{-25}$	

Table 4.41: Trapezoidal rule, IDeC method at $t = 1/8$ for 1.1a.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$8.2 \cdot 10^{-02}$	1.996	$2.6 \cdot 10^{-03}$	6.532
$1/5 \cdot 2^{-2}$	$2.0 \cdot 10^{-02}$	1.999	$2.8 \cdot 10^{-05}$	3.851
$1/5 \cdot 2^{-3}$	$5.1 \cdot 10^{-03}$	1.999	$2.0 \cdot 10^{-06}$	3.944
$1/5 \cdot 2^{-4}$	$1.2 \cdot 10^{-03}$	1.999	$1.3 \cdot 10^{-07}$	3.985
$1/5 \cdot 2^{-5}$	$3.2 \cdot 10^{-04}$	1.999	$8.2 \cdot 10^{-09}$	3.996
$1/5 \cdot 2^{-6}$	$8.0 \cdot 10^{-05}$	1.999	$5.1 \cdot 10^{-10}$	3.999
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-05}$	1.999	$3.2 \cdot 10^{-11}$	3.999
$1/5 \cdot 2^{-8}$	$5.0 \cdot 10^{-06}$	1.999	$2.0 \cdot 10^{-12}$	3.999
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-06}$	1.999	$1.2 \cdot 10^{-13}$	3.999
$1/5 \cdot 2^{-10}$	$3.1 \cdot 10^{-07}$	1.999	$7.8 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-11}$	$7.8 \cdot 10^{-08}$	1.999	$4.9 \cdot 10^{-16}$	3.999
$1/5 \cdot 2^{-12}$	$1.9 \cdot 10^{-08}$	2.000	$3.0 \cdot 10^{-17}$	4.000
$1/5 \cdot 2^{-13}$	$4.9 \cdot 10^{-09}$	1.999	$1.9 \cdot 10^{-18}$	3.999
$1/5 \cdot 2^{-14}$	$1.2 \cdot 10^{-09}$		$1.1 \cdot 10^{-19}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$8.9 \cdot 10^{-03}$	10.039	$2.9 \cdot 10^{-02}$	9.842
$1/5 \cdot 2^{-2}$	$8.4 \cdot 10^{-06}$	3.726	$3.2 \cdot 10^{-05}$	4.998
$1/5 \cdot 2^{-3}$	$6.4 \cdot 10^{-07}$	5.739	$1.0 \cdot 10^{-06}$	5.550
$1/5 \cdot 2^{-4}$	$1.1 \cdot 10^{-08}$	5.941	$2.1 \cdot 10^{-08}$	5.907
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-10}$	5.985	$3.5 \cdot 10^{-10}$	5.977
$1/5 \cdot 2^{-6}$	$3.0 \cdot 10^{-12}$	5.996	$5.7 \cdot 10^{-12}$	5.994
$1/5 \cdot 2^{-7}$	$4.8 \cdot 10^{-14}$	5.999	$8.9 \cdot 10^{-14}$	5.998
$1/5 \cdot 2^{-8}$	$7.5 \cdot 10^{-16}$	5.999	$1.4 \cdot 10^{-15}$	5.999
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-17}$	5.999	$2.1 \cdot 10^{-17}$	5.999
$1/5 \cdot 2^{-10}$	$1.8 \cdot 10^{-19}$	5.999	$3.4 \cdot 10^{-19}$	5.999
$1/5 \cdot 2^{-11}$	$2.8 \cdot 10^{-21}$	5.999	$5.3 \cdot 10^{-21}$	6.000
$1/5 \cdot 2^{-12}$	$4.4 \cdot 10^{-23}$	5.990	$8.3 \cdot 10^{-23}$	6.005
$1/5 \cdot 2^{-13}$	$7.0 \cdot 10^{-25}$		$1.3 \cdot 10^{-24}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$9.3 \cdot 10^{-02}$	8.881	$2.8 \cdot 10^{-01}$	8.533
$1/5 \cdot 2^{-2}$	$1.9 \cdot 10^{-04}$	7.208	$7.6 \cdot 10^{-04}$	9.689
$1/5 \cdot 2^{-3}$	$1.3 \cdot 10^{-06}$	5.146	$9.3 \cdot 10^{-07}$	3.866
$1/5 \cdot 2^{-4}$	$3.7 \cdot 10^{-08}$	5.848	$6.3 \cdot 10^{-08}$	5.746
$1/5 \cdot 2^{-5}$	$6.5 \cdot 10^{-10}$	5.964	$1.1 \cdot 10^{-09}$	5.943
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{-11}$	5.991	$1.9 \cdot 10^{-11}$	5.986
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-13}$	5.997	$3.0 \cdot 10^{-13}$	5.996
$1/5 \cdot 2^{-8}$	$2.5 \cdot 10^{-15}$	5.999	$4.7 \cdot 10^{-15}$	5.999
$1/5 \cdot 2^{-9}$	$4.0 \cdot 10^{-17}$	5.999	$7.4 \cdot 10^{-17}$	5.999
$1/5 \cdot 2^{-10}$	$6.3 \cdot 10^{-19}$	5.999	$1.1 \cdot 10^{-18}$	5.999
$1/5 \cdot 2^{-11}$	$9.8 \cdot 10^{-21}$	5.999	$1.8 \cdot 10^{-20}$	6.000
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-22}$		$2.8 \cdot 10^{-22}$	

Table 4.42: Trapezoidal rule, IDeC method at $t = 1$ for 1.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.4 \cdot 10^{-03}$	1.00000	1.998	$1.6 \cdot 10^{-03}$	0.10000	2.991
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-03}$	0.95000	1.999	$2.0 \cdot 10^{-04}$	0.05000	2.994
$1/5 \cdot 2^{-3}$	$2.7 \cdot 10^{-04}$	0.97500	1.999	$2.5 \cdot 10^{-05}$	0.02500	2.997
$1/5 \cdot 2^{-4}$	$6.9 \cdot 10^{-05}$	0.96250	1.999	$3.1 \cdot 10^{-06}$	0.01250	2.998
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-05}$	0.96875	1.999	$3.9 \cdot 10^{-07}$	0.00625	2.999
$1/5 \cdot 2^{-6}$	$4.3 \cdot 10^{-06}$	0.96875	1.999	$4.9 \cdot 10^{-08}$	0.00312	2.999
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-06}$	0.96875	1.999	$6.2 \cdot 10^{-09}$	0.00156	2.999
$1/5 \cdot 2^{-8}$	$2.7 \cdot 10^{-07}$	0.96796	1.999	$7.7 \cdot 10^{-10}$	0.00078	2.999
$1/5 \cdot 2^{-9}$	$6.7 \cdot 10^{-08}$	0.96796	1.999	$9.7 \cdot 10^{-11}$	0.00039	2.999
$1/5 \cdot 2^{-10}$	$1.6 \cdot 10^{-08}$	0.96796	1.999	$1.2 \cdot 10^{-11}$	0.00019	2.999
$1/5 \cdot 2^{-11}$	$4.2 \cdot 10^{-09}$	0.96806	1.999	$1.5 \cdot 10^{-12}$	0.00009	2.999
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-09}$	0.96806	2.000	$1.8 \cdot 10^{-13}$	0.00004	3.000
$1/5 \cdot 2^{-13}$	$2.6 \cdot 10^{-10}$	0.96806	1.999	$2.3 \cdot 10^{-14}$	0.00002	2.999
$1/5 \cdot 2^{-14}$	$6.6 \cdot 10^{-11}$	0.96805		$2.9 \cdot 10^{-15}$	0.00001	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.5 \cdot 10^{-03}$	0.10000	2.944	$3.2 \cdot 10^{-03}$	0.10000	2.822
$1/5 \cdot 2^{-2}$	$3.3 \cdot 10^{-04}$	0.05000	2.970	$4.5 \cdot 10^{-04}$	0.05000	2.915
$1/5 \cdot 2^{-3}$	$4.2 \cdot 10^{-05}$	0.02500	2.984	$6.0 \cdot 10^{-05}$	0.02500	2.958
$1/5 \cdot 2^{-4}$	$5.4 \cdot 10^{-06}$	0.01250	2.992	$7.8 \cdot 10^{-06}$	0.01250	2.979
$1/5 \cdot 2^{-5}$	$6.8 \cdot 10^{-07}$	0.00625	2.996	$9.8 \cdot 10^{-07}$	0.00625	2.989
$1/5 \cdot 2^{-6}$	$8.5 \cdot 10^{-08}$	0.00312	2.998	$1.2 \cdot 10^{-07}$	0.00312	2.994
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-08}$	0.00156	2.999	$1.5 \cdot 10^{-08}$	0.00156	2.997
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-09}$	0.00078	2.999	$1.9 \cdot 10^{-09}$	0.00078	2.998
$1/5 \cdot 2^{-9}$	$1.6 \cdot 10^{-10}$	0.00039	2.999	$2.4 \cdot 10^{-10}$	0.00039	2.999
$1/5 \cdot 2^{-10}$	$2.0 \cdot 10^{-11}$	0.00019	2.999	$3.0 \cdot 10^{-11}$	0.00019	2.999
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-12}$	0.00009	2.999	$3.8 \cdot 10^{-12}$	0.00009	2.999
$1/5 \cdot 2^{-12}$	$3.2 \cdot 10^{-13}$	0.00004	3.000	$4.7 \cdot 10^{-13}$	0.00004	3.000
$1/5 \cdot 2^{-13}$	$4.0 \cdot 10^{-14}$	0.00002		$5.9 \cdot 10^{-14}$	0.00002	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$3.4 \cdot 10^{-03}$	0.10000	2.619	$3.2 \cdot 10^{-03}$	0.10000	2.308
$1/5 \cdot 2^{-2}$	$5.6 \cdot 10^{-04}$	0.05000	2.831	$6.4 \cdot 10^{-04}$	0.05000	2.721
$1/5 \cdot 2^{-3}$	$7.9 \cdot 10^{-05}$	0.02500	2.920	$9.8 \cdot 10^{-05}$	0.02500	2.871
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-05}$	0.01250	2.960	$1.3 \cdot 10^{-05}$	0.01250	2.938
$1/5 \cdot 2^{-5}$	$1.3 \cdot 10^{-06}$	0.00625	2.980	$1.7 \cdot 10^{-06}$	0.00625	2.969
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-07}$	0.00312	2.990	$2.2 \cdot 10^{-07}$	0.00312	2.984
$1/5 \cdot 2^{-7}$	$2.1 \cdot 10^{-08}$	0.00156	2.995	$2.8 \cdot 10^{-08}$	0.00156	2.992
$1/5 \cdot 2^{-8}$	$2.6 \cdot 10^{-09}$	0.00078	2.997	$3.5 \cdot 10^{-09}$	0.00078	2.996
$1/5 \cdot 2^{-9}$	$3.3 \cdot 10^{-10}$	0.00039	2.998	$4.4 \cdot 10^{-10}$	0.00039	2.998
$1/5 \cdot 2^{-10}$	$4.1 \cdot 10^{-11}$	0.00019	2.999	$5.5 \cdot 10^{-11}$	0.00019	2.999
$1/5 \cdot 2^{-11}$	$5.2 \cdot 10^{-12}$	0.00009	2.999	$6.9 \cdot 10^{-12}$	0.00009	2.999
$1/5 \cdot 2^{-12}$	$6.5 \cdot 10^{-13}$	0.00004		$8.7 \cdot 10^{-13}$	0.00004	

Table 4.43: Trapezoidal rule, IDeC method on $[0, 1]$ for 1.2a.

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$2.4 \cdot 10^{-05}$	2.004	$1.2 \cdot 10^{-06}$	5.247
$1/5 \cdot 2^{-5}$	$6.0 \cdot 10^{-06}$	2.001	$3.4 \cdot 10^{-08}$	6.439
$1/5 \cdot 2^{-6}$	$1.5 \cdot 10^{-06}$	2.000	$3.9 \cdot 10^{-10}$	3.406
$1/5 \cdot 2^{-7}$	$3.7 \cdot 10^{-07}$	2.000	$3.7 \cdot 10^{-11}$	3.767
$1/5 \cdot 2^{-8}$	$9.4 \cdot 10^{-08}$	2.000	$2.7 \cdot 10^{-12}$	3.818
$1/5 \cdot 2^{-9}$	$2.3 \cdot 10^{-08}$	2.000	$1.9 \cdot 10^{-13}$	3.969
$1/5 \cdot 2^{-10}$	$5.8 \cdot 10^{-09}$	2.000	$1.2 \cdot 10^{-14}$	4.011
$1/5 \cdot 2^{-11}$	$1.4 \cdot 10^{-09}$	2.000	$7.6 \cdot 10^{-16}$	4.019
$1/5 \cdot 2^{-12}$	$3.6 \cdot 10^{-10}$	2.000	$4.6 \cdot 10^{-17}$	4.016
$1/5 \cdot 2^{-13}$	$9.2 \cdot 10^{-11}$	2.000	$2.9 \cdot 10^{-18}$	4.011
$1/5 \cdot 2^{-14}$	$2.3 \cdot 10^{-11}$	2.000	$1.7 \cdot 10^{-19}$	4.007
$1/5 \cdot 2^{-15}$	$5.7 \cdot 10^{-12}$	2.000	$1.1 \cdot 10^{-20}$	4.004
$1/5 \cdot 2^{-16}$	$1.4 \cdot 10^{-12}$	1.999	$6.9 \cdot 10^{-22}$	4.002
$1/5 \cdot 2^{-17}$	$3.5 \cdot 10^{-13}$		$4.3 \cdot 10^{-23}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$2.3 \cdot 10^{-06}$	5.147	$3.2 \cdot 10^{-06}$	4.986
$1/5 \cdot 2^{-5}$	$6.6 \cdot 10^{-08}$	6.082	$1.0 \cdot 10^{-07}$	5.566
$1/5 \cdot 2^{-6}$	$9.8 \cdot 10^{-10}$	4.068	$2.1 \cdot 10^{-09}$	5.552
$1/5 \cdot 2^{-7}$	$5.8 \cdot 10^{-11}$	3.813	$4.5 \cdot 10^{-11}$	3.782
$1/5 \cdot 2^{-8}$	$4.1 \cdot 10^{-12}$	3.765	$3.3 \cdot 10^{-12}$	3.847
$1/5 \cdot 2^{-9}$	$3.0 \cdot 10^{-13}$	3.953	$2.2 \cdot 10^{-13}$	3.899
$1/5 \cdot 2^{-10}$	$1.9 \cdot 10^{-14}$	4.007	$1.5 \cdot 10^{-14}$	4.001
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-15}$	4.018	$9.6 \cdot 10^{-16}$	4.025
$1/5 \cdot 2^{-12}$	$7.5 \cdot 10^{-17}$	4.017	$5.8 \cdot 10^{-17}$	4.025
$1/5 \cdot 2^{-13}$	$4.6 \cdot 10^{-18}$	4.012	$3.6 \cdot 10^{-18}$	4.019
$1/5 \cdot 2^{-14}$	$2.8 \cdot 10^{-19}$	4.008	$2.2 \cdot 10^{-19}$	4.013
$1/5 \cdot 2^{-15}$	$1.7 \cdot 10^{-20}$	4.005	$1.3 \cdot 10^{-20}$	4.008
$1/5 \cdot 2^{-16}$	$1.1 \cdot 10^{-21}$		$8.5 \cdot 10^{-22}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$4.1 \cdot 10^{-06}$	4.885	$5.2 \cdot 10^{-06}$	4.819
$1/5 \cdot 2^{-5}$	$1.4 \cdot 10^{-07}$	5.357	$1.8 \cdot 10^{-07}$	5.259
$1/5 \cdot 2^{-6}$	$3.4 \cdot 10^{-09}$	6.089	$4.8 \cdot 10^{-09}$	5.723
$1/5 \cdot 2^{-7}$	$5.0 \cdot 10^{-11}$	4.170	$9.1 \cdot 10^{-11}$	5.158
$1/5 \cdot 2^{-8}$	$2.7 \cdot 10^{-12}$	4.068	$2.5 \cdot 10^{-12}$	4.068
$1/5 \cdot 2^{-9}$	$1.6 \cdot 10^{-13}$	3.885	$1.5 \cdot 10^{-13}$	4.162
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-14}$	3.991	$8.5 \cdot 10^{-15}$	3.974
$1/5 \cdot 2^{-11}$	$7.0 \cdot 10^{-16}$	4.039	$5.4 \cdot 10^{-16}$	4.060
$1/5 \cdot 2^{-12}$	$4.2 \cdot 10^{-17}$	4.042	$3.2 \cdot 10^{-17}$	4.070
$1/5 \cdot 2^{-13}$	$2.6 \cdot 10^{-18}$	4.033	$1.9 \cdot 10^{-18}$	4.056
$1/5 \cdot 2^{-14}$	$1.5 \cdot 10^{-19}$	4.022	$1.1 \cdot 10^{-19}$	4.039
$1/5 \cdot 2^{-15}$	$9.7 \cdot 10^{-21}$		$7.0 \cdot 10^{-21}$	

Table 4.44: Trapezoidal rule, IDeC method at $t = 1/8$ for 1.2a.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$4.4 \cdot 10^{-03}$	2.000	$5.6 \cdot 10^{-04}$	5.754
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-03}$	1.999	$1.0 \cdot 10^{-05}$	3.645
$1/5 \cdot 2^{-3}$	$2.7 \cdot 10^{-04}$	1.999	$8.3 \cdot 10^{-07}$	4.183
$1/5 \cdot 2^{-4}$	$6.9 \cdot 10^{-05}$	1.999	$4.5 \cdot 10^{-08}$	5.280
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-05}$	1.999	$1.1 \cdot 10^{-09}$	5.320
$1/5 \cdot 2^{-6}$	$4.3 \cdot 10^{-06}$	1.999	$2.9 \cdot 10^{-11}$	3.699
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-06}$	1.999	$2.2 \cdot 10^{-12}$	3.462
$1/5 \cdot 2^{-8}$	$2.7 \cdot 10^{-07}$	1.999	$2.0 \cdot 10^{-13}$	3.830
$1/5 \cdot 2^{-9}$	$6.7 \cdot 10^{-08}$	1.999	$1.4 \cdot 10^{-14}$	3.943
$1/5 \cdot 2^{-10}$	$1.6 \cdot 10^{-08}$	1.999	$9.4 \cdot 10^{-16}$	3.983
$1/5 \cdot 2^{-11}$	$4.2 \cdot 10^{-09}$	1.999	$5.9 \cdot 10^{-17}$	3.997
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-09}$	2.000	$3.7 \cdot 10^{-18}$	4.001
$1/5 \cdot 2^{-13}$	$2.6 \cdot 10^{-10}$	1.999	$2.3 \cdot 10^{-19}$	4.001
$1/5 \cdot 2^{-14}$	$6.6 \cdot 10^{-11}$		$1.4 \cdot 10^{-20}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.0 \cdot 10^{-03}$	5.249	$1.3 \cdot 10^{-03}$	4.739
$1/5 \cdot 2^{-2}$	$2.7 \cdot 10^{-05}$	4.237	$5.0 \cdot 10^{-05}$	5.005
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-06}$	3.919	$1.5 \cdot 10^{-06}$	3.619
$1/5 \cdot 2^{-4}$	$9.7 \cdot 10^{-08}$	4.883	$1.2 \cdot 10^{-07}$	4.761
$1/5 \cdot 2^{-5}$	$3.2 \cdot 10^{-09}$	5.437	$4.6 \cdot 10^{-09}$	5.219
$1/5 \cdot 2^{-6}$	$7.6 \cdot 10^{-11}$	5.198	$1.2 \cdot 10^{-10}$	5.814
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-12}$	4.107	$2.2 \cdot 10^{-12}$	4.349
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-13}$	3.825	$1.0 \cdot 10^{-13}$	4.209
$1/5 \cdot 2^{-9}$	$8.4 \cdot 10^{-15}$	3.820	$5.9 \cdot 10^{-15}$	3.749
$1/5 \cdot 2^{-10}$	$5.9 \cdot 10^{-16}$	3.947	$4.4 \cdot 10^{-16}$	3.898
$1/5 \cdot 2^{-11}$	$3.8 \cdot 10^{-17}$	3.989	$2.9 \cdot 10^{-17}$	3.977
$1/5 \cdot 2^{-12}$	$2.4 \cdot 10^{-18}$	4.002	$1.8 \cdot 10^{-18}$	4.001
$1/5 \cdot 2^{-13}$	$1.5 \cdot 10^{-19}$		$1.1 \cdot 10^{-19}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.4 \cdot 10^{-03}$	4.337	$1.3 \cdot 10^{-03}$	3.912
$1/5 \cdot 2^{-2}$	$7.1 \cdot 10^{-05}$	5.432	$8.9 \cdot 10^{-05}$	5.649
$1/5 \cdot 2^{-3}$	$1.6 \cdot 10^{-06}$	3.360	$1.7 \cdot 10^{-06}$	3.152
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-07}$	4.675	$2.0 \cdot 10^{-07}$	4.615
$1/5 \cdot 2^{-5}$	$6.2 \cdot 10^{-09}$	5.105	$8.1 \cdot 10^{-09}$	5.044
$1/5 \cdot 2^{-6}$	$1.8 \cdot 10^{-10}$	5.460	$2.4 \cdot 10^{-10}$	5.322
$1/5 \cdot 2^{-7}$	$4.1 \cdot 10^{-12}$	5.340	$6.1 \cdot 10^{-12}$	5.753
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-13}$	4.266	$1.1 \cdot 10^{-13}$	4.457
$1/5 \cdot 2^{-9}$	$5.3 \cdot 10^{-15}$	4.214	$5.2 \cdot 10^{-15}$	4.370
$1/5 \cdot 2^{-10}$	$2.8 \cdot 10^{-16}$	3.792	$2.5 \cdot 10^{-16}$	4.120
$1/5 \cdot 2^{-11}$	$2.0 \cdot 10^{-17}$	3.952	$1.4 \cdot 10^{-17}$	3.907
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-18}$		$9.6 \cdot 10^{-19}$	

Table 4.45: Trapezoidal rule, IDeC method at $t = 1$ for 1.2a.

On the other hand, for example 2.1a IDeC shows its classical asymptotic behavior. The application of the iteration enhances the convergence order by 2 in every step up to the maximum $p = 10$ (no higher convergence order can be expected in general when a polynomial of degree 10 is used to construct the neighboring problems). Naturally, the same order sequence is observed at a fixed point. The fact that $t^{\max} \approx 1$ also supports the impression that the singularity does not influence the performance of IDeC for this problem. The numerical results are displayed in Tables 4.46 – 4.48.

For example 2.2a (where the coefficient matrix is variable), the result is essentially the same as for 2.1a which has the same spectral properties, see Table 4.49. As the behavior at $t = 1/8$ and $t = 1$ is identical, these results are omitted for this problem.

Even for the nonlinear example 2.3a, the classical order sequence is observed, see Table 4.50. Thus, for the problems tested IDeC proved robust with respect to the singularity for examples where $M(0)$ had one negative and one vanishing eigenvalue.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$3.6 \cdot 10^{-02}$	1.00000	2.000	$4.6 \cdot 10^{-04}$	1.00000	4.004
$1/5 \cdot 2^{-2}$	$9.2 \cdot 10^{-03}$	1.00000	2.000	$2.8 \cdot 10^{-05}$	1.00000	4.000
$1/5 \cdot 2^{-3}$	$2.3 \cdot 10^{-03}$	1.00000	2.000	$1.7 \cdot 10^{-06}$	1.00000	4.000
$1/5 \cdot 2^{-4}$	$5.7 \cdot 10^{-04}$	1.00000	2.000	$1.1 \cdot 10^{-07}$	1.00000	4.000
$1/5 \cdot 2^{-5}$	$1.4 \cdot 10^{-04}$	1.00000	2.000	$7.0 \cdot 10^{-09}$	1.00000	4.000
$1/5 \cdot 2^{-6}$	$3.6 \cdot 10^{-05}$	1.00000	2.000	$4.3 \cdot 10^{-10}$	1.00000	4.000
$1/5 \cdot 2^{-7}$	$9.0 \cdot 10^{-06}$	1.00000	2.000	$2.7 \cdot 10^{-11}$	1.00000	4.000
$1/5 \cdot 2^{-8}$	$2.2 \cdot 10^{-06}$	1.00000	2.000	$1.7 \cdot 10^{-12}$	1.00000	4.000
$1/5 \cdot 2^{-9}$	$5.6 \cdot 10^{-07}$	1.00000	2.000	$1.0 \cdot 10^{-13}$	1.00000	4.000
$1/5 \cdot 2^{-10}$	$1.4 \cdot 10^{-07}$	1.00000	2.000	$6.6 \cdot 10^{-15}$	1.00000	4.000
$1/5 \cdot 2^{-11}$	$3.5 \cdot 10^{-08}$	1.00000	2.000	$4.1 \cdot 10^{-16}$	1.00000	4.000
$1/5 \cdot 2^{-12}$	$8.8 \cdot 10^{-09}$	1.00000	2.000	$2.6 \cdot 10^{-17}$	1.00000	4.000
$1/5 \cdot 2^{-13}$	$2.2 \cdot 10^{-09}$	1.00000	1.999	$1.6 \cdot 10^{-18}$	1.00000	3.999
$1/5 \cdot 2^{-14}$	$5.5 \cdot 10^{-10}$	1.00000		$1.0 \cdot 10^{-19}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-06}$	1.00000	6.020	$4.0 \cdot 10^{-07}$	1.00000	9.866
$1/5 \cdot 2^{-2}$	$7.6 \cdot 10^{-08}$	1.00000	6.002	$4.3 \cdot 10^{-10}$	1.00000	8.828
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-09}$	1.00000	6.000	$9.4 \cdot 10^{-13}$	1.00000	8.279
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-11}$	1.00000	6.000	$3.0 \cdot 10^{-15}$	1.00000	8.077
$1/5 \cdot 2^{-5}$	$2.9 \cdot 10^{-13}$	1.00000	6.000	$1.1 \cdot 10^{-17}$	1.00000	8.019
$1/5 \cdot 2^{-6}$	$4.5 \cdot 10^{-15}$	1.00000	6.000	$4.3 \cdot 10^{-20}$	1.00000	8.005
$1/5 \cdot 2^{-7}$	$7.0 \cdot 10^{-17}$	1.00000	6.000	$1.6 \cdot 10^{-22}$	1.00000	8.001
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-18}$	1.00000	6.000	$6.6 \cdot 10^{-25}$	1.00000	7.995
$1/5 \cdot 2^{-9}$	$1.7 \cdot 10^{-20}$	1.00000	6.000	$2.5 \cdot 10^{-27}$	1.00000	6.477
$1/5 \cdot 2^{-10}$	$2.7 \cdot 10^{-22}$	1.00000	6.000	$2.9 \cdot 10^{-29}$	0.99980	-0.864
$1/5 \cdot 2^{-11}$	$4.2 \cdot 10^{-24}$	1.00000	6.000	$5.2 \cdot 10^{-29}$	0.87539	-0.794
$1/5 \cdot 2^{-12}$	$6.6 \cdot 10^{-26}$	1.00000	5.911	$9.1 \cdot 10^{-29}$	0.49750	-0.356
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-27}$	1.00000		$1.1 \cdot 10^{-28}$	0.67749	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$3.5 \cdot 10^{-07}$	0.10000	10.650	$3.1 \cdot 10^{-07}$	1.00000	10.433
$1/5 \cdot 2^{-2}$	$2.1 \cdot 10^{-10}$	1.00000	10.044	$2.2 \cdot 10^{-10}$	1.00000	10.093
$1/5 \cdot 2^{-3}$	$2.0 \cdot 10^{-13}$	1.00000	10.014	$2.0 \cdot 10^{-13}$	1.00000	10.021
$1/5 \cdot 2^{-4}$	$2.0 \cdot 10^{-16}$	1.00000	10.004	$2.0 \cdot 10^{-16}$	1.00000	10.005
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-19}$	1.00000	10.001	$1.9 \cdot 10^{-19}$	1.00000	10.001
$1/5 \cdot 2^{-6}$	$1.9 \cdot 10^{-22}$	1.00000	10.000	$1.9 \cdot 10^{-22}$	1.00000	10.000
$1/5 \cdot 2^{-7}$	$1.8 \cdot 10^{-25}$	1.00000	10.049	$1.8 \cdot 10^{-25}$	1.00000	10.005
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-28}$	1.00000	3.774	$1.8 \cdot 10^{-28}$	1.00000	3.872
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-29}$	0.66835	-1.624	$1.2 \cdot 10^{-29}$	0.64882	-1.229
$1/5 \cdot 2^{-10}$	$3.9 \cdot 10^{-29}$	0.91582	-0.515	$2.8 \cdot 10^{-29}$	0.94433	-1.051
$1/5 \cdot 2^{-11}$	$5.6 \cdot 10^{-29}$	0.86552	-0.590	$5.9 \cdot 10^{-29}$	0.92363	-0.097
$1/5 \cdot 2^{-12}$	$8.5 \cdot 10^{-29}$	0.51030		$6.4 \cdot 10^{-29}$	0.49750	

Table 4.46: Trapezoidal rule, IDeC method on $[0, 1]$ for 2.1a.

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$2.1 \cdot 10^{-05}$	2.000	$3.6 \cdot 10^{-09}$	4.000
$1/5 \cdot 2^{-5}$	$5.3 \cdot 10^{-06}$	2.000	$2.2 \cdot 10^{-10}$	4.000
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-06}$	2.000	$1.4 \cdot 10^{-11}$	4.000
$1/5 \cdot 2^{-7}$	$3.3 \cdot 10^{-07}$	2.000	$8.8 \cdot 10^{-13}$	4.000
$1/5 \cdot 2^{-8}$	$8.4 \cdot 10^{-08}$	2.000	$5.5 \cdot 10^{-14}$	4.000
$1/5 \cdot 2^{-9}$	$2.1 \cdot 10^{-08}$	2.000	$3.4 \cdot 10^{-15}$	4.000
$1/5 \cdot 2^{-10}$	$5.2 \cdot 10^{-09}$	2.000	$2.1 \cdot 10^{-16}$	4.000
$1/5 \cdot 2^{-11}$	$1.3 \cdot 10^{-09}$	2.000	$1.3 \cdot 10^{-17}$	4.000
$1/5 \cdot 2^{-12}$	$3.2 \cdot 10^{-10}$	2.000	$8.4 \cdot 10^{-19}$	4.000
$1/5 \cdot 2^{-13}$	$8.2 \cdot 10^{-11}$	2.000	$5.2 \cdot 10^{-20}$	4.000
$1/5 \cdot 2^{-14}$	$2.0 \cdot 10^{-11}$	2.000	$3.3 \cdot 10^{-21}$	4.000
$1/5 \cdot 2^{-15}$	$5.1 \cdot 10^{-12}$	2.000	$2.0 \cdot 10^{-22}$	4.000
$1/5 \cdot 2^{-16}$	$1.2 \cdot 10^{-12}$	1.999	$1.2 \cdot 10^{-23}$	4.002
$1/5 \cdot 2^{-17}$	$3.2 \cdot 10^{-13}$		$8.0 \cdot 10^{-25}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$5.9 \cdot 10^{-13}$	6.000	$9.9 \cdot 10^{-17}$	8.098
$1/5 \cdot 2^{-5}$	$9.3 \cdot 10^{-15}$	6.000	$3.6 \cdot 10^{-19}$	8.018
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-16}$	6.000	$1.3 \cdot 10^{-21}$	8.004
$1/5 \cdot 2^{-7}$	$2.2 \cdot 10^{-18}$	6.000	$5.4 \cdot 10^{-24}$	8.001
$1/5 \cdot 2^{-8}$	$3.5 \cdot 10^{-20}$	6.000	$2.1 \cdot 10^{-26}$	8.078
$1/5 \cdot 2^{-9}$	$5.5 \cdot 10^{-22}$	6.000	$7.8 \cdot 10^{-29}$	7.568
$1/5 \cdot 2^{-10}$	$8.7 \cdot 10^{-24}$	6.000	$4.1 \cdot 10^{-31}$	−4.555
$1/5 \cdot 2^{-11}$	$1.3 \cdot 10^{-25}$	6.008	$9.7 \cdot 10^{-30}$	−1.354
$1/5 \cdot 2^{-12}$	$2.1 \cdot 10^{-27}$	6.517	$2.4 \cdot 10^{-29}$	−0.685
$1/5 \cdot 2^{-13}$	$2.3 \cdot 10^{-29}$	−0.141	$3.9 \cdot 10^{-29}$	−0.941
$1/5 \cdot 2^{-14}$	$2.5 \cdot 10^{-29}$	−0.646	$7.6 \cdot 10^{-29}$	−0.995
$1/5 \cdot 2^{-15}$	$3.9 \cdot 10^{-29}$	−1.004	$1.5 \cdot 10^{-28}$	−1.132
$1/5 \cdot 2^{-16}$	$7.9 \cdot 10^{-29}$		$3.3 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$8.3 \cdot 10^{-18}$	9.969	$7.2 \cdot 10^{-18}$	9.778
$1/5 \cdot 2^{-5}$	$8.3 \cdot 10^{-21}$	9.974	$8.2 \cdot 10^{-21}$	9.952
$1/5 \cdot 2^{-6}$	$8.3 \cdot 10^{-24}$	9.991	$8.2 \cdot 10^{-24}$	9.988
$1/5 \cdot 2^{-7}$	$8.1 \cdot 10^{-27}$	9.993	$8.1 \cdot 10^{-27}$	9.999
$1/5 \cdot 2^{-8}$	$8.0 \cdot 10^{-30}$	1.119	$7.9 \cdot 10^{-30}$	2.830
$1/5 \cdot 2^{-9}$	$3.6 \cdot 10^{-30}$	−1.537	$1.1 \cdot 10^{-30}$	−0.849
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-29}$	0.547	$2.0 \cdot 10^{-30}$	−1.321
$1/5 \cdot 2^{-11}$	$7.3 \cdot 10^{-30}$	−1.927	$5.0 \cdot 10^{-30}$	−0.462
$1/5 \cdot 2^{-12}$	$2.7 \cdot 10^{-29}$	−0.612	$6.9 \cdot 10^{-30}$	−1.771
$1/5 \cdot 2^{-13}$	$4.2 \cdot 10^{-29}$	−1.042	$2.3 \cdot 10^{-29}$	−1.450
$1/5 \cdot 2^{-14}$	$8.7 \cdot 10^{-29}$	−0.727	$6.4 \cdot 10^{-29}$	−1.411
$1/5 \cdot 2^{-15}$	$1.4 \cdot 10^{-28}$		$1.7 \cdot 10^{-28}$	

Table 4.47: Trapezoidal rule, IDeC method at $t = 1/8$ for 2.1a.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$3.6 \cdot 10^{-02}$	2.000	$4.6 \cdot 10^{-04}$	4.004
$1/5 \cdot 2^{-2}$	$9.2 \cdot 10^{-03}$	2.000	$2.8 \cdot 10^{-05}$	4.000
$1/5 \cdot 2^{-3}$	$2.3 \cdot 10^{-03}$	2.000	$1.7 \cdot 10^{-06}$	4.000
$1/5 \cdot 2^{-4}$	$5.7 \cdot 10^{-04}$	2.000	$1.1 \cdot 10^{-07}$	4.000
$1/5 \cdot 2^{-5}$	$1.4 \cdot 10^{-04}$	2.000	$7.0 \cdot 10^{-09}$	4.000
$1/5 \cdot 2^{-6}$	$3.6 \cdot 10^{-05}$	2.000	$4.3 \cdot 10^{-10}$	4.000
$1/5 \cdot 2^{-7}$	$9.0 \cdot 10^{-06}$	2.000	$2.7 \cdot 10^{-11}$	4.000
$1/5 \cdot 2^{-8}$	$2.2 \cdot 10^{-06}$	2.000	$1.7 \cdot 10^{-12}$	4.000
$1/5 \cdot 2^{-9}$	$5.6 \cdot 10^{-07}$	2.000	$1.0 \cdot 10^{-13}$	4.000
$1/5 \cdot 2^{-10}$	$1.4 \cdot 10^{-07}$	2.000	$6.6 \cdot 10^{-15}$	4.000
$1/5 \cdot 2^{-11}$	$3.5 \cdot 10^{-08}$	2.000	$4.1 \cdot 10^{-16}$	4.000
$1/5 \cdot 2^{-12}$	$8.8 \cdot 10^{-09}$	2.000	$2.6 \cdot 10^{-17}$	4.000
$1/5 \cdot 2^{-13}$	$2.2 \cdot 10^{-09}$	1.999	$1.6 \cdot 10^{-18}$	3.999
$1/5 \cdot 2^{-14}$	$5.5 \cdot 10^{-10}$		$1.0 \cdot 10^{-19}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-06}$	6.020	$4.0 \cdot 10^{-07}$	9.866
$1/5 \cdot 2^{-2}$	$7.6 \cdot 10^{-08}$	6.002	$4.3 \cdot 10^{-10}$	8.828
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-09}$	6.000	$9.4 \cdot 10^{-13}$	8.279
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-11}$	6.000	$3.0 \cdot 10^{-15}$	8.077
$1/5 \cdot 2^{-5}$	$2.9 \cdot 10^{-13}$	6.000	$1.1 \cdot 10^{-17}$	8.019
$1/5 \cdot 2^{-6}$	$4.5 \cdot 10^{-15}$	6.000	$4.3 \cdot 10^{-20}$	8.005
$1/5 \cdot 2^{-7}$	$7.0 \cdot 10^{-17}$	6.000	$1.6 \cdot 10^{-22}$	8.001
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-18}$	6.000	$6.6 \cdot 10^{-25}$	7.996
$1/5 \cdot 2^{-9}$	$1.7 \cdot 10^{-20}$	6.000	$2.5 \cdot 10^{-27}$	7.398
$1/5 \cdot 2^{-10}$	$2.7 \cdot 10^{-22}$	6.000	$1.5 \cdot 10^{-29}$	-1.618
$1/5 \cdot 2^{-11}$	$4.2 \cdot 10^{-24}$	6.000	$4.7 \cdot 10^{-29}$	-1.856
$1/5 \cdot 2^{-12}$	$6.6 \cdot 10^{-26}$	5.931	$1.7 \cdot 10^{-28}$	-1.122
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-27}$		$3.7 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$3.2 \cdot 10^{-07}$	10.514	$3.1 \cdot 10^{-07}$	10.433
$1/5 \cdot 2^{-2}$	$2.1 \cdot 10^{-10}$	10.044	$2.2 \cdot 10^{-10}$	10.093
$1/5 \cdot 2^{-3}$	$2.0 \cdot 10^{-13}$	10.014	$2.0 \cdot 10^{-13}$	10.021
$1/5 \cdot 2^{-4}$	$2.0 \cdot 10^{-16}$	10.004	$2.0 \cdot 10^{-16}$	10.005
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-19}$	10.001	$1.9 \cdot 10^{-19}$	10.001
$1/5 \cdot 2^{-6}$	$1.9 \cdot 10^{-22}$	10.000	$1.9 \cdot 10^{-22}$	10.000
$1/5 \cdot 2^{-7}$	$1.8 \cdot 10^{-25}$	10.054	$1.8 \cdot 10^{-25}$	10.009
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-28}$	3.024	$1.8 \cdot 10^{-28}$	2.899
$1/5 \cdot 2^{-9}$	$2.1 \cdot 10^{-29}$	-0.754	$2.4 \cdot 10^{-29}$	-0.117
$1/5 \cdot 2^{-10}$	$3.6 \cdot 10^{-29}$	-0.483	$2.6 \cdot 10^{-29}$	-1.084
$1/5 \cdot 2^{-11}$	$5.0 \cdot 10^{-29}$	-1.672	$5.5 \cdot 10^{-29}$	-1.611
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-28}$		$1.6 \cdot 10^{-28}$	

Table 4.48: Trapezoidal rule, IDeC method at $t = 1$ for 2.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.4 \cdot 10^{-02}$	1.00000	2.005	$2.2 \cdot 10^{-04}$	1.00000	4.014
$1/5 \cdot 2^{-2}$	$5.9 \cdot 10^{-03}$	1.00000	2.001	$1.4 \cdot 10^{-05}$	1.00000	4.003
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-03}$	1.00000	2.000	$8.8 \cdot 10^{-07}$	1.00000	4.000
$1/5 \cdot 2^{-4}$	$3.7 \cdot 10^{-04}$	1.00000	2.000	$5.5 \cdot 10^{-08}$	1.00000	4.000
$1/5 \cdot 2^{-5}$	$9.3 \cdot 10^{-05}$	1.00000	2.000	$3.4 \cdot 10^{-09}$	1.00000	4.000
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-05}$	1.00000	2.000	$2.1 \cdot 10^{-10}$	1.00000	4.000
$1/5 \cdot 2^{-7}$	$5.8 \cdot 10^{-06}$	1.00000	2.000	$1.3 \cdot 10^{-11}$	1.00000	4.000
$1/5 \cdot 2^{-8}$	$1.4 \cdot 10^{-06}$	1.00000	2.000	$8.4 \cdot 10^{-13}$	1.00000	4.000
$1/5 \cdot 2^{-9}$	$3.6 \cdot 10^{-07}$	1.00000	2.000	$5.2 \cdot 10^{-14}$	1.00000	4.000
$1/5 \cdot 2^{-10}$	$9.1 \cdot 10^{-08}$	1.00000	2.000	$3.2 \cdot 10^{-15}$	1.00000	4.000
$1/5 \cdot 2^{-11}$	$2.2 \cdot 10^{-08}$	1.00000	2.000	$2.0 \cdot 10^{-16}$	1.00000	4.000
$1/5 \cdot 2^{-12}$	$5.6 \cdot 10^{-09}$	1.00000	2.000	$1.2 \cdot 10^{-17}$	1.00000	4.000
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-09}$	1.00000	1.999	$8.0 \cdot 10^{-19}$	1.00000	3.999
$1/5 \cdot 2^{-14}$	$3.5 \cdot 10^{-10}$	1.00000		$5.0 \cdot 10^{-20}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.7 \cdot 10^{-06}$	1.00000	6.024	$3.3 \cdot 10^{-08}$	1.00000	7.950
$1/5 \cdot 2^{-2}$	$4.1 \cdot 10^{-08}$	1.00000	6.006	$1.3 \cdot 10^{-10}$	1.00000	7.989
$1/5 \cdot 2^{-3}$	$6.5 \cdot 10^{-10}$	1.00000	6.001	$5.3 \cdot 10^{-13}$	1.00000	7.997
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-11}$	1.00000	6.000	$2.0 \cdot 10^{-15}$	1.00000	7.999
$1/5 \cdot 2^{-5}$	$1.5 \cdot 10^{-13}$	1.00000	6.000	$8.1 \cdot 10^{-18}$	1.00000	7.999
$1/5 \cdot 2^{-6}$	$2.4 \cdot 10^{-15}$	1.00000	6.000	$3.1 \cdot 10^{-20}$	1.00000	7.999
$1/5 \cdot 2^{-7}$	$3.8 \cdot 10^{-17}$	1.00000	6.000	$1.2 \cdot 10^{-22}$	1.00000	7.999
$1/5 \cdot 2^{-8}$	$6.0 \cdot 10^{-19}$	1.00000	6.000	$4.8 \cdot 10^{-25}$	1.00000	7.952
$1/5 \cdot 2^{-9}$	$9.4 \cdot 10^{-21}$	1.00000	5.999	$1.9 \cdot 10^{-27}$	1.00000	4.466
$1/5 \cdot 2^{-10}$	$1.4 \cdot 10^{-22}$	1.00000	5.999	$8.8 \cdot 10^{-29}$	0.88457	-1.260
$1/5 \cdot 2^{-11}$	$2.3 \cdot 10^{-24}$	1.00000	6.012	$2.1 \cdot 10^{-28}$	0.99970	-1.872
$1/5 \cdot 2^{-12}$	$3.5 \cdot 10^{-26}$	1.00000	8.412	$7.7 \cdot 10^{-28}$	1.00000	-0.802
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-28}$	0.89060		$1.3 \cdot 10^{-27}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$3.4 \cdot 10^{-09}$	1.00000	10.299	$2.5 \cdot 10^{-09}$	1.00000	10.159
$1/5 \cdot 2^{-2}$	$2.6 \cdot 10^{-12}$	1.00000	10.053	$2.2 \cdot 10^{-12}$	1.00000	10.061
$1/5 \cdot 2^{-3}$	$2.5 \cdot 10^{-15}$	1.00000	10.012	$2.0 \cdot 10^{-15}$	1.00000	10.016
$1/5 \cdot 2^{-4}$	$2.4 \cdot 10^{-18}$	1.00000	10.003	$2.0 \cdot 10^{-18}$	1.00000	10.004
$1/5 \cdot 2^{-5}$	$2.3 \cdot 10^{-21}$	1.00000	10.000	$1.9 \cdot 10^{-21}$	1.00000	10.001
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-24}$	1.00000	10.018	$1.9 \cdot 10^{-24}$	1.00000	10.002
$1/5 \cdot 2^{-7}$	$2.2 \cdot 10^{-27}$	1.00000	5.080	$1.8 \cdot 10^{-27}$	1.00000	6.225
$1/5 \cdot 2^{-8}$	$6.6 \cdot 10^{-29}$	0.99921	-0.590	$2.4 \cdot 10^{-29}$	0.99921	-2.086
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-28}$	0.98398	0.977	$1.0 \cdot 10^{-28}$	0.99960	-0.035
$1/5 \cdot 2^{-10}$	$5.0 \cdot 10^{-29}$	0.99199	-2.207	$1.0 \cdot 10^{-28}$	0.96699	-0.324
$1/5 \cdot 2^{-11}$	$2.3 \cdot 10^{-28}$	0.98818	-1.610	$1.3 \cdot 10^{-28}$	0.99130	-2.345
$1/5 \cdot 2^{-12}$	$7.1 \cdot 10^{-28}$	0.99951		$6.8 \cdot 10^{-28}$	0.99897	

Table 4.49: Trapezoidal rule, IDeC method on $[0, 1]$ for 2.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.3 \cdot 10^{-04}$	1.00000	1.998	$3.8 \cdot 10^{-06}$	0.80000	3.994
$1/5 \cdot 2^{-2}$	$1.0 \cdot 10^{-04}$	1.00000	1.999	$2.4 \cdot 10^{-07}$	0.80000	3.996
$1/5 \cdot 2^{-3}$	$2.7 \cdot 10^{-05}$	1.00000	1.999	$1.5 \cdot 10^{-08}$	0.82500	3.999
$1/5 \cdot 2^{-4}$	$6.7 \cdot 10^{-06}$	1.00000	1.999	$9.5 \cdot 10^{-10}$	0.82500	3.999
$1/5 \cdot 2^{-5}$	$1.6 \cdot 10^{-06}$	1.00000	1.999	$5.9 \cdot 10^{-11}$	0.82500	3.999
$1/5 \cdot 2^{-6}$	$4.2 \cdot 10^{-07}$	1.00000	1.999	$3.7 \cdot 10^{-12}$	0.82500	3.999
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-07}$	1.00000	1.999	$2.3 \cdot 10^{-13}$	0.82343	3.999
$1/5 \cdot 2^{-8}$	$2.6 \cdot 10^{-08}$	1.00000	1.999	$1.4 \cdot 10^{-14}$	0.82343	3.999
$1/5 \cdot 2^{-9}$	$6.6 \cdot 10^{-09}$	1.00000	1.999	$9.1 \cdot 10^{-16}$	0.82343	3.999
$1/5 \cdot 2^{-10}$	$1.6 \cdot 10^{-09}$	1.00000	1.999	$5.6 \cdot 10^{-17}$	0.82363	3.999
$1/5 \cdot 2^{-11}$	$4.1 \cdot 10^{-10}$	1.00000	1.999	$3.5 \cdot 10^{-18}$	0.82363	3.999
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-10}$	1.00000	2.000	$2.2 \cdot 10^{-19}$	0.82363	4.000
$1/5 \cdot 2^{-13}$	$2.5 \cdot 10^{-11}$	1.00000	1.999	$1.3 \cdot 10^{-20}$	0.82360	3.999
$1/5 \cdot 2^{-14}$	$6.4 \cdot 10^{-12}$	1.00000		$8.6 \cdot 10^{-22}$	0.82360	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$8.4 \cdot 10^{-08}$	1.00000	6.026	$1.0 \cdot 10^{-08}$	1.00000	8.651
$1/5 \cdot 2^{-2}$	$1.3 \cdot 10^{-09}$	1.00000	5.999	$2.7 \cdot 10^{-11}$	1.00000	9.147
$1/5 \cdot 2^{-3}$	$2.0 \cdot 10^{-11}$	1.00000	5.999	$4.7 \cdot 10^{-14}$	1.00000	8.337
$1/5 \cdot 2^{-4}$	$3.1 \cdot 10^{-13}$	1.00000	5.999	$1.4 \cdot 10^{-16}$	0.93750	8.092
$1/5 \cdot 2^{-5}$	$4.9 \cdot 10^{-15}$	1.00000	5.999	$5.4 \cdot 10^{-19}$	0.93750	8.023
$1/5 \cdot 2^{-6}$	$7.7 \cdot 10^{-17}$	1.00000	5.999	$2.0 \cdot 10^{-21}$	0.93750	8.006
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-18}$	1.00000	5.999	$8.0 \cdot 10^{-24}$	0.93750	8.001
$1/5 \cdot 2^{-8}$	$1.8 \cdot 10^{-20}$	1.00000	5.999	$3.1 \cdot 10^{-26}$	0.93750	8.128
$1/5 \cdot 2^{-9}$	$2.9 \cdot 10^{-22}$	1.00000	6.000	$1.1 \cdot 10^{-28}$	0.92187	2.397
$1/5 \cdot 2^{-10}$	$4.6 \cdot 10^{-24}$	1.00000	6.000	$2.1 \cdot 10^{-29}$	0.98554	-0.933
$1/5 \cdot 2^{-11}$	$7.2 \cdot 10^{-26}$	1.00000	6.006	$4.0 \cdot 10^{-29}$	0.98046	-0.558
$1/5 \cdot 2^{-12}$	$1.1 \cdot 10^{-27}$	1.00000	6.249	$6.0 \cdot 10^{-29}$	0.99897	-0.932
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-29}$	0.95063		$1.1 \cdot 10^{-28}$	0.99948	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$7.5 \cdot 10^{-09}$	1.00000	8.656	$8.0 \cdot 10^{-09}$	1.00000	8.964
$1/5 \cdot 2^{-2}$	$1.8 \cdot 10^{-11}$	0.05000	10.585	$1.6 \cdot 10^{-11}$	1.00000	10.306
$1/5 \cdot 2^{-3}$	$1.2 \cdot 10^{-14}$	1.00000	10.012	$1.2 \cdot 10^{-14}$	1.00000	10.075
$1/5 \cdot 2^{-4}$	$1.1 \cdot 10^{-17}$	1.00000	10.003	$1.1 \cdot 10^{-17}$	1.00000	10.010
$1/5 \cdot 2^{-5}$	$1.1 \cdot 10^{-20}$	0.99375	10.002	$1.1 \cdot 10^{-20}$	0.99375	10.003
$1/5 \cdot 2^{-6}$	$1.1 \cdot 10^{-23}$	0.99687	10.002	$1.1 \cdot 10^{-23}$	0.99687	10.002
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-26}$	0.99843	10.508	$1.0 \cdot 10^{-26}$	0.99843	10.182
$1/5 \cdot 2^{-8}$	$7.5 \cdot 10^{-30}$	0.34453	0.743	$9.3 \cdot 10^{-30}$	0.51562	0.591
$1/5 \cdot 2^{-9}$	$4.4 \cdot 10^{-30}$	0.96132	-1.830	$6.2 \cdot 10^{-30}$	0.42148	-1.053
$1/5 \cdot 2^{-10}$	$1.5 \cdot 10^{-29}$	0.99941	-0.176	$1.2 \cdot 10^{-29}$	0.74218	-1.419
$1/5 \cdot 2^{-11}$	$1.8 \cdot 10^{-29}$	0.92480	-1.461	$3.4 \cdot 10^{-29}$	0.99990	-0.812
$1/5 \cdot 2^{-12}$	$4.9 \cdot 10^{-29}$	0.99995		$6.0 \cdot 10^{-29}$	1.00000	

Table 4.50: Trapezoidal rule, IDeC method on $[0, 1]$ for 2.3a.

Now we examine the test problems with a double eigenvalue 0. For example 3.1a, IDeC performs satisfactorily, Table 4.51. Note that this problem has the same solution and a similar structure to that of 2.1a where the same convergence properties are observed.

For examples 3.1b and 3.1c, the order sequences $O(h^2)$, $O(h^3)$, ... and $O(h^2)$, $O(h^4)$, $O(h^5)$, ... are observed, see Tables 4.52 – 4.57. For 3.1b, there is a surprising improvement of the convergence order in the last iteration step. A similar phenomenon was already encountered for IDeC based on the box scheme for this problem, cf. Table 4.30. It is interesting to note that $t^{\max} \to 0$ as $h \to 0$ for the iteration steps where the order reductions occur, but for these two examples the order sequences at a fixed point are identical to those for the maximal error on $[0, 1]$, see Tables 4.53, 4.54 and 4.56, 4.57, respectively.

The variable coefficient problem 3.2a is another example for which IDeC loses its favorable convergence properties. In Table 4.58, an order sequence $O(h^2)$, $O(h^3)$, ... can be seen, where again the last iteration step spontaneously increases the convergence order to $p = 4$.

Finally, the nonlinear problem 3.3a can be solved satisfactorily using IDeC, see Table 4.59.

From the previous results we can conclude that in contrast to §4.2 no obvious classification of the problems for which IDeC works is available. Neither the spectral properties of $M(0)$ nor the smoothness of the solution seem to be critical. Nonetheless, the results in the next section support the previous experience that the asymptotic expansions of the global error are responsible for the behavior of IDeC.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$5.3 \cdot 10^{-02}$	1.00000	1.998	$7.6 \cdot 10^{-04}$	1.00000	4.003
$1/5 \cdot 2^{-2}$	$1.3 \cdot 10^{-02}$	1.00000	1.999	$4.7 \cdot 10^{-05}$	1.00000	3.999
$1/5 \cdot 2^{-3}$	$3.3 \cdot 10^{-03}$	1.00000	1.999	$2.9 \cdot 10^{-06}$	1.00000	3.999
$1/5 \cdot 2^{-4}$	$8.3 \cdot 10^{-04}$	1.00000	1.999	$1.8 \cdot 10^{-07}$	1.00000	3.999
$1/5 \cdot 2^{-5}$	$2.0 \cdot 10^{-04}$	1.00000	1.999	$1.1 \cdot 10^{-08}$	1.00000	3.999
$1/5 \cdot 2^{-6}$	$5.1 \cdot 10^{-05}$	1.00000	1.999	$7.2 \cdot 10^{-10}$	1.00000	3.999
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-05}$	1.00000	1.999	$4.5 \cdot 10^{-11}$	1.00000	3.999
$1/5 \cdot 2^{-8}$	$3.2 \cdot 10^{-06}$	1.00000	1.999	$2.8 \cdot 10^{-12}$	1.00000	3.999
$1/5 \cdot 2^{-9}$	$8.1 \cdot 10^{-07}$	1.00000	1.999	$1.7 \cdot 10^{-13}$	1.00000	3.999
$1/5 \cdot 2^{-10}$	$2.0 \cdot 10^{-07}$	1.00000	1.999	$1.1 \cdot 10^{-14}$	1.00000	3.999
$1/5 \cdot 2^{-11}$	$5.0 \cdot 10^{-08}$	1.00000	1.999	$6.9 \cdot 10^{-16}$	1.00000	3.999
$1/5 \cdot 2^{-12}$	$1.2 \cdot 10^{-08}$	1.00000	2.000	$4.3 \cdot 10^{-17}$	1.00000	4.000
$1/5 \cdot 2^{-13}$	$3.1 \cdot 10^{-09}$	1.00000	1.999	$2.7 \cdot 10^{-18}$	1.00000	3.999
$1/5 \cdot 2^{-14}$	$7.9 \cdot 10^{-10}$	1.00000		$1.6 \cdot 10^{-19}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$9.5 \cdot 10^{-06}$	1.00000	6.118	$1.0 \cdot 10^{-06}$	1.00000	10.112
$1/5 \cdot 2^{-2}$	$1.3 \cdot 10^{-07}$	1.00000	6.004	$9.0 \cdot 10^{-10}$	1.00000	8.987
$1/5 \cdot 2^{-3}$	$2.1 \cdot 10^{-09}$	1.00000	6.000	$1.7 \cdot 10^{-12}$	1.00000	8.275
$1/5 \cdot 2^{-4}$	$3.3 \cdot 10^{-11}$	1.00000	6.000	$5.7 \cdot 10^{-15}$	1.00000	8.067
$1/5 \cdot 2^{-5}$	$5.2 \cdot 10^{-13}$	1.00000	6.000	$2.1 \cdot 10^{-17}$	1.00000	8.016
$1/5 \cdot 2^{-6}$	$8.1 \cdot 10^{-15}$	1.00000	6.000	$8.2 \cdot 10^{-20}$	1.00000	8.004
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-16}$	1.00000	6.000	$3.2 \cdot 10^{-22}$	1.00000	8.001
$1/5 \cdot 2^{-8}$	$1.9 \cdot 10^{-18}$	1.00000	6.000	$1.2 \cdot 10^{-24}$	1.00000	7.999
$1/5 \cdot 2^{-9}$	$3.1 \cdot 10^{-20}$	1.00000	5.999	$4.9 \cdot 10^{-27}$	1.00000	6.738
$1/5 \cdot 2^{-10}$	$4.8 \cdot 10^{-22}$	1.00000	6.000	$4.6 \cdot 10^{-29}$	0.99980	-0.340
$1/5 \cdot 2^{-11}$	$7.6 \cdot 10^{-24}$	1.00000	6.000	$5.8 \cdot 10^{-29}$	0.87070	-0.636
$1/5 \cdot 2^{-12}$	$1.1 \cdot 10^{-25}$	1.00000	5.925	$9.0 \cdot 10^{-29}$	0.49506	-0.919
$1/5 \cdot 2^{-13}$	$1.9 \cdot 10^{-27}$	1.00000		$1.7 \cdot 10^{-28}$	0.81765	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$9.9 \cdot 10^{-07}$	1.00000	10.775	$9.7 \cdot 10^{-07}$	1.00000	10.763
$1/5 \cdot 2^{-2}$	$5.6 \cdot 10^{-10}$	1.00000	10.504	$5.6 \cdot 10^{-10}$	1.00000	10.494
$1/5 \cdot 2^{-3}$	$3.9 \cdot 10^{-13}$	1.00000	10.203	$3.8 \cdot 10^{-13}$	1.00000	10.199
$1/5 \cdot 2^{-4}$	$3.3 \cdot 10^{-16}$	1.00000	10.032	$3.3 \cdot 10^{-16}$	1.00000	10.028
$1/5 \cdot 2^{-5}$	$3.1 \cdot 10^{-19}$	0.93750	9.999	$3.1 \cdot 10^{-19}$	0.93750	9.999
$1/5 \cdot 2^{-6}$	$3.0 \cdot 10^{-22}$	0.96875	10.000	$3.0 \cdot 10^{-22}$	0.96875	10.000
$1/5 \cdot 2^{-7}$	$3.0 \cdot 10^{-25}$	0.95468	9.994	$3.0 \cdot 10^{-25}$	0.95468	9.993
$1/5 \cdot 2^{-8}$	$2.9 \cdot 10^{-28}$	0.93046	5.122	$2.9 \cdot 10^{-28}$	0.94609	4.402
$1/5 \cdot 2^{-9}$	$8.4 \cdot 10^{-30}$	0.74804	-2.419	$1.4 \cdot 10^{-29}$	0.82460	-1.201
$1/5 \cdot 2^{-10}$	$4.5 \cdot 10^{-29}$	0.99980	-0.054	$3.2 \cdot 10^{-29}$	0.99980	-0.597
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-29}$	0.99882	-0.598	$4.8 \cdot 10^{-29}$	0.82695	-0.651
$1/5 \cdot 2^{-12}$	$7.1 \cdot 10^{-29}$	0.49511		$7.6 \cdot 10^{-29}$	0.80424	

Table 4.51: Trapezoidal rule, IDeC method on $[0, 1]$ for 3.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$5.7 \cdot 10^{-02}$	1.00000	1.999	$3.9 \cdot 10^{-04}$	0.20000	2.943
$1/5 \cdot 2^{-2}$	$1.4 \cdot 10^{-02}$	1.00000	1.999	$5.1 \cdot 10^{-05}$	0.10000	2.987
$1/5 \cdot 2^{-3}$	$3.6 \cdot 10^{-03}$	1.00000	1.999	$6.5 \cdot 10^{-06}$	0.05000	2.996
$1/5 \cdot 2^{-4}$	$9.0 \cdot 10^{-04}$	1.00000	1.999	$8.1 \cdot 10^{-07}$	0.02500	2.999
$1/5 \cdot 2^{-5}$	$2.2 \cdot 10^{-04}$	1.00000	1.999	$1.0 \cdot 10^{-07}$	0.01250	2.999
$1/5 \cdot 2^{-6}$	$5.6 \cdot 10^{-05}$	1.00000	1.999	$1.2 \cdot 10^{-08}$	0.00625	2.999
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-05}$	1.00000	1.999	$1.5 \cdot 10^{-09}$	0.00312	2.999
$1/5 \cdot 2^{-8}$	$3.5 \cdot 10^{-06}$	1.00000	1.999	$1.9 \cdot 10^{-10}$	0.00156	2.999
$1/5 \cdot 2^{-9}$	$8.8 \cdot 10^{-07}$	1.00000	1.999	$2.4 \cdot 10^{-11}$	0.00078	2.999
$1/5 \cdot 2^{-10}$	$2.2 \cdot 10^{-07}$	1.00000	1.999	$3.1 \cdot 10^{-12}$	0.00039	2.999
$1/5 \cdot 2^{-11}$	$5.5 \cdot 10^{-08}$	1.00000	1.999	$3.9 \cdot 10^{-13}$	0.00019	2.999
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-08}$	1.00000	2.000	$4.8 \cdot 10^{-14}$	0.00009	3.000
$1/5 \cdot 2^{-13}$	$3.4 \cdot 10^{-09}$	1.00000	1.999	$6.1 \cdot 10^{-15}$	0.00004	2.999
$1/5 \cdot 2^{-14}$	$8.6 \cdot 10^{-10}$	1.00000		$7.6 \cdot 10^{-16}$	0.00002	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.1 \cdot 10^{-04}$	0.30000	3.016	$1.3 \cdot 10^{-05}$	0.40000	3.040
$1/5 \cdot 2^{-2}$	$1.4 \cdot 10^{-05}$	0.15000	3.004	$1.6 \cdot 10^{-06}$	0.20000	3.010
$1/5 \cdot 2^{-3}$	$1.7 \cdot 10^{-06}$	0.07500	3.001	$2.0 \cdot 10^{-07}$	0.10000	3.002
$1/5 \cdot 2^{-4}$	$2.2 \cdot 10^{-07}$	0.03750	3.000	$2.6 \cdot 10^{-08}$	0.05000	3.000
$1/5 \cdot 2^{-5}$	$2.7 \cdot 10^{-08}$	0.01875	3.000	$3.2 \cdot 10^{-09}$	0.02500	3.000
$1/5 \cdot 2^{-6}$	$3.4 \cdot 10^{-09}$	0.00937	3.000	$4.0 \cdot 10^{-10}$	0.01250	3.000
$1/5 \cdot 2^{-7}$	$4.3 \cdot 10^{-10}$	0.00468	3.000	$5.0 \cdot 10^{-11}$	0.00625	3.000
$1/5 \cdot 2^{-8}$	$5.3 \cdot 10^{-11}$	0.00234	3.000	$6.3 \cdot 10^{-12}$	0.00312	3.000
$1/5 \cdot 2^{-9}$	$6.7 \cdot 10^{-12}$	0.00117	3.000	$7.9 \cdot 10^{-13}$	0.00156	3.000
$1/5 \cdot 2^{-10}$	$8.4 \cdot 10^{-13}$	0.00058	3.000	$9.9 \cdot 10^{-14}$	0.00078	3.000
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-13}$	0.00029	3.000	$1.2 \cdot 10^{-14}$	0.00039	3.000
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-14}$	0.00014	3.000	$1.5 \cdot 10^{-15}$	0.00034	3.000
$1/5 \cdot 2^{-13}$	$1.6 \cdot 10^{-15}$	0.00007		$1.9 \cdot 10^{-16}$	0.00017	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$6.0 \cdot 10^{-07}$	0.50000	3.113	$3.6 \cdot 10^{-09}$	0.60000	5.513
$1/5 \cdot 2^{-2}$	$6.9 \cdot 10^{-08}$	0.25000	3.029	$8.0 \cdot 10^{-11}$	0.30000	5.022
$1/5 \cdot 2^{-3}$	$8.5 \cdot 10^{-09}$	0.12500	3.007	$2.4 \cdot 10^{-12}$	0.15000	5.004
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-09}$	0.06250	3.001	$7.6 \cdot 10^{-14}$	0.07500	5.000
$1/5 \cdot 2^{-5}$	$1.3 \cdot 10^{-10}$	0.03125	3.000	$2.3 \cdot 10^{-15}$	0.03750	5.000
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-11}$	0.01562	3.000	$7.4 \cdot 10^{-17}$	0.01875	5.000
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-12}$	0.00781	3.000	$2.3 \cdot 10^{-18}$	0.00937	5.000
$1/5 \cdot 2^{-8}$	$2.5 \cdot 10^{-13}$	0.00390	3.000	$7.3 \cdot 10^{-20}$	0.00468	5.000
$1/5 \cdot 2^{-9}$	$3.2 \cdot 10^{-14}$	0.00234	3.000	$2.2 \cdot 10^{-21}$	0.00234	5.000
$1/5 \cdot 2^{-10}$	$4.0 \cdot 10^{-15}$	0.00117	3.000	$7.1 \cdot 10^{-23}$	0.00097	5.000
$1/5 \cdot 2^{-11}$	$5.0 \cdot 10^{-16}$	0.00058	3.000	$2.2 \cdot 10^{-24}$	0.00048	5.000
$1/5 \cdot 2^{-12}$	$6.3 \cdot 10^{-17}$	0.00029		$6.9 \cdot 10^{-26}$	0.00024	

Table 4.52: Trapezoidal rule, IDeC method on $[0,1]$ for 3.1b.

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$4.1 \cdot 10^{-05}$	1.974	$7.5 \cdot 10^{-07}$	2.996
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-05}$	1.987	$9.3 \cdot 10^{-08}$	2.998
$1/5 \cdot 2^{-6}$	$2.6 \cdot 10^{-06}$	1.993	$1.1 \cdot 10^{-08}$	2.999
$1/5 \cdot 2^{-7}$	$6.6 \cdot 10^{-07}$	1.996	$1.4 \cdot 10^{-09}$	2.999
$1/5 \cdot 2^{-8}$	$1.6 \cdot 10^{-07}$	1.998	$1.8 \cdot 10^{-10}$	2.999
$1/5 \cdot 2^{-9}$	$4.1 \cdot 10^{-08}$	1.999	$2.2 \cdot 10^{-11}$	2.999
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-08}$	1.999	$2.8 \cdot 10^{-12}$	2.999
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-09}$	1.999	$3.5 \cdot 10^{-13}$	2.999
$1/5 \cdot 2^{-12}$	$6.5 \cdot 10^{-10}$	1.999	$4.4 \cdot 10^{-14}$	2.999
$1/5 \cdot 2^{-13}$	$1.6 \cdot 10^{-10}$	1.999	$5.6 \cdot 10^{-15}$	2.999
$1/5 \cdot 2^{-14}$	$4.0 \cdot 10^{-11}$	1.999	$7.0 \cdot 10^{-16}$	2.999
$1/5 \cdot 2^{-15}$	$1.0 \cdot 10^{-11}$	2.000	$8.7 \cdot 10^{-17}$	3.000
$1/5 \cdot 2^{-16}$	$2.5 \cdot 10^{-12}$	1.999	$1.0 \cdot 10^{-17}$	2.999
$1/5 \cdot 2^{-17}$	$6.4 \cdot 10^{-13}$		$1.3 \cdot 10^{-18}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$1.7 \cdot 10^{-07}$	3.000	$1.5 \cdot 10^{-08}$	3.000
$1/5 \cdot 2^{-5}$	$2.1 \cdot 10^{-08}$	3.000	$1.9 \cdot 10^{-09}$	3.000
$1/5 \cdot 2^{-6}$	$2.7 \cdot 10^{-09}$	3.000	$2.4 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-7}$	$3.4 \cdot 10^{-10}$	3.000	$3.0 \cdot 10^{-11}$	3.000
$1/5 \cdot 2^{-8}$	$4.2 \cdot 10^{-11}$	3.000	$3.8 \cdot 10^{-12}$	3.000
$1/5 \cdot 2^{-9}$	$5.3 \cdot 10^{-12}$	3.000	$4.7 \cdot 10^{-13}$	3.000
$1/5 \cdot 2^{-10}$	$6.6 \cdot 10^{-13}$	3.000	$5.9 \cdot 10^{-14}$	3.000
$1/5 \cdot 2^{-11}$	$8.3 \cdot 10^{-14}$	3.000	$7.4 \cdot 10^{-15}$	3.000
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-14}$	3.000	$9.3 \cdot 10^{-16}$	3.000
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-15}$	3.000	$1.1 \cdot 10^{-16}$	3.000
$1/5 \cdot 2^{-14}$	$1.6 \cdot 10^{-16}$	3.000	$1.4 \cdot 10^{-17}$	3.000
$1/5 \cdot 2^{-15}$	$2.0 \cdot 10^{-17}$	3.000	$1.8 \cdot 10^{-18}$	3.000
$1/5 \cdot 2^{-16}$	$2.5 \cdot 10^{-18}$		$2.2 \cdot 10^{-19}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-10}$	3.003	$2.5 \cdot 10^{-14}$	5.001
$1/5 \cdot 2^{-5}$	$4.4 \cdot 10^{-11}$	3.000	$7.9 \cdot 10^{-16}$	5.000
$1/5 \cdot 2^{-6}$	$5.5 \cdot 10^{-12}$	3.000	$2.4 \cdot 10^{-17}$	5.000
$1/5 \cdot 2^{-7}$	$6.8 \cdot 10^{-13}$	3.000	$7.7 \cdot 10^{-19}$	5.000
$1/5 \cdot 2^{-8}$	$8.6 \cdot 10^{-14}$	3.000	$2.4 \cdot 10^{-20}$	5.000
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-14}$	3.000	$7.6 \cdot 10^{-22}$	5.000
$1/5 \cdot 2^{-10}$	$1.3 \cdot 10^{-15}$	3.000	$2.3 \cdot 10^{-23}$	5.000
$1/5 \cdot 2^{-11}$	$1.6 \cdot 10^{-16}$	3.000	$7.4 \cdot 10^{-25}$	5.000
$1/5 \cdot 2^{-12}$	$2.1 \cdot 10^{-17}$	3.000	$2.3 \cdot 10^{-26}$	5.000
$1/5 \cdot 2^{-13}$	$2.6 \cdot 10^{-18}$	3.000	$7.2 \cdot 10^{-28}$	5.022
$1/5 \cdot 2^{-14}$	$3.2 \cdot 10^{-19}$	3.000	$2.2 \cdot 10^{-29}$	6.253
$1/5 \cdot 2^{-15}$	$4.1 \cdot 10^{-20}$		$2.9 \cdot 10^{-31}$	

Table 4.53: Trapezoidal rule, IDeC method at $t = 1/8$ for 3.1b.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$5.7 \cdot 10^{-02}$	1.999	$1.9 \cdot 10^{-04}$	2.419
$1/5 \cdot 2^{-2}$	$1.4 \cdot 10^{-02}$	1.999	$3.6 \cdot 10^{-05}$	2.776
$1/5 \cdot 2^{-3}$	$3.6 \cdot 10^{-03}$	1.999	$5.2 \cdot 10^{-06}$	2.899
$1/5 \cdot 2^{-4}$	$9.0 \cdot 10^{-04}$	1.999	$7.0 \cdot 10^{-07}$	2.952
$1/5 \cdot 2^{-5}$	$2.2 \cdot 10^{-04}$	1.999	$9.1 \cdot 10^{-08}$	2.976
$1/5 \cdot 2^{-6}$	$5.6 \cdot 10^{-05}$	1.999	$1.1 \cdot 10^{-08}$	2.988
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-05}$	1.999	$1.4 \cdot 10^{-09}$	2.994
$1/5 \cdot 2^{-8}$	$3.5 \cdot 10^{-06}$	1.999	$1.8 \cdot 10^{-10}$	2.997
$1/5 \cdot 2^{-9}$	$8.8 \cdot 10^{-07}$	1.999	$2.2 \cdot 10^{-11}$	2.998
$1/5 \cdot 2^{-10}$	$2.2 \cdot 10^{-07}$	1.999	$2.8 \cdot 10^{-12}$	2.999
$1/5 \cdot 2^{-11}$	$5.5 \cdot 10^{-08}$	1.999	$3.5 \cdot 10^{-13}$	2.999
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-08}$	2.000	$4.4 \cdot 10^{-14}$	2.999
$1/5 \cdot 2^{-13}$	$3.4 \cdot 10^{-09}$	1.999	$5.6 \cdot 10^{-15}$	2.999
$1/5 \cdot 2^{-14}$	$8.6 \cdot 10^{-10}$		$7.0 \cdot 10^{-16}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$9.0 \cdot 10^{-05}$	3.013	$8.4 \cdot 10^{-06}$	3.053
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-05}$	3.004	$1.0 \cdot 10^{-06}$	3.013
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-06}$	3.001	$1.2 \cdot 10^{-07}$	3.003
$1/5 \cdot 2^{-4}$	$1.7 \cdot 10^{-07}$	3.000	$1.5 \cdot 10^{-08}$	3.000
$1/5 \cdot 2^{-5}$	$2.1 \cdot 10^{-08}$	3.000	$1.9 \cdot 10^{-09}$	3.000
$1/5 \cdot 2^{-6}$	$2.7 \cdot 10^{-09}$	3.000	$2.4 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-7}$	$3.4 \cdot 10^{-10}$	3.000	$3.0 \cdot 10^{-11}$	3.000
$1/5 \cdot 2^{-8}$	$4.2 \cdot 10^{-11}$	3.000	$3.8 \cdot 10^{-12}$	3.000
$1/5 \cdot 2^{-9}$	$5.3 \cdot 10^{-12}$	3.000	$4.7 \cdot 10^{-13}$	3.000
$1/5 \cdot 2^{-10}$	$6.6 \cdot 10^{-13}$	3.000	$5.9 \cdot 10^{-14}$	3.000
$1/5 \cdot 2^{-11}$	$8.3 \cdot 10^{-14}$	3.000	$7.4 \cdot 10^{-15}$	3.000
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-14}$	3.000	$9.3 \cdot 10^{-16}$	3.000
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-15}$		$1.1 \cdot 10^{-16}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$2.1 \cdot 10^{-07}$	3.194	$2.0 \cdot 10^{-09}$	6.245
$1/5 \cdot 2^{-2}$	$2.3 \cdot 10^{-08}$	3.054	$2.7 \cdot 10^{-11}$	5.068
$1/5 \cdot 2^{-3}$	$2.8 \cdot 10^{-09}$	3.013	$8.2 \cdot 10^{-13}$	5.008
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-10}$	3.003	$2.5 \cdot 10^{-14}$	5.001
$1/5 \cdot 2^{-5}$	$4.4 \cdot 10^{-11}$	3.000	$7.9 \cdot 10^{-16}$	5.000
$1/5 \cdot 2^{-6}$	$5.5 \cdot 10^{-12}$	3.000	$2.4 \cdot 10^{-17}$	5.000
$1/5 \cdot 2^{-7}$	$6.8 \cdot 10^{-13}$	3.000	$7.7 \cdot 10^{-19}$	5.000
$1/5 \cdot 2^{-8}$	$8.6 \cdot 10^{-14}$	3.000	$2.4 \cdot 10^{-20}$	5.000
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-14}$	3.000	$7.6 \cdot 10^{-22}$	5.000
$1/5 \cdot 2^{-10}$	$1.3 \cdot 10^{-15}$	3.000	$2.3 \cdot 10^{-23}$	5.000
$1/5 \cdot 2^{-11}$	$1.6 \cdot 10^{-16}$	3.000	$7.4 \cdot 10^{-25}$	5.004
$1/5 \cdot 2^{-12}$	$2.1 \cdot 10^{-17}$		$2.3 \cdot 10^{-26}$	

Table 4.54: Trapezoidal rule, IDeC method at $t = 1$ for 3.1b.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.4 \cdot 10^{-01}$	1.00000	1.996	$6.6 \cdot 10^{-03}$	1.00000	3.996
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-01}$	1.00000	1.999	$4.1 \cdot 10^{-04}$	1.00000	3.999
$1/5 \cdot 2^{-3}$	$2.8 \cdot 10^{-02}$	1.00000	1.999	$2.6 \cdot 10^{-05}$	1.00000	3.999
$1/5 \cdot 2^{-4}$	$7.0 \cdot 10^{-03}$	1.00000	1.999	$1.6 \cdot 10^{-06}$	1.00000	3.999
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-03}$	1.00000	1.999	$1.0 \cdot 10^{-07}$	1.00000	3.999
$1/5 \cdot 2^{-6}$	$4.3 \cdot 10^{-04}$	1.00000	1.999	$6.3 \cdot 10^{-09}$	1.00000	3.999
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-04}$	1.00000	1.999	$3.9 \cdot 10^{-10}$	1.00000	3.999
$1/5 \cdot 2^{-8}$	$2.7 \cdot 10^{-05}$	1.00000	1.999	$2.4 \cdot 10^{-11}$	1.00000	3.999
$1/5 \cdot 2^{-9}$	$6.8 \cdot 10^{-06}$	1.00000	1.999	$1.5 \cdot 10^{-12}$	1.00000	3.999
$1/5 \cdot 2^{-10}$	$1.7 \cdot 10^{-06}$	1.00000	1.999	$9.7 \cdot 10^{-14}$	1.00000	3.999
$1/5 \cdot 2^{-11}$	$4.2 \cdot 10^{-07}$	1.00000	1.999	$6.0 \cdot 10^{-15}$	1.00000	3.999
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-07}$	1.00000	2.000	$3.8 \cdot 10^{-16}$	1.00000	4.000
$1/5 \cdot 2^{-13}$	$2.6 \cdot 10^{-08}$	1.00000	1.999	$2.3 \cdot 10^{-17}$	1.00000	3.999
$1/5 \cdot 2^{-14}$	$6.6 \cdot 10^{-09}$	1.00000		$1.4 \cdot 10^{-18}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$5.0 \cdot 10^{-05}$	1.00000	6.002	$3.7 \cdot 10^{-06}$	0.80000	5.269
$1/5 \cdot 2^{-2}$	$7.8 \cdot 10^{-07}$	1.00000	5.999	$9.7 \cdot 10^{-08}$	0.15000	5.022
$1/5 \cdot 2^{-3}$	$1.2 \cdot 10^{-08}$	1.00000	5.196	$3.0 \cdot 10^{-09}$	0.07500	5.005
$1/5 \cdot 2^{-4}$	$3.3 \cdot 10^{-10}$	0.02500	4.995	$9.3 \cdot 10^{-11}$	0.03750	5.001
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-11}$	0.01250	4.998	$2.9 \cdot 10^{-12}$	0.01875	5.000
$1/5 \cdot 2^{-6}$	$3.2 \cdot 10^{-13}$	0.00625	4.999	$9.1 \cdot 10^{-14}$	0.00937	5.000
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-14}$	0.00312	4.999	$2.8 \cdot 10^{-15}$	0.00468	5.000
$1/5 \cdot 2^{-8}$	$3.2 \cdot 10^{-16}$	0.00156	4.999	$8.9 \cdot 10^{-17}$	0.00234	5.000
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-17}$	0.00078	4.999	$2.7 \cdot 10^{-18}$	0.00117	5.000
$1/5 \cdot 2^{-10}$	$3.1 \cdot 10^{-19}$	0.00039	4.999	$8.6 \cdot 10^{-20}$	0.00058	5.000
$1/5 \cdot 2^{-11}$	$9.7 \cdot 10^{-21}$	0.00019	4.999	$2.7 \cdot 10^{-21}$	0.00029	5.000
$1/5 \cdot 2^{-12}$	$3.0 \cdot 10^{-22}$	0.00009	5.000	$8.4 \cdot 10^{-23}$	0.00014	5.000
$1/5 \cdot 2^{-13}$	$9.5 \cdot 10^{-24}$	0.00004		$2.6 \cdot 10^{-24}$	0.00007	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-07}$	1.00000	5.364	$7.6 \cdot 10^{-07}$	1.00000	10.007
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-08}$	0.20000	5.030	$7.4 \cdot 10^{-10}$	0.30000	5.602
$1/5 \cdot 2^{-3}$	$3.6 \cdot 10^{-10}$	0.10000	5.012	$1.5 \cdot 10^{-11}$	0.15000	5.046
$1/5 \cdot 2^{-4}$	$1.1 \cdot 10^{-11}$	0.05000	5.003	$4.6 \cdot 10^{-13}$	0.07500	5.010
$1/5 \cdot 2^{-5}$	$3.5 \cdot 10^{-13}$	0.02500	5.000	$1.4 \cdot 10^{-14}$	0.03750	5.002
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{-14}$	0.01250	5.000	$4.4 \cdot 10^{-16}$	0.01875	5.000
$1/5 \cdot 2^{-7}$	$3.4 \cdot 10^{-16}$	0.00625	5.000	$1.4 \cdot 10^{-17}$	0.00937	5.000
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-17}$	0.00312	5.000	$4.3 \cdot 10^{-19}$	0.00468	5.000
$1/5 \cdot 2^{-9}$	$3.3 \cdot 10^{-19}$	0.00156	5.000	$1.3 \cdot 10^{-20}$	0.00234	5.000
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-20}$	0.00078	5.000	$4.2 \cdot 10^{-22}$	0.00117	5.000
$1/5 \cdot 2^{-11}$	$3.2 \cdot 10^{-22}$	0.00039	5.000	$1.3 \cdot 10^{-23}$	0.00058	5.000
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-23}$	0.00019		$4.1 \cdot 10^{-25}$	0.00029	

Table 4.55: Trapezoidal rule, IDeC method on $[0, 1]$ for 3.1c.

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$3.1 \cdot 10^{-06}$	1.998	$2.4 \cdot 10^{-08}$	3.983
$1/5 \cdot 2^{-5}$	$7.9 \cdot 10^{-07}$	1.999	$1.5 \cdot 10^{-09}$	3.991
$1/5 \cdot 2^{-6}$	$1.9 \cdot 10^{-07}$	1.999	$9.8 \cdot 10^{-11}$	3.995
$1/5 \cdot 2^{-7}$	$4.9 \cdot 10^{-08}$	1.999	$6.1 \cdot 10^{-12}$	3.997
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-08}$	1.999	$3.8 \cdot 10^{-13}$	3.998
$1/5 \cdot 2^{-9}$	$3.1 \cdot 10^{-09}$	1.999	$2.4 \cdot 10^{-14}$	3.999
$1/5 \cdot 2^{-10}$	$7.7 \cdot 10^{-10}$	1.999	$1.5 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-11}$	$1.9 \cdot 10^{-10}$	1.999	$9.4 \cdot 10^{-17}$	3.999
$1/5 \cdot 2^{-12}$	$4.8 \cdot 10^{-11}$	1.999	$5.9 \cdot 10^{-18}$	3.999
$1/5 \cdot 2^{-13}$	$1.2 \cdot 10^{-11}$	1.999	$3.7 \cdot 10^{-19}$	3.999
$1/5 \cdot 2^{-14}$	$3.0 \cdot 10^{-12}$	1.999	$2.3 \cdot 10^{-20}$	3.999
$1/5 \cdot 2^{-15}$	$7.5 \cdot 10^{-13}$	2.000	$1.4 \cdot 10^{-21}$	4.000
$1/5 \cdot 2^{-16}$	$1.8 \cdot 10^{-13}$	1.999	$9.0 \cdot 10^{-23}$	3.999
$1/5 \cdot 2^{-17}$	$4.7 \cdot 10^{-14}$		$5.6 \cdot 10^{-24}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$3.0 \cdot 10^{-10}$	4.978	$7.4 \cdot 10^{-11}$	5.001
$1/5 \cdot 2^{-5}$	$9.6 \cdot 10^{-12}$	4.989	$2.3 \cdot 10^{-12}$	5.000
$1/5 \cdot 2^{-6}$	$3.0 \cdot 10^{-13}$	4.994	$7.2 \cdot 10^{-14}$	5.000
$1/5 \cdot 2^{-7}$	$9.4 \cdot 10^{-15}$	4.997	$2.2 \cdot 10^{-15}$	5.000
$1/5 \cdot 2^{-8}$	$2.9 \cdot 10^{-16}$	4.998	$7.1 \cdot 10^{-17}$	5.000
$1/5 \cdot 2^{-9}$	$9.2 \cdot 10^{-18}$	4.999	$2.2 \cdot 10^{-18}$	5.000
$1/5 \cdot 2^{-10}$	$2.9 \cdot 10^{-19}$	4.999	$6.9 \cdot 10^{-20}$	5.000
$1/5 \cdot 2^{-11}$	$9.0 \cdot 10^{-21}$	4.999	$2.1 \cdot 10^{-21}$	5.000
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-22}$	4.999	$6.7 \cdot 10^{-23}$	5.000
$1/5 \cdot 2^{-13}$	$8.8 \cdot 10^{-24}$	4.999	$2.1 \cdot 10^{-24}$	5.000
$1/5 \cdot 2^{-14}$	$2.7 \cdot 10^{-25}$	4.999	$6.6 \cdot 10^{-26}$	5.000
$1/5 \cdot 2^{-15}$	$8.6 \cdot 10^{-27}$	5.000	$2.0 \cdot 10^{-27}$	5.000
$1/5 \cdot 2^{-16}$	$2.7 \cdot 10^{-28}$		$6.4 \cdot 10^{-29}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-4}$	$6.8 \cdot 10^{-12}$	5.004	$1.5 \cdot 10^{-13}$	5.018
$1/5 \cdot 2^{-5}$	$2.1 \cdot 10^{-13}$	5.001	$4.8 \cdot 10^{-15}$	5.004
$1/5 \cdot 2^{-6}$	$6.6 \cdot 10^{-15}$	5.000	$1.4 \cdot 10^{-16}$	5.001
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-16}$	5.000	$4.6 \cdot 10^{-18}$	5.000
$1/5 \cdot 2^{-8}$	$6.4 \cdot 10^{-18}$	5.000	$1.4 \cdot 10^{-19}$	5.000
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-19}$	5.000	$4.5 \cdot 10^{-21}$	5.000
$1/5 \cdot 2^{-10}$	$6.3 \cdot 10^{-21}$	5.000	$1.4 \cdot 10^{-22}$	5.000
$1/5 \cdot 2^{-11}$	$1.9 \cdot 10^{-22}$	5.000	$4.4 \cdot 10^{-24}$	5.000
$1/5 \cdot 2^{-12}$	$6.1 \cdot 10^{-24}$	5.000	$1.3 \cdot 10^{-25}$	5.000
$1/5 \cdot 2^{-13}$	$1.9 \cdot 10^{-25}$	4.999	$4.3 \cdot 10^{-27}$	5.000
$1/5 \cdot 2^{-14}$	$6.0 \cdot 10^{-27}$	4.999	$1.3 \cdot 10^{-28}$	5.000
$1/5 \cdot 2^{-15}$	$1.8 \cdot 10^{-28}$		$4.2 \cdot 10^{-30}$	

Table 4.56: Trapezoidal rule, IDeC method at $t = 1/8$ for 3.1c.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$4.4 \cdot 10^{-01}$	1.996	$6.6 \cdot 10^{-03}$	3.996
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-01}$	1.999	$4.1 \cdot 10^{-04}$	3.999
$1/5 \cdot 2^{-3}$	$2.8 \cdot 10^{-02}$	1.999	$2.6 \cdot 10^{-05}$	3.999
$1/5 \cdot 2^{-4}$	$7.0 \cdot 10^{-03}$	1.999	$1.6 \cdot 10^{-06}$	3.999
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-03}$	1.999	$1.0 \cdot 10^{-07}$	3.999
$1/5 \cdot 2^{-6}$	$4.3 \cdot 10^{-04}$	1.999	$6.3 \cdot 10^{-09}$	3.999
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-04}$	1.999	$3.9 \cdot 10^{-10}$	3.999
$1/5 \cdot 2^{-8}$	$2.7 \cdot 10^{-05}$	1.999	$2.4 \cdot 10^{-11}$	3.999
$1/5 \cdot 2^{-9}$	$6.8 \cdot 10^{-06}$	1.999	$1.5 \cdot 10^{-12}$	3.999
$1/5 \cdot 2^{-10}$	$1.7 \cdot 10^{-06}$	1.999	$9.7 \cdot 10^{-14}$	3.999
$1/5 \cdot 2^{-11}$	$4.2 \cdot 10^{-07}$	1.999	$6.0 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-07}$	2.000	$3.8 \cdot 10^{-16}$	4.000
$1/5 \cdot 2^{-13}$	$2.6 \cdot 10^{-08}$	1.999	$2.3 \cdot 10^{-17}$	3.999
$1/5 \cdot 2^{-14}$	$6.6 \cdot 10^{-09}$		$1.4 \cdot 10^{-18}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$5.0 \cdot 10^{-05}$	6.002	$3.1 \cdot 10^{-06}$	5.332
$1/5 \cdot 2^{-2}$	$7.8 \cdot 10^{-07}$	5.999	$7.7 \cdot 10^{-08}$	5.017
$1/5 \cdot 2^{-3}$	$1.2 \cdot 10^{-08}$	5.999	$2.3 \cdot 10^{-09}$	5.004
$1/5 \cdot 2^{-4}$	$1.9 \cdot 10^{-10}$	4.832	$7.4 \cdot 10^{-11}$	5.001
$1/5 \cdot 2^{-5}$	$6.7 \cdot 10^{-12}$	4.709	$2.3 \cdot 10^{-12}$	5.000
$1/5 \cdot 2^{-6}$	$2.5 \cdot 10^{-13}$	4.874	$7.2 \cdot 10^{-14}$	5.000
$1/5 \cdot 2^{-7}$	$8.7 \cdot 10^{-15}$	4.940	$2.2 \cdot 10^{-15}$	5.000
$1/5 \cdot 2^{-8}$	$2.8 \cdot 10^{-16}$	4.971	$7.1 \cdot 10^{-17}$	5.000
$1/5 \cdot 2^{-9}$	$9.1 \cdot 10^{-18}$	4.985	$2.2 \cdot 10^{-18}$	5.000
$1/5 \cdot 2^{-10}$	$2.8 \cdot 10^{-19}$	4.992	$6.9 \cdot 10^{-20}$	5.000
$1/5 \cdot 2^{-11}$	$9.0 \cdot 10^{-21}$	4.996	$2.1 \cdot 10^{-21}$	4.999
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-22}$	4.998	$6.7 \cdot 10^{-23}$	5.000
$1/5 \cdot 2^{-13}$	$8.8 \cdot 10^{-24}$		$2.1 \cdot 10^{-24}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-07}$	6.146	$7.6 \cdot 10^{-07}$	10.153
$1/5 \cdot 2^{-2}$	$6.9 \cdot 10^{-09}$	4.975	$6.7 \cdot 10^{-10}$	6.906
$1/5 \cdot 2^{-3}$	$2.2 \cdot 10^{-10}$	5.015	$5.6 \cdot 10^{-12}$	5.168
$1/5 \cdot 2^{-4}$	$6.8 \cdot 10^{-12}$	5.004	$1.5 \cdot 10^{-13}$	5.021
$1/5 \cdot 2^{-5}$	$2.1 \cdot 10^{-13}$	5.001	$4.8 \cdot 10^{-15}$	5.004
$1/5 \cdot 2^{-6}$	$6.6 \cdot 10^{-15}$	5.000	$1.4 \cdot 10^{-16}$	5.001
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-16}$	5.000	$4.6 \cdot 10^{-18}$	5.000
$1/5 \cdot 2^{-8}$	$6.4 \cdot 10^{-18}$	5.000	$1.4 \cdot 10^{-19}$	5.000
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-19}$	5.000	$4.5 \cdot 10^{-21}$	5.000
$1/5 \cdot 2^{-10}$	$6.3 \cdot 10^{-21}$	5.000	$1.4 \cdot 10^{-22}$	5.000
$1/5 \cdot 2^{-11}$	$1.9 \cdot 10^{-22}$	5.000	$4.4 \cdot 10^{-24}$	4.999
$1/5 \cdot 2^{-12}$	$6.1 \cdot 10^{-24}$		$1.3 \cdot 10^{-25}$	

Table 4.57: Trapezoidal rule, IDeC method at $t = 1$ for 3.1c.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.2 \cdot 10^{-01}$	1.00000	1.974	$1.3 \cdot 10^{-03}$	1.00000	0.820
$1/5 \cdot 2^{-2}$	$5.7 \cdot 10^{-02}$	1.00000	1.976	$7.7 \cdot 10^{-04}$	1.00000	2.392
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-02}$	1.00000	1.984	$1.4 \cdot 10^{-04}$	1.00000	2.711
$1/5 \cdot 2^{-4}$	$3.7 \cdot 10^{-03}$	1.00000	1.991	$2.2 \cdot 10^{-05}$	1.00000	2.849
$1/5 \cdot 2^{-5}$	$9.3 \cdot 10^{-04}$	1.00000	1.995	$3.1 \cdot 10^{-06}$	1.00000	2.919
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-04}$	1.00000	1.997	$4.1 \cdot 10^{-07}$	1.00000	2.956
$1/5 \cdot 2^{-7}$	$5.8 \cdot 10^{-05}$	1.00000	1.998	$5.3 \cdot 10^{-08}$	1.00000	2.976
$1/5 \cdot 2^{-8}$	$1.4 \cdot 10^{-05}$	1.00000	1.999	$6.7 \cdot 10^{-09}$	1.00000	2.987
$1/5 \cdot 2^{-9}$	$3.6 \cdot 10^{-06}$	1.00000	1.999	$8.5 \cdot 10^{-10}$	1.00000	2.993
$1/5 \cdot 2^{-10}$	$9.1 \cdot 10^{-07}$	1.00000	1.999	$1.0 \cdot 10^{-10}$	1.00000	2.996
$1/5 \cdot 2^{-11}$	$2.2 \cdot 10^{-07}$	1.00000	1.999	$1.3 \cdot 10^{-11}$	1.00000	2.998
$1/5 \cdot 2^{-12}$	$5.7 \cdot 10^{-08}$	1.00000	2.000	$1.6 \cdot 10^{-12}$	1.00000	2.999
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-08}$	1.00000	1.999	$2.0 \cdot 10^{-13}$	1.00000	2.999
$1/5 \cdot 2^{-14}$	$3.5 \cdot 10^{-09}$	1.00000		$2.6 \cdot 10^{-14}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.8 \cdot 10^{-03}$	1.00000	2.768	$6.0 \cdot 10^{-04}$	1.00000	3.493
$1/5 \cdot 2^{-2}$	$2.6 \cdot 10^{-04}$	1.00000	2.768	$5.4 \cdot 10^{-05}$	1.00000	3.304
$1/5 \cdot 2^{-3}$	$3.8 \cdot 10^{-05}$	1.00000	2.825	$5.4 \cdot 10^{-06}$	1.00000	3.158
$1/5 \cdot 2^{-4}$	$5.4 \cdot 10^{-06}$	1.00000	2.887	$6.1 \cdot 10^{-07}$	1.00000	3.062
$1/5 \cdot 2^{-5}$	$7.3 \cdot 10^{-07}$	1.00000	2.934	$7.3 \cdot 10^{-08}$	1.00000	3.016
$1/5 \cdot 2^{-6}$	$9.6 \cdot 10^{-08}$	1.00000	2.963	$9.0 \cdot 10^{-09}$	1.00000	3.000
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-08}$	1.00000	2.980	$1.1 \cdot 10^{-09}$	1.00000	2.996
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-09}$	1.00000	2.989	$1.4 \cdot 10^{-10}$	1.00000	2.996
$1/5 \cdot 2^{-9}$	$1.9 \cdot 10^{-10}$	1.00000	2.994	$1.7 \cdot 10^{-11}$	1.00000	2.997
$1/5 \cdot 2^{-10}$	$2.4 \cdot 10^{-11}$	1.00000	2.997	$2.2 \cdot 10^{-12}$	1.00000	2.998
$1/5 \cdot 2^{-11}$	$3.1 \cdot 10^{-12}$	1.00000	2.998	$2.7 \cdot 10^{-13}$	1.00000	2.999
$1/5 \cdot 2^{-12}$	$3.8 \cdot 10^{-13}$	1.00000	2.999	$3.4 \cdot 10^{-14}$	1.00000	2.999
$1/5 \cdot 2^{-13}$	$4.8 \cdot 10^{-14}$	1.00000		$4.3 \cdot 10^{-15}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.4 \cdot 10^{-04}$	1.00000	3.911	$9.1 \cdot 10^{-05}$	1.00000	4.350
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-05}$	1.00000	3.839	$4.5 \cdot 10^{-06}$	1.00000	4.149
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-06}$	1.00000	3.835	$2.5 \cdot 10^{-07}$	1.00000	4.056
$1/5 \cdot 2^{-4}$	$8.0 \cdot 10^{-08}$	1.00000	3.795	$1.5 \cdot 10^{-08}$	1.00000	4.020
$1/5 \cdot 2^{-5}$	$5.7 \cdot 10^{-09}$	1.00000	3.679	$9.4 \cdot 10^{-10}$	1.00000	4.014
$1/5 \cdot 2^{-6}$	$4.5 \cdot 10^{-10}$	1.00000	3.501	$5.8 \cdot 10^{-11}$	1.00000	4.018
$1/5 \cdot 2^{-7}$	$3.9 \cdot 10^{-11}$	1.00000	3.313	$3.5 \cdot 10^{-12}$	1.00000	4.025
$1/5 \cdot 2^{-8}$	$4.0 \cdot 10^{-12}$	1.00000	3.170	$2.2 \cdot 10^{-13}$	1.00000	4.030
$1/5 \cdot 2^{-9}$	$4.4 \cdot 10^{-13}$	1.00000	3.083	$1.3 \cdot 10^{-14}$	1.00000	4.034
$1/5 \cdot 2^{-10}$	$5.2 \cdot 10^{-14}$	1.00000	3.038	$8.2 \cdot 10^{-16}$	1.00000	4.037
$1/5 \cdot 2^{-11}$	$6.4 \cdot 10^{-15}$	1.00000	3.016	$5.0 \cdot 10^{-17}$	1.00000	4.039
$1/5 \cdot 2^{-12}$	$7.9 \cdot 10^{-16}$	1.00000		$3.0 \cdot 10^{-18}$	1.00000	

Table 4.58: Trapezoidal rule, IDeC method on $[0, 1]$ for 3.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.0 \cdot 10^{-03}$	0.80000	1.995	$5.8 \cdot 10^{-05}$	1.00000	3.677
$1/5 \cdot 2^{-2}$	$5.0 \cdot 10^{-04}$	0.80000	1.996	$4.5 \cdot 10^{-06}$	1.00000	3.998
$1/5 \cdot 2^{-3}$	$1.2 \cdot 10^{-04}$	0.77500	1.999	$2.8 \cdot 10^{-07}$	1.00000	3.998
$1/5 \cdot 2^{-4}$	$3.1 \cdot 10^{-05}$	0.77500	1.999	$1.7 \cdot 10^{-08}$	1.00000	3.999
$1/5 \cdot 2^{-5}$	$7.9 \cdot 10^{-06}$	0.77500	1.999	$1.1 \cdot 10^{-09}$	1.00000	3.999
$1/5 \cdot 2^{-6}$	$1.9 \cdot 10^{-06}$	0.77187	1.999	$6.9 \cdot 10^{-11}$	1.00000	3.999
$1/5 \cdot 2^{-7}$	$4.9 \cdot 10^{-07}$	0.77343	1.999	$4.3 \cdot 10^{-12}$	1.00000	3.999
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-07}$	0.77343	1.999	$2.7 \cdot 10^{-13}$	1.00000	3.999
$1/5 \cdot 2^{-9}$	$3.0 \cdot 10^{-08}$	0.77304	1.999	$1.7 \cdot 10^{-14}$	1.00000	3.999
$1/5 \cdot 2^{-10}$	$7.7 \cdot 10^{-09}$	0.77324	1.999	$1.0 \cdot 10^{-15}$	1.00000	3.999
$1/5 \cdot 2^{-11}$	$1.9 \cdot 10^{-09}$	0.77324	1.999	$6.6 \cdot 10^{-17}$	1.00000	3.999
$1/5 \cdot 2^{-12}$	$4.8 \cdot 10^{-10}$	0.77324	2.000	$4.1 \cdot 10^{-18}$	1.00000	4.000
$1/5 \cdot 2^{-13}$	$1.2 \cdot 10^{-10}$	0.77321	1.999	$2.5 \cdot 10^{-19}$	1.00000	3.999
$1/5 \cdot 2^{-14}$	$3.0 \cdot 10^{-11}$	0.77321		$1.6 \cdot 10^{-20}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$5.1 \cdot 10^{-06}$	0.10000	6.034	$6.3 \cdot 10^{-06}$	0.90000	9.859
$1/5 \cdot 2^{-2}$	$7.8 \cdot 10^{-08}$	0.90000	6.037	$6.8 \cdot 10^{-09}$	0.95000	8.401
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-09}$	0.87500	6.007	$2.0 \cdot 10^{-11}$	0.97500	9.164
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-11}$	0.87500	5.999	$3.5 \cdot 10^{-14}$	0.92500	8.308
$1/5 \cdot 2^{-5}$	$2.9 \cdot 10^{-13}$	0.87500	5.999	$1.1 \cdot 10^{-16}$	0.85625	8.064
$1/5 \cdot 2^{-6}$	$4.5 \cdot 10^{-15}$	0.87812	5.999	$4.1 \cdot 10^{-19}$	0.83437	8.014
$1/5 \cdot 2^{-7}$	$7.1 \cdot 10^{-17}$	0.87656	5.999	$1.6 \cdot 10^{-21}$	0.82968	8.003
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-18}$	0.87656	5.999	$6.2 \cdot 10^{-24}$	0.82890	8.001
$1/5 \cdot 2^{-9}$	$1.7 \cdot 10^{-20}$	0.87695	5.999	$2.4 \cdot 10^{-26}$	0.82851	8.266
$1/5 \cdot 2^{-10}$	$2.7 \cdot 10^{-22}$	0.87675	6.000	$7.9 \cdot 10^{-29}$	0.36152	0.877
$1/5 \cdot 2^{-11}$	$4.2 \cdot 10^{-24}$	0.87675	6.000	$4.3 \cdot 10^{-29}$	0.99472	-0.926
$1/5 \cdot 2^{-12}$	$6.6 \cdot 10^{-26}$	0.87749	6.045	$8.2 \cdot 10^{-29}$	0.99750	-0.927
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-27}$	0.86687		$1.5 \cdot 10^{-28}$	0.99848	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$7.9 \cdot 10^{-06}$	0.90000	10.419	$7.9 \cdot 10^{-06}$	0.90000	10.471
$1/5 \cdot 2^{-2}$	$5.8 \cdot 10^{-09}$	0.95000	8.679	$5.6 \cdot 10^{-09}$	0.95000	8.649
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-11}$	0.97500	10.637	$1.4 \cdot 10^{-11}$	0.97500	10.632
$1/5 \cdot 2^{-4}$	$8.9 \cdot 10^{-15}$	1.00000	10.364	$8.8 \cdot 10^{-15}$	1.00000	10.359
$1/5 \cdot 2^{-5}$	$6.7 \cdot 10^{-18}$	1.00000	10.137	$6.7 \cdot 10^{-18}$	1.00000	10.134
$1/5 \cdot 2^{-6}$	$6.0 \cdot 10^{-21}$	1.00000	10.042	$5.9 \cdot 10^{-21}$	1.00000	10.042
$1/5 \cdot 2^{-7}$	$5.7 \cdot 10^{-24}$	1.00000	10.013	$5.6 \cdot 10^{-24}$	1.00000	10.012
$1/5 \cdot 2^{-8}$	$5.5 \cdot 10^{-27}$	1.00000	10.188	$5.4 \cdot 10^{-27}$	1.00000	10.550
$1/5 \cdot 2^{-9}$	$4.7 \cdot 10^{-30}$	0.94687	-1.311	$3.6 \cdot 10^{-30}$	0.25781	-2.849
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-29}$	0.99628	-1.454	$2.6 \cdot 10^{-29}$	0.99785	-0.733
$1/5 \cdot 2^{-11}$	$3.2 \cdot 10^{-29}$	0.99794	-1.094	$4.3 \cdot 10^{-29}$	1.00000	-0.741
$1/5 \cdot 2^{-12}$	$6.8 \cdot 10^{-29}$	0.99995		$7.3 \cdot 10^{-29}$	0.99536	

Table 4.59: Trapezoidal rule, IDeC method on $[0, 1]$ for 3.3a.

4.3.2   Extrapolation

The aim of this section is to compare the performance of the extrapolation method based on the trapezoidal rule with that of IDeC. From §3.3 it is clear that the order sequences that can be obtained by Richardson extrapolation are tightly linked to the existence of an asymptotic error expansion (3.13). Thus we can conclude that the reason for the failure of IDeC is indeed the break-down of the error expansion if the analogous asymptotic properties are observed for extrapolation.

It turns out that this is indeed the case, extrapolation breaks down in exactly the same manner as IDeC[12]. Even the points $t^{\max}$ where the maximum of the error occurs show the same behavior. For the detailed results, see Tables 4.60 – 4.70.

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[12]Note however that the spontaneous increase in the convergence order in the last step of the IDeC iteration is not observed for extrapolation. This phenomenon can clearly be attributed to technical features of IDeC and is independent of the asymptotic expansion of the global error.

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.326E+00$	1.00000	1.987	$0.800E+01$
$1/5 \cdot 2^{-1}$	$0.823E-01$	1.00000	1.997	$0.817E+01$
$1/5 \cdot 2^{-2}$	$0.206E-01$	1.00000	1.999	$0.823E+01$
$1/5 \cdot 2^{-3}$	$0.516E-02$	1.00000	2.000	$0.825E+01$
$1/5 \cdot 2^{-4}$	$0.129E-02$	1.00000	2.000	$0.825E+01$
$1/5 \cdot 2^{-5}$	$0.322E-03$	1.00000	2.000	$0.825E+01$
$1/5 \cdot 2^{-6}$	$0.806E-04$	1.00000	2.000	$0.825E+01$
$1/5 \cdot 2^{-7}$	$0.202E-04$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.950E-03$	1.00000	3.172	$0.141E+01$
$1/5 \cdot 2^{-2}$	$0.105E-03$	0.10000	2.974	$0.780E+00$
$1/5 \cdot 2^{-3}$	$0.134E-04$	0.05000	2.994	$0.840E+00$
$1/5 \cdot 2^{-4}$	$0.168E-05$	0.02500	2.999	$0.857E+00$
$1/5 \cdot 2^{-5}$	$0.211E-06$	0.01250	3.000	$0.862E+00$
$1/5 \cdot 2^{-6}$	$0.263E-07$	0.00625	3.000	$0.863E+00$
$1/5 \cdot 2^{-7}$	$0.329E-08$	0.00313		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.299E-03$	0.20000	2.975	$0.222E+01$
$1/5 \cdot 2^{-3}$	$0.381E-04$	0.10000	2.994	$0.238E+01$
$1/5 \cdot 2^{-4}$	$0.478E-05$	0.05000	2.998	$0.243E+01$
$1/5 \cdot 2^{-5}$	$0.598E-06$	0.02500	3.000	$0.245E+01$
$1/5 \cdot 2^{-6}$	$0.748E-07$	0.01250	3.000	$0.245E+01$
$1/5 \cdot 2^{-7}$	$0.935E-08$	0.00625		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.224E-04$	0.20000	2.984	$0.135E+01$
$1/5 \cdot 2^{-4}$	$0.283E-05$	0.10000	2.996	$0.142E+01$
$1/5 \cdot 2^{-5}$	$0.355E-06$	0.05000	2.999	$0.145E+01$
$1/5 \cdot 2^{-6}$	$0.444E-07$	0.02500	3.000	$0.145E+01$
$1/5 \cdot 2^{-7}$	$0.555E-08$	0.01250		

Table 4.60: Trapezoidal rule, basic sol. and extrapolation steps $1-3$ for 1.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.140E-01$	0.40000	2.301	$0.568E+00$
$1/5 \cdot 2^{-1}$	$0.284E-02$	0.40000	2.040	$0.311E+00$
$1/5 \cdot 2^{-2}$	$0.689E-03$	0.45000	2.001	$0.277E+00$
$1/5 \cdot 2^{-3}$	$0.172E-03$	0.42500	2.000	$0.276E+00$
$1/5 \cdot 2^{-4}$	$0.430E-04$	0.43750	2.000	$0.275E+00$
$1/5 \cdot 2^{-5}$	$0.108E-04$	0.43125	2.000	$0.275E+00$
$1/5 \cdot 2^{-6}$	$0.269E-05$	0.43437	2.000	$0.275E+00$
$1/5 \cdot 2^{-7}$	$0.672E-06$	0.43281		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.240E-02$	0.20000	2.990	$0.234E+01$
$1/5 \cdot 2^{-2}$	$0.302E-03$	0.10000	2.998	$0.240E+01$
$1/5 \cdot 2^{-3}$	$0.377E-04$	0.05000	3.000	$0.241E+01$
$1/5 \cdot 2^{-4}$	$0.472E-05$	0.02500	3.000	$0.242E+01$
$1/5 \cdot 2^{-5}$	$0.590E-06$	0.01250	3.000	$0.242E+01$
$1/5 \cdot 2^{-6}$	$0.737E-07$	0.00625	3.000	$0.242E+01$
$1/5 \cdot 2^{-7}$	$0.922E-08$	0.00313		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.313E-03$	0.20000	3.068	$0.307E+01$
$1/5 \cdot 2^{-3}$	$0.373E-04$	0.10000	3.036	$0.272E+01$
$1/5 \cdot 2^{-4}$	$0.455E-05$	0.05000	3.018	$0.252E+01$
$1/5 \cdot 2^{-5}$	$0.562E-06$	0.02500	3.009	$0.241E+01$
$1/5 \cdot 2^{-6}$	$0.697E-07$	0.01250	3.005	$0.235E+01$
$1/5 \cdot 2^{-7}$	$0.869E-08$	0.00625		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.491E-05$	0.20000	3.029	$0.349E+00$
$1/5 \cdot 2^{-4}$	$0.602E-06$	0.10000	2.987	$0.291E+00$
$1/5 \cdot 2^{-5}$	$0.759E-07$	0.05000	2.986	$0.290E+00$
$1/5 \cdot 2^{-6}$	$0.957E-08$	0.02500	2.991	$0.299E+00$
$1/5 \cdot 2^{-7}$	$0.120E-08$	0.01250		

Table 4.61: Trapezoidal rule, basic sol. and extrapolation steps $1-3$ for 1.2a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.148 \cdot 10^{+00}$	1.00000	2.000	$0.370 \cdot 10^{+01}$
$1/5 \cdot 2^{-1}$	$0.370 \cdot 10^{-01}$	1.00000	2.000	$0.370 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.925 \cdot 10^{-02}$	1.00000	2.000	$0.370 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.231 \cdot 10^{-02}$	1.00000	2.000	$0.370 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.578 \cdot 10^{-03}$	1.00000	2.000	$0.370 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.145 \cdot 10^{-03}$	1.00000	2.000	$0.370 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.361 \cdot 10^{-04}$	1.00000	2.000	$0.370 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.903 \cdot 10^{-05}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.471 \cdot 10^{-03}$	1.00000	4.026	$0.500 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.289 \cdot 10^{-04}$	1.00000	4.006	$0.471 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.180 \cdot 10^{-05}$	1.00000	3.961	$0.399 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.115 \cdot 10^{-06}$	0.97500	4.139	$0.869 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.655 \cdot 10^{-08}$	0.65000	3.777	$0.138 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.478 \cdot 10^{-09}$	0.85625	4.000	$0.501 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.299 \cdot 10^{-10}$	0.85312		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.561 \cdot 10^{-06}$	1.00000	6.067	$0.439 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.837 \cdot 10^{-08}$	0.80000	5.960	$0.296 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.134 \cdot 10^{-09}$	1.00000	5.979	$0.321 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.213 \cdot 10^{-11}$	0.90000	6.023	$0.401 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.328 \cdot 10^{-13}$	1.00000	5.975	$0.304 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.521 \cdot 10^{-15}$	0.92500		

Table 4.62: Trapezoidal rule, basic sol. and extrapolation steps 1,2 for 2.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.135 \cdot 10^{-09}$	1.00000	8.020	$0.949 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.518 \cdot 10^{-12}$	1.00000	8.005	$0.889 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.202 \cdot 10^{-14}$	1.00000	8.417	$0.720 \cdot 10^{+04}$
$1/5 \cdot 2^{-6}$	$0.590 \cdot 10^{-17}$	0.65000	7.581	$0.578 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.308 \cdot 10^{-19}$	0.97500		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-4}$	$0.649 \cdot 10^{-14}$	0.80000	9.867	$0.388 \cdot 10^{+05}$
$1/5 \cdot 2^{-5}$	$0.695 \cdot 10^{-17}$	1.00000	10.043	$0.950 \cdot 10^{+05}$
$1/5 \cdot 2^{-6}$	$0.659 \cdot 10^{-20}$	0.90000	9.963	$0.599 \cdot 10^{+05}$
$1/5 \cdot 2^{-7}$	$0.660 \cdot 10^{-23}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-5}$	$0.899 \cdot 10^{-19}$	1.00000	12.018	$0.277 \cdot 10^{+08}$
$1/5 \cdot 2^{-6}$	$0.217 \cdot 10^{-22}$	1.00000	12.517	$0.492 \cdot 10^{+09}$
$1/5 \cdot 2^{-7}$	$0.370 \cdot 10^{-26}$	0.65000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-6}$	$0.267 \cdot 10^{-24}$	1.00000	13.956	$0.244 \cdot 10^{+11}$
$1/5 \cdot 2^{-7}$	$0.168 \cdot 10^{-28}$	0.90000		

Table 4.63: Trapezoidal rule, extrapolation steps 3 – 6 for 2.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.976 \cdot 10^{-01}$	1.00000	2.024	$0.254 \cdot 10^{+01}$
$1/5 \cdot 2^{-1}$	$0.240 \cdot 10^{-01}$	1.00000	2.006	$0.243 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.598 \cdot 10^{-02}$	1.00000	2.001	$0.240 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.149 \cdot 10^{-02}$	1.00000	2.000	$0.239 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.373 \cdot 10^{-03}$	1.00000	2.000	$0.239 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.933 \cdot 10^{-04}$	1.00000	2.000	$0.239 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.233 \cdot 10^{-04}$	1.00000	2.000	$0.239 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.583 \cdot 10^{-05}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.539 \cdot 10^{-03}$	1.00000	4.032	$0.580 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.329 \cdot 10^{-04}$	1.00000	4.008	$0.540 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.205 \cdot 10^{-05}$	1.00000	4.002	$0.528 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.128 \cdot 10^{-06}$	1.00000	4.000	$0.524 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.798 \cdot 10^{-08}$	1.00000	4.000	$0.523 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.499 \cdot 10^{-09}$	1.00000	4.000	$0.523 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.312 \cdot 10^{-10}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.790 \cdot 10^{-06}$	1.00000	6.035	$0.561 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.120 \cdot 10^{-07}$	1.00000	6.009	$0.509 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.187 \cdot 10^{-09}$	1.00000	6.002	$0.495 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.292 \cdot 10^{-11}$	1.00000	6.001	$0.491 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.456 \cdot 10^{-13}$	1.00000	6.000	$0.490 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.712 \cdot 10^{-15}$	1.00000		

Table 4.64: Trapezoidal rule, basic sol. and extrapolation steps 1,2 for 2.2a

4.4   The Explicit Euler Method

In the view of the failure of higher order methods, it seems natural to try the (computationally cheaper) explicit Euler method instead of the implicit Euler rule as a basis for IDeC. Although the classical convergence order of the explicit Euler, $O(h)$, holds (see [14]), IDeC based on this method does not work satisfactorily for (4.1). Due to an apparent break-down of the asymptotic error expansion, for the majority of test problems no higher convergence order than 2 was observed. As in §4.3 this was confirmed by the identical behavior of the extrapolation method. In the following we give the complete numerical results for our test problems to illustrate this statement, where we apply IDeC with polynomial degree $m = 5$ for the construction of the neighboring problems.

First we look at Tables 4.71 – 4.73 for problem 1.1a. As for many previous examples, $t^{\max} \to 0$ as $h \to 0$ for the iteration steps where order reductions (compared to classical results) are observed. Thus, at $t = 1/8$ and $t = 1$ the classical convergence order $O(h^5)$ can be obtained with polynomials of degree 5. Note that the order sequence appears to be $O(h)$, $O(h^2)$, $O(h^3)$, $O(h^5)$, $O(h^5)$, ... This is no contradiction to the theory, however.

Tables 4.74 – 4.76 contain similar results. In this case, the recovery of the convergence order at a fixed point only takes place up to $O(h^3)$.

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.297 \cdot 10^{-09}$	1.00000	8.036	$0.222 \cdot 10^{+04}$
$1/5 \cdot 2^{-4}$	$0.113 \cdot 10^{-11}$	1.00000	8.009	$0.198 \cdot 10^{+04}$
$1/5 \cdot 2^{-5}$	$0.440 \cdot 10^{-14}$	1.00000	8.002	$0.191 \cdot 10^{+04}$
$1/5 \cdot 2^{-6}$	$0.172 \cdot 10^{-16}$	1.00000	8.001	$0.189 \cdot 10^{+04}$
$1/5 \cdot 2^{-7}$	$0.670 \cdot 10^{-19}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-4}$	$0.284 \cdot 10^{-13}$	1.00000	10.036	$0.357 \cdot 10^{+06}$
$1/5 \cdot 2^{-5}$	$0.270 \cdot 10^{-16}$	1.00000	10.009	$0.311 \cdot 10^{+06}$
$1/5 \cdot 2^{-6}$	$0.263 \cdot 10^{-19}$	1.00000	10.002	$0.299 \cdot 10^{+06}$
$1/5 \cdot 2^{-7}$	$0.256 \cdot 10^{-22}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-5}$	$0.683 \cdot 10^{-18}$	1.00000	12.036	$0.231 \cdot 10^{+09}$
$1/5 \cdot 2^{-6}$	$0.163 \cdot 10^{-21}$	1.00000	12.008	$0.197 \cdot 10^{+09}$
$1/5 \cdot 2^{-7}$	$0.395 \cdot 10^{-25}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-6}$	$0.413 \cdot 10^{-23}$	1.00000	14.119	$0.969 \cdot 10^{+12}$
$1/5 \cdot 2^{-7}$	$0.232 \cdot 10^{-27}$	1.00000		

Table 4.65: Trapezoidal rule, extrapolation steps 3 – 6 for 2.2a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.212 \cdot 10^{+00}$	1.00000	1.994	$0.524 \cdot 10^{+01}$
$1/5 \cdot 2^{-1}$	$0.531 \cdot 10^{-01}$	1.00000	1.999	$0.529 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.133 \cdot 10^{-01}$	1.00000	2.000	$0.531 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.332 \cdot 10^{-02}$	1.00000	2.000	$0.532 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.831 \cdot 10^{-03}$	1.00000	2.000	$0.532 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.208 \cdot 10^{-03}$	1.00000	2.000	$0.532 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.519 \cdot 10^{-04}$	1.00000	2.000	$0.532 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.130 \cdot 10^{-04}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.383 \cdot 10^{-03}$	1.00000	4.017	$0.399 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.237 \cdot 10^{-04}$	1.00000	4.004	$0.384 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.148 \cdot 10^{-05}$	1.00000	4.001	$0.379 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.921 \cdot 10^{-07}$	1.00000	3.986	$0.356 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.581 \cdot 10^{-08}$	0.98750	3.948	$0.292 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.377 \cdot 10^{-09}$	0.88750	4.000	$0.395 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.235 \cdot 10^{-10}$	0.88437		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.319 \cdot 10^{-06}$	1.00000	6.021	$0.217 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.491 \cdot 10^{-08}$	1.00000	6.005	$0.205 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.764 \cdot 10^{-10}$	1.00000	6.001	$0.201 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.119 \cdot 10^{-11}$	1.00000	6.000	$0.200 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.186 \cdot 10^{-13}$	1.00000	6.000	$0.200 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.291 \cdot 10^{-15}$	1.00000		

Table 4.66: Trapezoidal rule, basic sol. and extrapolation steps 1,2 for 3.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.719 \cdot 10^{-10}$	1.00000	8.019	$0.505 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.277 \cdot 10^{-12}$	1.00000	8.005	$0.475 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.108 \cdot 10^{-14}$	1.00000	8.001	$0.466 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.421 \cdot 10^{-17}$	1.00000	8.000	$0.464 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.165 \cdot 10^{-19}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-4}$	$0.365 \cdot 10^{-14}$	1.00000	10.018	$0.423 \cdot 10^{+05}$
$1/5 \cdot 2^{-5}$	$0.352 \cdot 10^{-17}$	1.00000	10.004	$0.396 \cdot 10^{+05}$
$1/5 \cdot 2^{-6}$	$0.343 \cdot 10^{-20}$	1.00000	10.001	$0.388 \cdot 10^{+05}$
$1/5 \cdot 2^{-7}$	$0.334 \cdot 10^{-23}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-5}$	$0.431 \cdot 10^{-19}$	1.00000	12.017	$0.132 \cdot 10^{+08}$
$1/5 \cdot 2^{-6}$	$0.104 \cdot 10^{-22}$	1.00000	12.004	$0.123 \cdot 10^{+08}$
$1/5 \cdot 2^{-7}$	$0.253 \cdot 10^{-26}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-6}$	$0.121 \cdot 10^{-24}$	1.00000	14.026	$0.167 \cdot 10^{+11}$
$1/5 \cdot 2^{-7}$	$0.727 \cdot 10^{-29}$	1.00000		

Table 4.67: Trapezoidal rule, extrapolation steps 3 – 6 for 3.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.178 \cdot 10^{+00}$	1.00000	1.978	$0.429 \cdot 10^{+01}$
$1/5 \cdot 2^{-1}$	$0.452 \cdot 10^{-01}$	1.00000	1.989	$0.440 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.114 \cdot 10^{-01}$	1.00000	1.994	$0.447 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.286 \cdot 10^{-02}$	1.00000	1.997	$0.452 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.716 \cdot 10^{-03}$	1.00000	1.664	$0.105 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.226 \cdot 10^{-03}$	1.00000	2.000	$0.578 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.565 \cdot 10^{-04}$	1.00000	2.000	$0.578 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.141 \cdot 10^{-04}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.987 \cdot 10^{-03}$	0.20000	2.986	$0.955 \cdot 10^{+00}$
$1/5 \cdot 2^{-2}$	$0.125 \cdot 10^{-03}$	0.10000	2.997	$0.987 \cdot 10^{+00}$
$1/5 \cdot 2^{-3}$	$0.156 \cdot 10^{-04}$	0.05000	2.999	$0.996 \cdot 10^{+00}$
$1/5 \cdot 2^{-4}$	$0.195 \cdot 10^{-05}$	0.02500	3.000	$0.999 \cdot 10^{+00}$
$1/5 \cdot 2^{-5}$	$0.244 \cdot 10^{-06}$	0.01250	3.000	$0.100 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.305 \cdot 10^{-07}$	0.00625	3.000	$0.100 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.381 \cdot 10^{-08}$	0.00313		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.669 \cdot 10^{-04}$	0.20000	3.003	$0.541 \cdot 10^{+00}$
$1/5 \cdot 2^{-3}$	$0.834 \cdot 10^{-05}$	0.10000	3.001	$0.535 \cdot 10^{+00}$
$1/5 \cdot 2^{-4}$	$0.104 \cdot 10^{-05}$	0.05000	3.000	$0.534 \cdot 10^{+00}$
$1/5 \cdot 2^{-5}$	$0.130 \cdot 10^{-06}$	0.02500	3.000	$0.534 \cdot 10^{+00}$
$1/5 \cdot 2^{-6}$	$0.163 \cdot 10^{-07}$	0.01250	3.000	$0.533 \cdot 10^{+00}$
$1/5 \cdot 2^{-7}$	$0.203 \cdot 10^{-08}$	0.00625		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.741 \cdot 10^{-05}$	0.20000	3.001	$0.475 \cdot 10^{+00}$
$1/5 \cdot 2^{-4}$	$0.926 \cdot 10^{-06}$	0.10000	3.000	$0.474 \cdot 10^{+00}$
$1/5 \cdot 2^{-5}$	$0.116 \cdot 10^{-06}$	0.05000	3.000	$0.474 \cdot 10^{+00}$
$1/5 \cdot 2^{-6}$	$0.145 \cdot 10^{-07}$	0.02500	3.000	$0.474 \cdot 10^{+00}$
$1/5 \cdot 2^{-7}$	$0.181 \cdot 10^{-08}$	0.01250		

Table 4.68: Trapezoidal rule, basic sol. and extrapolation steps 1 – 3 for 3.1b

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.709 \cdot 10^{+00}$	1.00000	2.016	$0.182 \cdot 10^{+02}$
$1/5 \cdot 2^{-1}$	$0.175 \cdot 10^{+00}$	1.00000	2.004	$0.177 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.437 \cdot 10^{-01}$	1.00000	2.001	$0.175 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.109 \cdot 10^{-01}$	1.00000	2.000	$0.175 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.273 \cdot 10^{-02}$	1.00000	0.639	$0.448 \cdot 10^{-01}$
$1/5 \cdot 2^{-5}$	$0.175 \cdot 10^{-02}$	1.00000	2.000	$0.448 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.438 \cdot 10^{-03}$	1.00000	2.000	$0.449 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.110 \cdot 10^{-03}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.264 \cdot 10^{-02}$	1.00000	3.993	$0.260 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.166 \cdot 10^{-03}$	1.00000	4.001	$0.266 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.104 \cdot 10^{-04}$	1.00000	4.002	$0.267 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.647 \cdot 10^{-06}$	1.00000	2.984	$0.309 \cdot 10^{+00}$
$1/5 \cdot 2^{-5}$	$0.818 \cdot 10^{-07}$	1.00000	4.000	$0.536 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.511 \cdot 10^{-08}$	1.00000	4.000	$0.536 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.320 \cdot 10^{-09}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.111 \cdot 10^{-05}$	0.20000	4.823	$0.209 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.392 \cdot 10^{-07}$	0.10000	4.963	$0.350 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.126 \cdot 10^{-08}$	0.05000	4.991	$0.396 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.395 \cdot 10^{-10}$	0.02500	4.998	$0.410 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.124 \cdot 10^{-11}$	0.01250	4.999	$0.413 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.386 \cdot 10^{-13}$	0.00625		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.211 \cdot 10^{-07}$	0.20000	5.027	$0.239 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.647 \cdot 10^{-09}$	0.10000	5.007	$0.219 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.201 \cdot 10^{-10}$	0.05000	5.002	$0.213 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.628 \cdot 10^{-12}$	0.02500	5.000	$0.211 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.196 \cdot 10^{-13}$	0.01250		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-4}$	$0.567 \cdot 10^{-09}$	0.20000	5.004	$0.189 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.177 \cdot 10^{-10}$	0.10000	5.001	$0.186 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.552 \cdot 10^{-12}$	0.05000	5.000	$0.185 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.172 \cdot 10^{-13}$	0.02500		

Table 4.69: Trapezoidal rule, basic sol. and extrapolation steps 1 – 4 for 3.1c

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.917 \cdot 10^{+00}$	1.00000	2.016	$0.235 \cdot 10^{+02}$
$1/5 \cdot 2^{-1}$	$0.227 \cdot 10^{+00}$	1.00000	1.975	$0.214 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.577 \cdot 10^{-01}$	1.00000	1.977	$0.215 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.147 \cdot 10^{-01}$	1.00000	1.985	$0.222 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.370 \cdot 10^{-02}$	1.00000	1.991	$0.228 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.931 \cdot 10^{-03}$	1.00000	1.995	$0.233 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.234 \cdot 10^{-03}$	1.00000	1.998	$0.236 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.585 \cdot 10^{-04}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.348 \cdot 10^{-02}$	1.00000	1.389	$0.851 \cdot 10^{-01}$
$1/5 \cdot 2^{-2}$	$0.133 \cdot 10^{-02}$	1.00000	2.083	$0.681 \cdot 10^{+00}$
$1/5 \cdot 2^{-3}$	$0.313 \cdot 10^{-03}$	1.00000	2.597	$0.453 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.518 \cdot 10^{-04}$	1.00000	2.785	$0.103 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.752 \cdot 10^{-05}$	1.00000	2.879	$0.167 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.102 \cdot 10^{-05}$	1.00000	2.931	$0.225 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.134 \cdot 10^{-06}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.165 \cdot 10^{-02}$	1.00000	2.746	$0.615 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.246 \cdot 10^{-03}$	1.00000	2.838	$0.865 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.344 \cdot 10^{-04}$	1.00000	2.913	$0.120 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.456 \cdot 10^{-05}$	1.00000	2.955	$0.149 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.589 \cdot 10^{-06}$	1.00000	2.977	$0.169 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.748 \cdot 10^{-07}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.223 \cdot 10^{-03}$	1.00000	2.849	$0.820 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.310 \cdot 10^{-04}$	1.00000	2.922	$0.113 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.409 \cdot 10^{-05}$	1.00000	2.961	$0.137 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.526 \cdot 10^{-06}$	1.00000	2.980	$0.154 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.666 \cdot 10^{-07}$	1.00000		

Table 4.70: Trapezoidal rule, basic sol. and extrapolation steps $1 - 3$ for 3.2a

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	2.8	0.60000	1.033	$1.5\cdot10^{+02}$	0.60000	2.138
$1/5\cdot2^{-1}$	1.3	1.00000	0.980	$3.5\cdot10^{+01}$	0.30000	2.088
$1/5\cdot2^{-2}$	$7.0\cdot10^{-01}$	1.00000	0.991	8.2	0.15000	2.047
$1/5\cdot2^{-3}$	$3.5\cdot10^{-01}$	1.00000	0.995	2.0	0.07500	2.024
$1/5\cdot2^{-4}$	$1.7\cdot10^{-01}$	1.00000	0.998	$4.9\cdot10^{-01}$	0.03750	2.012
$1/5\cdot2^{-5}$	$8.9\cdot10^{-02}$	1.00000	0.999	$1.2\cdot10^{-01}$	0.01875	2.006
$1/5\cdot2^{-6}$	$4.4\cdot10^{-02}$	1.00000	0.999	$3.0\cdot10^{-02}$	0.00937	2.003
$1/5\cdot2^{-7}$	$2.2\cdot10^{-02}$	1.00000	0.999	$7.5\cdot10^{-03}$	0.00468	2.001
$1/5\cdot2^{-8}$	$1.1\cdot10^{-02}$	1.00000	0.999	$1.8\cdot10^{-03}$	0.00234	2.000
$1/5\cdot2^{-9}$	$5.5\cdot10^{-03}$	1.00000	0.999	$4.7\cdot10^{-04}$	0.00117	2.000
$1/5\cdot2^{-10}$	$2.7\cdot10^{-03}$	1.00000	0.999	$1.1\cdot10^{-04}$	0.00058	2.000
$1/5\cdot2^{-11}$	$1.3\cdot10^{-03}$	1.00000	1.000	$2.9\cdot10^{-05}$	0.00029	-17.611
$1/5\cdot2^{-12}$	$6.9\cdot10^{-04}$	0.99998	0.999	5.9	0.99984	21.609
$1/5\cdot2^{-13}$	$3.4\cdot10^{-04}$	1.00000	0.999	$1.8\cdot10^{-06}$	0.00007	2.000
$1/5\cdot2^{-14}$	$1.7\cdot10^{-04}$	1.00000		$4.6\cdot10^{-07}$	0.00003	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$8.1\cdot10^{+03}$	0.60000	2.134	$4.1\cdot10^{+05}$	0.60000	2.133
$1/5\cdot2^{-1}$	$1.8\cdot10^{+03}$	0.30000	2.086	$9.5\cdot10^{+04}$	0.30000	2.086
$1/5\cdot2^{-2}$	$4.3\cdot10^{+02}$	0.15000	2.046	$2.2\cdot10^{+04}$	0.15000	2.046
$1/5\cdot2^{-3}$	$1.0\cdot10^{+02}$	0.07500	2.023	$5.4\cdot10^{+03}$	0.07500	2.023
$1/5\cdot2^{-4}$	$2.5\cdot10^{+01}$	0.03750	2.012	$1.3\cdot10^{+03}$	0.03750	2.012
$1/5\cdot2^{-5}$	6.4	0.01875	2.006	$3.3\cdot10^{+02}$	0.01875	2.006
$1/5\cdot2^{-6}$	1.5	0.00937	2.003	$8.2\cdot10^{+01}$	0.00937	2.003
$1/5\cdot2^{-7}$	$3.9\cdot10^{-01}$	0.00468	2.001	$2.0\cdot10^{+01}$	0.00468	2.001
$1/5\cdot2^{-8}$	$9.9\cdot10^{-02}$	0.00234	2.000	5.1	0.00234	2.000
$1/5\cdot2^{-9}$	$2.4\cdot10^{-02}$	0.00117	2.000	1.2	0.00117	2.000
$1/5\cdot2^{-10}$	$6.2\cdot10^{-03}$	0.00058	2.000	$3.2\cdot10^{-01}$	0.00058	2.000
$1/5\cdot2^{-11}$	$1.5\cdot10^{-03}$	0.00029	-12.892	$8.0\cdot10^{-02}$	0.00029	-6.396
$1/5\cdot2^{-12}$	$1.1\cdot10^{+01}$	0.99998	16.891	6.7	0.99993	10.395
$1/5\cdot2^{-13}$	$9.7\cdot10^{-05}$	0.00007		$5.0\cdot10^{-03}$	0.00007	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$2.1\cdot10^{+07}$	0.60000	2.133	$1.1\cdot10^{+09}$	0.60000	2.133
$1/5\cdot2^{-1}$	$4.8\cdot10^{+06}$	0.30000	2.086	$2.5\cdot10^{+08}$	0.30000	2.086
$1/5\cdot2^{-2}$	$1.1\cdot10^{+06}$	0.15000	2.046	$5.9\cdot10^{+07}$	0.15000	2.046
$1/5\cdot2^{-3}$	$2.7\cdot10^{+05}$	0.07500	2.023	$1.4\cdot10^{+07}$	0.07500	2.023
$1/5\cdot2^{-4}$	$6.8\cdot10^{+04}$	0.03750	2.012	$3.5\cdot10^{+06}$	0.03750	2.012
$1/5\cdot2^{-5}$	$1.7\cdot10^{+04}$	0.01875	2.006	$8.7\cdot10^{+05}$	0.01875	2.006
$1/5\cdot2^{-6}$	$4.2\cdot10^{+03}$	0.00937	2.003	$2.1\cdot10^{+05}$	0.00937	2.003
$1/5\cdot2^{-7}$	$1.0\cdot10^{+03}$	0.00468	2.001	$5.4\cdot10^{+04}$	0.00468	2.001
$1/5\cdot2^{-8}$	$2.6\cdot10^{+02}$	0.00234	2.000	$1.3\cdot10^{+04}$	0.00234	2.000
$1/5\cdot2^{-9}$	$6.5\cdot10^{+01}$	0.00117	2.000	$3.3\cdot10^{+03}$	0.00117	2.000
$1/5\cdot2^{-10}$	$1.6\cdot10^{+01}$	0.00058	2.000	$8.4\cdot10^{+02}$	0.00058	2.000
$1/5\cdot2^{-11}$	4.1	0.00029	-0.851	$2.1\cdot10^{+02}$	0.00029	2.000
$1/5\cdot2^{-12}$	7.4	0.99984		$5.2\cdot10^{+01}$	0.00014	

Table 4.71: Explicit Euler, IDeC method on $[0,1]$ for 1.1a.

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$4.4 \cdot 10^{-03}$	1.834	$1.7 \cdot 10^{-01}$	9.218
$1/5 \cdot 2^{-4}$	$1.2 \cdot 10^{-03}$	1.002	$2.8 \cdot 10^{-04}$	4.237
$1/5 \cdot 2^{-5}$	$6.2 \cdot 10^{-04}$	1.001	$1.5 \cdot 10^{-05}$	2.445
$1/5 \cdot 2^{-6}$	$3.1 \cdot 10^{-04}$	1.000	$2.7 \cdot 10^{-06}$	2.053
$1/5 \cdot 2^{-7}$	$1.5 \cdot 10^{-04}$	1.000	$6.6 \cdot 10^{-07}$	2.006
$1/5 \cdot 2^{-8}$	$7.8 \cdot 10^{-05}$	1.000	$1.6 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-9}$	$3.9 \cdot 10^{-05}$	1.000	$4.1 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-10}$	$1.9 \cdot 10^{-05}$	1.000	$1.0 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-11}$	$9.8 \cdot 10^{-06}$	1.000	$2.6 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-12}$	$4.9 \cdot 10^{-06}$	1.000	$6.5 \cdot 10^{-10}$	1.999
$1/5 \cdot 2^{-13}$	$2.4 \cdot 10^{-06}$	1.000	$1.6 \cdot 10^{-10}$	1.999
$1/5 \cdot 2^{-14}$	$1.2 \cdot 10^{-06}$	0.088	$4.0 \cdot 10^{-11}$	-27.137
$1/5 \cdot 2^{-15}$	$1.1 \cdot 10^{-06}$	1.911	$6.0 \cdot 10^{-03}$	31.134
$1/5 \cdot 2^{-16}$	$3.0 \cdot 10^{-07}$	1.000	$2.5 \cdot 10^{-12}$	1.999
$1/5 \cdot 2^{-17}$	$1.5 \cdot 10^{-07}$		$6.3 \cdot 10^{-13}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	8.3	11.181	$4.1 \cdot 10^{+02}$	12.063
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-03}$	5.990	$9.7 \cdot 10^{-02}$	6.426
$1/5 \cdot 2^{-5}$	$5.6 \cdot 10^{-05}$	5.325	$1.1 \cdot 10^{-03}$	5.394
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-06}$	5.179	$2.6 \cdot 10^{-05}$	5.148
$1/5 \cdot 2^{-7}$	$3.8 \cdot 10^{-08}$	5.262	$7.5 \cdot 10^{-07}$	5.065
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-09}$	6.362	$2.2 \cdot 10^{-08}$	5.030
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-11}$	2.660	$6.9 \cdot 10^{-10}$	5.014
$1/5 \cdot 2^{-10}$	$1.9 \cdot 10^{-12}$	2.471	$2.1 \cdot 10^{-11}$	5.007
$1/5 \cdot 2^{-11}$	$3.5 \cdot 10^{-13}$	2.894	$6.6 \cdot 10^{-13}$	5.003
$1/5 \cdot 2^{-12}$	$4.7 \cdot 10^{-14}$	2.974	$2.0 \cdot 10^{-14}$	5.001
$1/5 \cdot 2^{-13}$	$5.9 \cdot 10^{-15}$	2.993	$6.4 \cdot 10^{-16}$	5.000
$1/5 \cdot 2^{-14}$	$7.5 \cdot 10^{-16}$	-44.449	$2.0 \cdot 10^{-17}$	-47.274
$1/5 \cdot 2^{-15}$	$1.8 \cdot 10^{-02}$	50.442	$3.4 \cdot 10^{-03}$	57.266
$1/5 \cdot 2^{-16}$	$1.1 \cdot 10^{-17}$		$1.9 \cdot 10^{-20}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$2.1 \cdot 10^{+04}$	12.615	$1.0 \cdot 10^{+06}$	12.872
$1/5 \cdot 2^{-4}$	3.3	7.110	$1.4 \cdot 10^{+02}$	7.786
$1/5 \cdot 2^{-5}$	$2.4 \cdot 10^{-02}$	5.608	$6.5 \cdot 10^{-01}$	6.026
$1/5 \cdot 2^{-6}$	$5.0 \cdot 10^{-04}$	5.196	$1.0 \cdot 10^{-02}$	5.321
$1/5 \cdot 2^{-7}$	$1.3 \cdot 10^{-05}$	5.076	$2.5 \cdot 10^{-04}$	5.106
$1/5 \cdot 2^{-8}$	$4.0 \cdot 10^{-07}$	5.033	$7.2 \cdot 10^{-06}$	5.040
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-08}$	5.015	$2.2 \cdot 10^{-07}$	5.017
$1/5 \cdot 2^{-10}$	$3.8 \cdot 10^{-10}$	5.007	$6.8 \cdot 10^{-09}$	5.007
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-11}$	5.003	$2.1 \cdot 10^{-10}$	5.003
$1/5 \cdot 2^{-12}$	$3.7 \cdot 10^{-13}$	5.001	$6.6 \cdot 10^{-12}$	5.001
$1/5 \cdot 2^{-13}$	$1.1 \cdot 10^{-14}$	5.000	$2.0 \cdot 10^{-13}$	5.000
$1/5 \cdot 2^{-14}$	$3.6 \cdot 10^{-16}$	-41.551	$6.4 \cdot 10^{-15}$	-41.501
$1/5 \cdot 2^{-15}$	$1.1 \cdot 10^{-03}$		$2.0 \cdot 10^{-02}$	

Table 4.72: Explicit Euler, IDeC method at $t = 1/8$ for 1.1a.

h	δ	p	δ	p
$1/5$	2.5	0.892	$1.3 \cdot 10^{+01}$	6.072
$1/5 \cdot 2^{-1}$	1.3	0.980	$1.9 \cdot 10^{-01}$	2.013
$1/5 \cdot 2^{-2}$	$7.0 \cdot 10^{-01}$	0.991	$4.8 \cdot 10^{-02}$	1.992
$1/5 \cdot 2^{-3}$	$3.5 \cdot 10^{-01}$	0.995	$1.2 \cdot 10^{-02}$	1.995
$1/5 \cdot 2^{-4}$	$1.7 \cdot 10^{-01}$	0.998	$3.0 \cdot 10^{-03}$	1.997
$1/5 \cdot 2^{-5}$	$8.9 \cdot 10^{-02}$	0.999	$7.6 \cdot 10^{-04}$	1.998
$1/5 \cdot 2^{-6}$	$4.4 \cdot 10^{-02}$	0.999	$1.9 \cdot 10^{-04}$	1.999
$1/5 \cdot 2^{-7}$	$2.2 \cdot 10^{-02}$	0.999	$4.7 \cdot 10^{-05}$	1.999
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-02}$	0.999	$1.1 \cdot 10^{-05}$	1.999
$1/5 \cdot 2^{-9}$	$5.5 \cdot 10^{-03}$	0.999	$2.9 \cdot 10^{-06}$	1.999
$1/5 \cdot 2^{-10}$	$2.7 \cdot 10^{-03}$	0.999	$7.4 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-11}$	$1.3 \cdot 10^{-03}$	-0.316	$1.8 \cdot 10^{-07}$	-24.331
$1/5 \cdot 2^{-12}$	$1.7 \cdot 10^{-03}$	2.316	3.9	28.328
$1/5 \cdot 2^{-13}$	$3.4 \cdot 10^{-04}$	0.999	$1.1 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-14}$	$1.7 \cdot 10^{-04}$		$2.9 \cdot 10^{-09}$	

h	δ	p	δ	p
$1/5$	$6.3 \cdot 10^{+02}$	11.489	$3.2 \cdot 10^{+04}$	12.203
$1/5 \cdot 2^{-1}$	$2.2 \cdot 10^{-01}$	7.081	6.7	6.493
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-03}$	1.809	$7.5 \cdot 10^{-02}$	5.411
$1/5 \cdot 2^{-3}$	$4.6 \cdot 10^{-04}$	2.769	$1.7 \cdot 10^{-03}$	5.141
$1/5 \cdot 2^{-4}$	$6.8 \cdot 10^{-05}$	2.948	$5.0 \cdot 10^{-05}$	5.032
$1/5 \cdot 2^{-5}$	$8.8 \cdot 10^{-06}$	2.985	$1.5 \cdot 10^{-06}$	4.959
$1/5 \cdot 2^{-6}$	$1.1 \cdot 10^{-06}$	2.995	$4.9 \cdot 10^{-08}$	4.878
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-07}$	2.998	$1.6 \cdot 10^{-09}$	4.766
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-08}$	2.999	$6.1 \cdot 10^{-11}$	4.615
$1/5 \cdot 2^{-9}$	$2.2 \cdot 10^{-09}$	2.999	$2.5 \cdot 10^{-12}$	4.444
$1/5 \cdot 2^{-10}$	$2.7 \cdot 10^{-10}$	2.999	$1.1 \cdot 10^{-13}$	4.286
$1/5 \cdot 2^{-11}$	$3.4 \cdot 10^{-11}$	-38.321	$5.9 \cdot 10^{-15}$	-48.439
$1/5 \cdot 2^{-12}$	$1.1 \cdot 10^{+01}$	44.317	2.2	56.693
$1/5 \cdot 2^{-13}$	$5.3 \cdot 10^{-13}$		$1.9 \cdot 10^{-17}$	

h	δ	p	δ	p
$1/5$	$1.6 \cdot 10^{+06}$	12.740	$8.3 \cdot 10^{+07}$	12.988
$1/5 \cdot 2^{-1}$	$2.3 \cdot 10^{+02}$	7.191	$1.0 \cdot 10^{+04}$	7.869
$1/5 \cdot 2^{-2}$	1.6	5.639	$4.3 \cdot 10^{+01}$	6.064
$1/5 \cdot 2^{-3}$	$3.2 \cdot 10^{-02}$	5.210	$6.5 \cdot 10^{-01}$	5.335
$1/5 \cdot 2^{-4}$	$8.8 \cdot 10^{-04}$	5.082	$1.6 \cdot 10^{-02}$	5.113
$1/5 \cdot 2^{-5}$	$2.6 \cdot 10^{-05}$	5.036	$4.6 \cdot 10^{-04}$	5.043
$1/5 \cdot 2^{-6}$	$7.9 \cdot 10^{-07}$	5.017	$1.4 \cdot 10^{-05}$	5.018
$1/5 \cdot 2^{-7}$	$2.4 \cdot 10^{-08}$	5.008	$4.3 \cdot 10^{-07}$	5.008
$1/5 \cdot 2^{-8}$	$7.6 \cdot 10^{-10}$	5.004	$1.3 \cdot 10^{-08}$	5.004
$1/5 \cdot 2^{-9}$	$2.3 \cdot 10^{-11}$	5.002	$4.2 \cdot 10^{-10}$	5.002
$1/5 \cdot 2^{-10}$	$7.4 \cdot 10^{-13}$	5.001	$1.3 \cdot 10^{-11}$	5.001
$1/5 \cdot 2^{-11}$	$2.3 \cdot 10^{-14}$	-44.920	$4.1 \cdot 10^{-13}$	-44.856
$1/5 \cdot 2^{-12}$	$7.6 \cdot 10^{-01}$		$1.3 \cdot 10^{+01}$	

Table 4.73: Explicit Euler, IDeC method at $t = 1$ for 1.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$6.0 \cdot 10^{-01}$	0.40000	2.157	3.6	0.60000	2.444
$1/5 \cdot 2^{-1}$	$1.3 \cdot 10^{-01}$	0.20000	2.129	$6.7 \cdot 10^{-01}$	0.20000	1.625
$1/5 \cdot 2^{-2}$	$3.0 \cdot 10^{-02}$	0.10000	1.172	$2.1 \cdot 10^{-01}$	0.10000	1.832
$1/5 \cdot 2^{-3}$	$1.3 \cdot 10^{-02}$	0.62500	1.014	$6.1 \cdot 10^{-02}$	0.05000	1.920
$1/5 \cdot 2^{-4}$	$6.7 \cdot 10^{-03}$	0.63750	1.008	$1.6 \cdot 10^{-02}$	0.02500	1.961
$1/5 \cdot 2^{-5}$	$3.3 \cdot 10^{-03}$	0.63750	1.004	$4.1 \cdot 10^{-03}$	0.01250	1.980
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-03}$	0.64062	1.002	$1.0 \cdot 10^{-03}$	0.00625	1.990
$1/5 \cdot 2^{-7}$	$8.3 \cdot 10^{-04}$	0.64062	1.001	$2.6 \cdot 10^{-04}$	0.00312	1.995
$1/5 \cdot 2^{-8}$	$4.1 \cdot 10^{-04}$	0.64140	1.000	$6.6 \cdot 10^{-05}$	0.00156	1.997
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-04}$	0.64140	1.000	$1.6 \cdot 10^{-05}$	0.00078	1.998
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-04}$	0.64160	1.000	$4.1 \cdot 10^{-06}$	0.00039	1.999
$1/5 \cdot 2^{-11}$	$5.2 \cdot 10^{-05}$	0.64160	1.000	$1.0 \cdot 10^{-06}$	0.00019	-17.552
$1/5 \cdot 2^{-12}$	$2.6 \cdot 10^{-05}$	0.64157	1.000	$2.0 \cdot 10^{-01}$	0.99984	21.550
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-05}$	0.64157	1.000	$6.5 \cdot 10^{-08}$	0.00004	1.999
$1/5 \cdot 2^{-14}$	$6.5 \cdot 10^{-06}$	0.64158		$1.6 \cdot 10^{-08}$	0.00002	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$2.0 \cdot 10^{+01}$	0.80000	2.273	$1.4 \cdot 10^{+02}$	0.40000	3.318
$1/5 \cdot 2^{-1}$	4.2	0.10000	1.502	$1.4 \cdot 10^{+01}$	0.10000	0.551
$1/5 \cdot 2^{-2}$	1.4	0.05000	1.706	$1.0 \cdot 10^{+01}$	0.05000	1.500
$1/5 \cdot 2^{-3}$	$4.5 \cdot 10^{-01}$	0.05000	1.777	3.5	0.02500	1.710
$1/5 \cdot 2^{-4}$	$1.3 \cdot 10^{-01}$	0.02500	1.896	1.0	0.02500	1.821
$1/5 \cdot 2^{-5}$	$3.5 \cdot 10^{-02}$	0.01250	1.949	$3.0 \cdot 10^{-01}$	0.01250	1.914
$1/5 \cdot 2^{-6}$	$9.3 \cdot 10^{-03}$	0.00625	1.975	$8.1 \cdot 10^{-02}$	0.00625	1.958
$1/5 \cdot 2^{-7}$	$2.3 \cdot 10^{-03}$	0.00312	1.987	$2.1 \cdot 10^{-02}$	0.00312	1.979
$1/5 \cdot 2^{-8}$	$5.9 \cdot 10^{-04}$	0.00156	1.993	$5.3 \cdot 10^{-03}$	0.00156	1.989
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-04}$	0.00078	1.996	$1.3 \cdot 10^{-03}$	0.00078	1.994
$1/5 \cdot 2^{-10}$	$3.7 \cdot 10^{-05}$	0.00039	1.998	$3.3 \cdot 10^{-04}$	0.00039	1.997
$1/5 \cdot 2^{-11}$	$9.3 \cdot 10^{-06}$	0.00019	-15.379	$8.4 \cdot 10^{-05}$	0.00019	-11.406
$1/5 \cdot 2^{-12}$	$4.0 \cdot 10^{-01}$	0.99998	19.376	$2.2 \cdot 10^{-01}$	0.99993	15.403
$1/5 \cdot 2^{-13}$	$5.8 \cdot 10^{-07}$	0.00004		$5.2 \cdot 10^{-06}$	0.00004	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$9.2 \cdot 10^{+02}$	0.60000	2.481	$5.2 \cdot 10^{+03}$	0.40000	2.404
$1/5 \cdot 2^{-1}$	$1.6 \cdot 10^{+02}$	0.20000	1.544	$9.9 \cdot 10^{+02}$	0.20000	1.900
$1/5 \cdot 2^{-2}$	$5.6 \cdot 10^{+01}$	0.05000	1.076	$2.6 \cdot 10^{+02}$	0.05000	0.445
$1/5 \cdot 2^{-3}$	$2.7 \cdot 10^{+01}$	0.02500	1.617	$1.9 \cdot 10^{+02}$	0.02500	1.452
$1/5 \cdot 2^{-4}$	8.8	0.02500	1.740	$7.1 \cdot 10^{+01}$	0.01250	1.673
$1/5 \cdot 2^{-5}$	2.6	0.01250	1.878	$2.2 \cdot 10^{+01}$	0.01250	1.840
$1/5 \cdot 2^{-6}$	$7.1 \cdot 10^{-01}$	0.00625	1.940	6.2	0.00625	1.922
$1/5 \cdot 2^{-7}$	$1.8 \cdot 10^{-01}$	0.00312	1.970	1.6	0.00312	1.962
$1/5 \cdot 2^{-8}$	$4.7 \cdot 10^{-02}$	0.00156	1.985	$4.2 \cdot 10^{-01}$	0.00156	1.981
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-02}$	0.00078	1.992	$1.0 \cdot 10^{-01}$	0.00078	1.990
$1/5 \cdot 2^{-10}$	$3.0 \cdot 10^{-03}$	0.00039	1.996	$2.7 \cdot 10^{-02}$	0.00039	1.995
$1/5 \cdot 2^{-11}$	$7.5 \cdot 10^{-04}$	0.00019	-8.380	$6.8 \cdot 10^{-03}$	0.00019	-6.031
$1/5 \cdot 2^{-12}$	$2.5 \cdot 10^{-01}$	0.99984		$4.4 \cdot 10^{-01}$	0.99998	

Table 4.74: Explicit Euler, IDeC method on $[0, 1]$ for 1.2a.

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$5.8 \cdot 10^{-03}$	0.967	$3.3 \cdot 10^{-03}$	6.104
$1/5 \cdot 2^{-4}$	$3.0 \cdot 10^{-03}$	1.001	$4.8 \cdot 10^{-05}$	3.582
$1/5 \cdot 2^{-5}$	$1.5 \cdot 10^{-03}$	1.003	$4.0 \cdot 10^{-06}$	0.330
$1/5 \cdot 2^{-6}$	$7.5 \cdot 10^{-04}$	1.001	$3.2 \cdot 10^{-06}$	1.608
$1/5 \cdot 2^{-7}$	$3.7 \cdot 10^{-04}$	1.000	$1.0 \cdot 10^{-06}$	1.859
$1/5 \cdot 2^{-8}$	$1.8 \cdot 10^{-04}$	1.000	$2.9 \cdot 10^{-07}$	1.941
$1/5 \cdot 2^{-9}$	$9.3 \cdot 10^{-05}$	1.000	$7.6 \cdot 10^{-08}$	1.973
$1/5 \cdot 2^{-10}$	$4.6 \cdot 10^{-05}$	1.000	$1.9 \cdot 10^{-08}$	1.987
$1/5 \cdot 2^{-11}$	$2.3 \cdot 10^{-05}$	1.000	$4.8 \cdot 10^{-09}$	1.994
$1/5 \cdot 2^{-12}$	$1.1 \cdot 10^{-05}$	1.000	$1.2 \cdot 10^{-09}$	1.997
$1/5 \cdot 2^{-13}$	$5.8 \cdot 10^{-06}$	1.000	$3.0 \cdot 10^{-10}$	1.998
$1/5 \cdot 2^{-14}$	$2.9 \cdot 10^{-06}$	0.344	$7.6 \cdot 10^{-11}$	-26.863
$1/5 \cdot 2^{-15}$	$2.3 \cdot 10^{-06}$	1.655	$9.3 \cdot 10^{-03}$	30.859
$1/5 \cdot 2^{-16}$	$7.3 \cdot 10^{-07}$	1.000	$4.8 \cdot 10^{-12}$	1.999
$1/5 \cdot 2^{-17}$	$3.6 \cdot 10^{-07}$		$1.2 \cdot 10^{-12}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$2.3 \cdot 10^{-02}$	5.436	$2.4 \cdot 10^{-01}$	5.523
$1/5 \cdot 2^{-4}$	$5.3 \cdot 10^{-04}$	3.171	$5.3 \cdot 10^{-03}$	3.793
$1/5 \cdot 2^{-5}$	$5.9 \cdot 10^{-05}$	3.303	$3.8 \cdot 10^{-04}$	3.298
$1/5 \cdot 2^{-6}$	$6.0 \cdot 10^{-06}$	3.128	$3.9 \cdot 10^{-05}$	3.287
$1/5 \cdot 2^{-7}$	$6.9 \cdot 10^{-07}$	3.162	$4.0 \cdot 10^{-06}$	3.150
$1/5 \cdot 2^{-8}$	$7.7 \cdot 10^{-08}$	3.133	$4.5 \cdot 10^{-07}$	3.168
$1/5 \cdot 2^{-9}$	$8.7 \cdot 10^{-09}$	3.093	$5.0 \cdot 10^{-08}$	3.136
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-09}$	3.059	$5.7 \cdot 10^{-09}$	3.096
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-10}$	3.036	$6.6 \cdot 10^{-10}$	3.062
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-11}$	3.021	$7.9 \cdot 10^{-11}$	3.038
$1/5 \cdot 2^{-13}$	$1.8 \cdot 10^{-12}$	3.011	$9.7 \cdot 10^{-12}$	3.022
$1/5 \cdot 2^{-14}$	$2.3 \cdot 10^{-13}$	-36.831	$1.1 \cdot 10^{-12}$	-32.057
$1/5 \cdot 2^{-15}$	$2.8 \cdot 10^{-02}$	42.837	$5.3 \cdot 10^{-03}$	38.073
$1/5 \cdot 2^{-16}$	$3.5 \cdot 10^{-15}$		$1.8 \cdot 10^{-14}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	2.0	5.108	$1.5 \cdot 10^{+01}$	4.850
$1/5 \cdot 2^{-4}$	$5.8 \cdot 10^{-02}$	4.546	$5.3 \cdot 10^{-01}$	4.836
$1/5 \cdot 2^{-5}$	$2.5 \cdot 10^{-03}$	3.261	$1.8 \cdot 10^{-02}$	3.207
$1/5 \cdot 2^{-6}$	$2.6 \cdot 10^{-04}$	3.484	$2.0 \cdot 10^{-03}$	3.512
$1/5 \cdot 2^{-7}$	$2.3 \cdot 10^{-05}$	3.393	$1.7 \cdot 10^{-04}$	3.604
$1/5 \cdot 2^{-8}$	$2.2 \cdot 10^{-06}$	3.287	$1.4 \cdot 10^{-05}$	3.445
$1/5 \cdot 2^{-9}$	$2.2 \cdot 10^{-07}$	3.264	$1.3 \cdot 10^{-06}$	3.409
$1/5 \cdot 2^{-10}$	$2.3 \cdot 10^{-08}$	3.206	$1.2 \cdot 10^{-07}$	3.358
$1/5 \cdot 2^{-11}$	$2.5 \cdot 10^{-09}$	3.144	$1.2 \cdot 10^{-08}$	3.278
$1/5 \cdot 2^{-12}$	$2.9 \cdot 10^{-10}$	3.093	$1.2 \cdot 10^{-09}$	3.196
$1/5 \cdot 2^{-13}$	$3.4 \cdot 10^{-11}$	3.056	$1.3 \cdot 10^{-10}$	3.127
$1/5 \cdot 2^{-14}$	$4.1 \cdot 10^{-12}$	-28.771	$1.5 \cdot 10^{-11}$	-30.875
$1/5 \cdot 2^{-15}$	$1.8 \cdot 10^{-03}$		$3.1 \cdot 10^{-02}$	

Table 4.75: Explicit Euler, IDeC method at $t = 1/8$ for 1.2a.

h	δ	p	δ	p
$1/5$	$2.2 \cdot 10^{-01}$	3.087	$8.1 \cdot 10^{-01}$	2.869
$1/5 \cdot 2^{-1}$	$2.6 \cdot 10^{-02}$	1.025	$1.1 \cdot 10^{-01}$	7.783
$1/5 \cdot 2^{-2}$	$1.3 \cdot 10^{-02}$	0.949	$5.0 \cdot 10^{-04}$	0.636
$1/5 \cdot 2^{-3}$	$6.7 \cdot 10^{-03}$	0.980	$3.2 \cdot 10^{-04}$	1.763
$1/5 \cdot 2^{-4}$	$3.4 \cdot 10^{-03}$	0.990	$9.6 \cdot 10^{-05}$	1.965
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-03}$	0.995	$2.4 \cdot 10^{-05}$	1.999
$1/5 \cdot 2^{-6}$	$8.6 \cdot 10^{-04}$	0.997	$6.1 \cdot 10^{-06}$	2.002
$1/5 \cdot 2^{-7}$	$4.3 \cdot 10^{-04}$	0.998	$1.5 \cdot 10^{-06}$	2.001
$1/5 \cdot 2^{-8}$	$2.1 \cdot 10^{-04}$	0.999	$3.8 \cdot 10^{-07}$	2.001
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-04}$	0.999	$9.5 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-10}$	$5.4 \cdot 10^{-05}$	0.999	$2.3 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-11}$	$2.7 \cdot 10^{-05}$	0.999	$5.9 \cdot 10^{-09}$	-24.409
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-05}$	1.000	$1.3 \cdot 10^{-01}$	28.406
$1/5 \cdot 2^{-13}$	$6.7 \cdot 10^{-06}$	0.999	$3.7 \cdot 10^{-10}$	2.000
$1/5 \cdot 2^{-14}$	$3.3 \cdot 10^{-06}$		$9.3 \cdot 10^{-11}$	

h	δ	p	δ	p
$1/5$	$1.6 \cdot 10^{+01}$	4.415	$1.0 \cdot 10^{+02}$	5.368
$1/5 \cdot 2^{-1}$	$7.7 \cdot 10^{-01}$	5.340	2.5	3.841
$1/5 \cdot 2^{-2}$	$1.9 \cdot 10^{-02}$	5.841	$1.8 \cdot 10^{-01}$	5.034
$1/5 \cdot 2^{-3}$	$3.3 \cdot 10^{-04}$	3.771	$5.5 \cdot 10^{-03}$	4.840
$1/5 \cdot 2^{-4}$	$2.4 \cdot 10^{-05}$	3.685	$1.9 \cdot 10^{-04}$	3.655
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-06}$	3.384	$1.5 \cdot 10^{-05}$	3.550
$1/5 \cdot 2^{-6}$	$1.8 \cdot 10^{-07}$	2.989	$1.3 \cdot 10^{-06}$	3.527
$1/5 \cdot 2^{-7}$	$2.2 \cdot 10^{-08}$	3.061	$1.1 \cdot 10^{-07}$	3.186
$1/5 \cdot 2^{-8}$	$2.7 \cdot 10^{-09}$	3.066	$1.2 \cdot 10^{-08}$	3.051
$1/5 \cdot 2^{-9}$	$3.2 \cdot 10^{-10}$	3.051	$1.4 \cdot 10^{-09}$	3.070
$1/5 \cdot 2^{-10}$	$3.9 \cdot 10^{-11}$	3.035	$1.7 \cdot 10^{-10}$	3.059
$1/5 \cdot 2^{-11}$	$4.8 \cdot 10^{-12}$	-36.271	$2.1 \cdot 10^{-11}$	-31.732
$1/5 \cdot 2^{-12}$	$4.0 \cdot 10^{-01}$	42.302	$7.6 \cdot 10^{-02}$	37.798
$1/5 \cdot 2^{-13}$	$7.3 \cdot 10^{-14}$		$3.1 \cdot 10^{-13}$	

h	δ	p	δ	p
$1/5$	$2.0 \cdot 10^{+02}$	4.823	$4.9 \cdot 10^{+03}$	5.134
$1/5 \cdot 2^{-1}$	7.1	2.495	$1.4 \cdot 10^{+02}$	4.253
$1/5 \cdot 2^{-2}$	1.2	4.643	7.4	4.184
$1/5 \cdot 2^{-3}$	$5.0 \cdot 10^{-02}$	4.571	$4.0 \cdot 10^{-01}$	4.384
$1/5 \cdot 2^{-4}$	$2.1 \cdot 10^{-03}$	4.248	$1.9 \cdot 10^{-02}$	4.450
$1/5 \cdot 2^{-5}$	$1.1 \cdot 10^{-04}$	3.577	$8.9 \cdot 10^{-04}$	3.562
$1/5 \cdot 2^{-6}$	$9.3 \cdot 10^{-06}$	3.622	$7.5 \cdot 10^{-05}$	3.657
$1/5 \cdot 2^{-7}$	$7.6 \cdot 10^{-07}$	3.607	$5.9 \cdot 10^{-06}$	3.687
$1/5 \cdot 2^{-8}$	$6.2 \cdot 10^{-08}$	3.360	$4.6 \cdot 10^{-07}$	3.672
$1/5 \cdot 2^{-9}$	$6.0 \cdot 10^{-09}$	3.135	$3.6 \cdot 10^{-08}$	3.449
$1/5 \cdot 2^{-10}$	$6.9 \cdot 10^{-10}$	3.127	$3.3 \cdot 10^{-09}$	3.235
$1/5 \cdot 2^{-11}$	$7.9 \cdot 10^{-11}$	-28.358	$3.5 \cdot 10^{-10}$	-30.225
$1/5 \cdot 2^{-12}$	$2.7 \cdot 10^{-02}$		$4.4 \cdot 10^{-01}$	

Table 4.76: Explicit Euler, IDeC method at $t = 1$ for 1.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$9.5 \cdot 10^{-01}$	1.00000	1.050	$7.4 \cdot 10^{-01}$	0.20000	2.036
$1/5 \cdot 2^{-1}$	$4.5 \cdot 10^{-01}$	1.00000	1.022	$1.8 \cdot 10^{-01}$	0.10000	2.011
$1/5 \cdot 2^{-2}$	$2.2 \cdot 10^{-01}$	1.00000	1.010	$4.5 \cdot 10^{-02}$	0.05000	2.003
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-01}$	1.00000	1.005	$1.1 \cdot 10^{-02}$	0.02500	2.000
$1/5 \cdot 2^{-4}$	$5.5 \cdot 10^{-02}$	1.00000	1.002	$2.8 \cdot 10^{-03}$	0.01250	2.000
$1/5 \cdot 2^{-5}$	$2.7 \cdot 10^{-02}$	1.00000	1.001	$7.0 \cdot 10^{-04}$	0.00625	2.000
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-02}$	1.00000	1.000	$1.7 \cdot 10^{-04}$	0.00312	2.000
$1/5 \cdot 2^{-7}$	$6.9 \cdot 10^{-03}$	1.00000	1.000	$4.3 \cdot 10^{-05}$	0.00156	2.000
$1/5 \cdot 2^{-8}$	$3.4 \cdot 10^{-03}$	1.00000	1.000	$1.0 \cdot 10^{-05}$	0.00078	2.000
$1/5 \cdot 2^{-9}$	$1.7 \cdot 10^{-03}$	1.00000	1.000	$2.7 \cdot 10^{-06}$	0.00039	2.000
$1/5 \cdot 2^{-10}$	$8.7 \cdot 10^{-04}$	1.00000	1.000	$6.8 \cdot 10^{-07}$	0.00019	2.000
$1/5 \cdot 2^{-11}$	$4.3 \cdot 10^{-04}$	1.00000	1.000	$1.7 \cdot 10^{-07}$	0.00009	-18.912
$1/5 \cdot 2^{-12}$	$2.1 \cdot 10^{-04}$	0.99998	0.999	$8.4 \cdot 10^{-02}$	0.99984	22.910
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-04}$	1.00000	1.000	$1.0 \cdot 10^{-08}$	0.00002	2.000
$1/5 \cdot 2^{-14}$	$5.4 \cdot 10^{-05}$	1.00000		$2.6 \cdot 10^{-09}$	0.00001	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	2.6	0.20000	2.021	9.2	0.20000	1.978
$1/5 \cdot 2^{-1}$	$6.6 \cdot 10^{-01}$	0.10000	2.006	2.3	0.10000	1.997
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-01}$	0.05000	2.002	$5.9 \cdot 10^{-01}$	0.05000	2.000
$1/5 \cdot 2^{-3}$	$4.1 \cdot 10^{-02}$	0.02500	2.000	$1.4 \cdot 10^{-01}$	0.02500	2.000
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-02}$	0.01250	2.000	$3.6 \cdot 10^{-02}$	0.01250	2.000
$1/5 \cdot 2^{-5}$	$2.5 \cdot 10^{-03}$	0.00625	2.000	$9.2 \cdot 10^{-03}$	0.00625	2.000
$1/5 \cdot 2^{-6}$	$6.4 \cdot 10^{-04}$	0.00312	2.000	$2.3 \cdot 10^{-03}$	0.00312	2.000
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-04}$	0.00156	2.000	$5.7 \cdot 10^{-04}$	0.00156	2.000
$1/5 \cdot 2^{-8}$	$4.0 \cdot 10^{-05}$	0.00078	2.000	$1.4 \cdot 10^{-04}$	0.00078	2.000
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-05}$	0.00039	2.000	$3.5 \cdot 10^{-05}$	0.00039	2.000
$1/5 \cdot 2^{-10}$	$2.5 \cdot 10^{-06}$	0.00019	2.000	$8.9 \cdot 10^{-06}$	0.00019	2.000
$1/5 \cdot 2^{-11}$	$6.2 \cdot 10^{-07}$	0.00009	-18.038	$2.2 \cdot 10^{-06}$	0.00009	-15.391
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-01}$	0.99998	22.036	$9.6 \cdot 10^{-02}$	0.99993	19.390
$1/5 \cdot 2^{-13}$	$3.9 \cdot 10^{-08}$	0.00002		$1.4 \cdot 10^{-07}$	0.00002	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$3.3 \cdot 10^{+01}$	0.20000	1.974	$1.1 \cdot 10^{+02}$	0.20000	1.974
$1/5 \cdot 2^{-1}$	8.4	0.10000	1.997	$3.0 \cdot 10^{+01}$	0.10000	1.997
$1/5 \cdot 2^{-2}$	2.1	0.05000	2.000	7.5	0.05000	2.000
$1/5 \cdot 2^{-3}$	$5.2 \cdot 10^{-01}$	0.02500	2.000	1.8	0.02500	2.000
$1/5 \cdot 2^{-4}$	$1.3 \cdot 10^{-01}$	0.01250	2.000	$4.6 \cdot 10^{-01}$	0.01250	2.000
$1/5 \cdot 2^{-5}$	$3.2 \cdot 10^{-02}$	0.00625	2.000	$1.1 \cdot 10^{-01}$	0.00625	2.000
$1/5 \cdot 2^{-6}$	$8.2 \cdot 10^{-03}$	0.00312	2.000	$2.9 \cdot 10^{-02}$	0.00312	2.000
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-03}$	0.00156	2.000	$7.3 \cdot 10^{-03}$	0.00156	2.000
$1/5 \cdot 2^{-8}$	$5.1 \cdot 10^{-04}$	0.00078	2.000	$1.8 \cdot 10^{-03}$	0.00078	2.000
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-04}$	0.00039	2.000	$4.5 \cdot 10^{-04}$	0.00039	2.000
$1/5 \cdot 2^{-10}$	$3.2 \cdot 10^{-05}$	0.00019	2.000	$1.1 \cdot 10^{-04}$	0.00019	2.000
$1/5 \cdot 2^{-11}$	$8.0 \cdot 10^{-06}$	0.00009	-13.698	$2.8 \cdot 10^{-05}$	0.00009	-12.681
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-01}$	0.99984		$1.8 \cdot 10^{-01}$	0.99998	

Table 4.77: Explicit Euler, IDeC method on $[0,1]$ for 2.1a.

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$2.7 \cdot 10^{-02}$	1.003	$1.4 \cdot 10^{-03}$	2.841
$1/5 \cdot 2^{-4}$	$1.3 \cdot 10^{-02}$	1.001	$1.9 \cdot 10^{-04}$	2.578
$1/5 \cdot 2^{-5}$	$6.8 \cdot 10^{-03}$	1.000	$3.2 \cdot 10^{-05}$	2.370
$1/5 \cdot 2^{-6}$	$3.4 \cdot 10^{-03}$	1.000	$6.3 \cdot 10^{-06}$	2.218
$1/5 \cdot 2^{-7}$	$1.7 \cdot 10^{-03}$	1.000	$1.3 \cdot 10^{-06}$	2.120
$1/5 \cdot 2^{-8}$	$8.5 \cdot 10^{-04}$	1.000	$3.1 \cdot 10^{-07}$	2.063
$1/5 \cdot 2^{-9}$	$4.2 \cdot 10^{-04}$	1.000	$7.4 \cdot 10^{-08}$	2.032
$1/5 \cdot 2^{-10}$	$2.1 \cdot 10^{-04}$	1.000	$1.8 \cdot 10^{-08}$	2.016
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-04}$	1.000	$4.5 \cdot 10^{-09}$	2.008
$1/5 \cdot 2^{-12}$	$5.3 \cdot 10^{-05}$	1.000	$1.1 \cdot 10^{-09}$	2.004
$1/5 \cdot 2^{-13}$	$2.6 \cdot 10^{-05}$	1.000	$2.8 \cdot 10^{-10}$	2.002
$1/5 \cdot 2^{-14}$	$1.3 \cdot 10^{-05}$	0.683	$6.9 \cdot 10^{-11}$	-31.777
$1/5 \cdot 2^{-15}$	$8.3 \cdot 10^{-06}$	1.316	$2.5 \cdot 10^{-01}$	35.775
$1/5 \cdot 2^{-16}$	$3.3 \cdot 10^{-06}$	1.000	$4.3 \cdot 10^{-12}$	2.000
$1/5 \cdot 2^{-17}$	$1.6 \cdot 10^{-06}$		$1.0 \cdot 10^{-12}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$2.5 \cdot 10^{-03}$	3.282	$1.2 \cdot 10^{-02}$	3.229
$1/5 \cdot 2^{-4}$	$2.6 \cdot 10^{-04}$	3.123	$1.2 \cdot 10^{-03}$	3.098
$1/5 \cdot 2^{-5}$	$2.9 \cdot 10^{-05}$	3.058	$1.4 \cdot 10^{-04}$	3.045
$1/5 \cdot 2^{-6}$	$3.5 \cdot 10^{-06}$	3.028	$1.8 \cdot 10^{-05}$	3.022
$1/5 \cdot 2^{-7}$	$4.4 \cdot 10^{-07}$	3.013	$2.2 \cdot 10^{-06}$	3.010
$1/5 \cdot 2^{-8}$	$5.4 \cdot 10^{-08}$	3.006	$2.7 \cdot 10^{-07}$	3.005
$1/5 \cdot 2^{-9}$	$6.7 \cdot 10^{-09}$	3.003	$3.4 \cdot 10^{-08}$	3.002
$1/5 \cdot 2^{-10}$	$8.4 \cdot 10^{-10}$	3.001	$4.2 \cdot 10^{-09}$	3.001
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-10}$	3.000	$5.3 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-11}$	3.000	$6.7 \cdot 10^{-11}$	3.000
$1/5 \cdot 2^{-13}$	$1.6 \cdot 10^{-12}$	3.000	$8.3 \cdot 10^{-12}$	3.000
$1/5 \cdot 2^{-14}$	$2.0 \cdot 10^{-13}$	-41.768	$1.0 \cdot 10^{-12}$	-37.029
$1/5 \cdot 2^{-15}$	$7.7 \cdot 10^{-01}$	47.764	$1.4 \cdot 10^{-01}$	43.024
$1/5 \cdot 2^{-16}$	$3.2 \cdot 10^{-15}$		$1.6 \cdot 10^{-14}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$4.0 \cdot 10^{-02}$	3.248	$1.4 \cdot 10^{-01}$	3.245
$1/5 \cdot 2^{-4}$	$4.2 \cdot 10^{-03}$	3.106	$1.5 \cdot 10^{-02}$	3.105
$1/5 \cdot 2^{-5}$	$4.9 \cdot 10^{-04}$	3.049	$1.7 \cdot 10^{-03}$	3.049
$1/5 \cdot 2^{-6}$	$6.0 \cdot 10^{-05}$	3.024	$2.1 \cdot 10^{-04}$	3.023
$1/5 \cdot 2^{-7}$	$7.3 \cdot 10^{-06}$	3.011	$2.6 \cdot 10^{-05}$	3.011
$1/5 \cdot 2^{-8}$	$9.1 \cdot 10^{-07}$	3.005	$3.2 \cdot 10^{-06}$	3.005
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-07}$	3.002	$4.1 \cdot 10^{-07}$	3.002
$1/5 \cdot 2^{-10}$	$1.4 \cdot 10^{-08}$	3.001	$5.1 \cdot 10^{-08}$	3.001
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-09}$	3.000	$6.3 \cdot 10^{-09}$	3.000
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-10}$	3.000	$7.9 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-13}$	$2.7 \cdot 10^{-11}$	3.000	$9.9 \cdot 10^{-11}$	3.000
$1/5 \cdot 2^{-14}$	$3.4 \cdot 10^{-12}$	-33.815	$1.2 \cdot 10^{-11}$	-36.002
$1/5 \cdot 2^{-15}$	$5.2 \cdot 10^{-02}$		$8.5 \cdot 10^{-01}$	

Table 4.78: Explicit Euler, IDeC method at $t = 1/8$ for 2.1a.

h	δ	p	δ	p
$1/5$	$9.5 \cdot 10^{-01}$	1.050	$5.8 \cdot 10^{-01}$	2.039
$1/5 \cdot 2^{-1}$	$4.5 \cdot 10^{-01}$	1.022	$1.4 \cdot 10^{-01}$	2.039
$1/5 \cdot 2^{-2}$	$2.2 \cdot 10^{-01}$	1.010	$3.4 \cdot 10^{-02}$	2.021
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-01}$	1.005	$8.5 \cdot 10^{-03}$	2.011
$1/5 \cdot 2^{-4}$	$5.5 \cdot 10^{-02}$	1.002	$2.1 \cdot 10^{-03}$	2.005
$1/5 \cdot 2^{-5}$	$2.7 \cdot 10^{-02}$	1.001	$5.2 \cdot 10^{-04}$	2.002
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-02}$	1.000	$1.3 \cdot 10^{-04}$	2.001
$1/5 \cdot 2^{-7}$	$6.9 \cdot 10^{-03}$	1.000	$3.2 \cdot 10^{-05}$	2.000
$1/5 \cdot 2^{-8}$	$3.4 \cdot 10^{-03}$	1.000	$8.2 \cdot 10^{-06}$	2.000
$1/5 \cdot 2^{-9}$	$1.7 \cdot 10^{-03}$	1.000	$2.0 \cdot 10^{-06}$	2.000
$1/5 \cdot 2^{-10}$	$8.7 \cdot 10^{-04}$	1.000	$5.1 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-11}$	$4.3 \cdot 10^{-04}$	0.447	$1.2 \cdot 10^{-07}$	-18.752
$1/5 \cdot 2^{-12}$	$3.1 \cdot 10^{-04}$	1.552	$5.6 \cdot 10^{-02}$	22.750
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-04}$	1.000	$8.0 \cdot 10^{-09}$	2.000
$1/5 \cdot 2^{-14}$	$5.4 \cdot 10^{-05}$		$2.0 \cdot 10^{-09}$	

h	δ	p	δ	p
$1/5$	$6.8 \cdot 10^{-02}$	3.057	$7.6 \cdot 10^{-01}$	3.235
$1/5 \cdot 2^{-1}$	$8.1 \cdot 10^{-03}$	2.668	$8.1 \cdot 10^{-02}$	3.088
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-03}$	2.876	$9.5 \cdot 10^{-03}$	3.044
$1/5 \cdot 2^{-3}$	$1.7 \cdot 10^{-04}$	2.943	$1.1 \cdot 10^{-03}$	3.022
$1/5 \cdot 2^{-4}$	$2.2 \cdot 10^{-05}$	2.972	$1.4 \cdot 10^{-04}$	3.011
$1/5 \cdot 2^{-5}$	$2.9 \cdot 10^{-06}$	2.986	$1.7 \cdot 10^{-05}$	3.005
$1/5 \cdot 2^{-6}$	$3.6 \cdot 10^{-07}$	2.993	$2.2 \cdot 10^{-06}$	3.002
$1/5 \cdot 2^{-7}$	$4.6 \cdot 10^{-08}$	2.996	$2.7 \cdot 10^{-07}$	3.001
$1/5 \cdot 2^{-8}$	$5.7 \cdot 10^{-09}$	2.998	$3.4 \cdot 10^{-08}$	3.000
$1/5 \cdot 2^{-9}$	$7.2 \cdot 10^{-10}$	2.999	$4.2 \cdot 10^{-09}$	3.000
$1/5 \cdot 2^{-10}$	$9.0 \cdot 10^{-11}$	2.999	$5.3 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-11}$	-33.804	$6.6 \cdot 10^{-11}$	-28.848
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-01}$	39.800	$3.2 \cdot 10^{-02}$	34.844
$1/5 \cdot 2^{-13}$	$1.7 \cdot 10^{-13}$		$1.0 \cdot 10^{-12}$	

h	δ	p	δ	p
$1/5$	2.5	3.228	9.1	3.220
$1/5 \cdot 2^{-1}$	$2.7 \cdot 10^{-01}$	3.106	$9.8 \cdot 10^{-01}$	3.101
$1/5 \cdot 2^{-2}$	$3.1 \cdot 10^{-02}$	3.050	$1.1 \cdot 10^{-01}$	3.049
$1/5 \cdot 2^{-3}$	$3.8 \cdot 10^{-03}$	3.024	$1.3 \cdot 10^{-02}$	3.023
$1/5 \cdot 2^{-4}$	$4.7 \cdot 10^{-04}$	3.012	$1.7 \cdot 10^{-03}$	3.011
$1/5 \cdot 2^{-5}$	$5.8 \cdot 10^{-05}$	3.005	$2.1 \cdot 10^{-04}$	3.005
$1/5 \cdot 2^{-6}$	$7.3 \cdot 10^{-06}$	3.002	$2.6 \cdot 10^{-05}$	3.002
$1/5 \cdot 2^{-7}$	$9.1 \cdot 10^{-07}$	3.001	$3.2 \cdot 10^{-06}$	3.001
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-07}$	3.000	$4.0 \cdot 10^{-07}$	3.000
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-08}$	3.000	$5.1 \cdot 10^{-08}$	3.000
$1/5 \cdot 2^{-10}$	$1.7 \cdot 10^{-09}$	3.000	$6.3 \cdot 10^{-09}$	3.000
$1/5 \cdot 2^{-11}$	$2.2 \cdot 10^{-10}$	-25.631	$7.9 \cdot 10^{-10}$	-27.811
$1/5 \cdot 2^{-12}$	$1.1 \cdot 10^{-02}$		$1.8 \cdot 10^{-01}$	

Table 4.79: Explicit Euler, IDeC method at $t = 1$ for 2.1a.

For problems, 2.1a, 2.2a and 2.3a, the same properties as for 1.2a are observed. These results are listed in Tables 4.77 – 4.85.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	1.6	1.00000	0.823	$4.3\cdot10^{-01}$	1.00000	1.656
$1/5\cdot2^{-1}$	$9.0\cdot10^{-01}$	1.00000	0.905	$1.3\cdot10^{-01}$	1.00000	1.756
$1/5\cdot2^{-2}$	$4.8\cdot10^{-01}$	1.00000	0.950	$4.0\cdot10^{-02}$	1.00000	1.864
$1/5\cdot2^{-3}$	$2.5\cdot10^{-01}$	1.00000	0.974	$1.1\cdot10^{-02}$	1.00000	1.928
$1/5\cdot2^{-4}$	$1.2\cdot10^{-01}$	1.00000	0.987	$2.9\cdot10^{-03}$	1.00000	1.963
$1/5\cdot2^{-5}$	$6.4\cdot10^{-02}$	1.00000	0.993	$7.5\cdot10^{-04}$	1.00000	1.981
$1/5\cdot2^{-6}$	$3.2\cdot10^{-02}$	1.00000	0.996	$1.9\cdot10^{-04}$	1.00000	1.990
$1/5\cdot2^{-7}$	$1.6\cdot10^{-02}$	1.00000	0.998	$4.8\cdot10^{-05}$	1.00000	1.995
$1/5\cdot2^{-8}$	$8.1\cdot10^{-03}$	1.00000	0.999	$1.2\cdot10^{-05}$	1.00000	1.997
$1/5\cdot2^{-9}$	$4.0\cdot10^{-03}$	1.00000	0.999	$3.0\cdot10^{-06}$	1.00000	1.998
$1/5\cdot2^{-10}$	$2.0\cdot10^{-03}$	1.00000	0.999	$7.5\cdot10^{-07}$	1.00000	1.999
$1/5\cdot2^{-11}$	$1.0\cdot10^{-03}$	1.00000	0.999	$1.8\cdot10^{-07}$	1.00000	-22.470
$1/5\cdot2^{-12}$	$5.0\cdot10^{-04}$	0.99998	0.999	1.0	0.99984	26.467
$1/5\cdot2^{-13}$	$2.5\cdot10^{-04}$	1.00000	0.999	$1.1\cdot10^{-08}$	1.00000	1.999
$1/5\cdot2^{-14}$	$1.2\cdot10^{-04}$	1.00000		$2.9\cdot10^{-09}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	1.4	0.20000	2.323	6.0	0.20000	2.489
$1/5\cdot2^{-1}$	$2.9\cdot10^{-01}$	0.10000	2.085	1.0	0.10000	2.128
$1/5\cdot2^{-2}$	$6.8\cdot10^{-02}$	0.05000	2.021	$2.4\cdot10^{-01}$	0.05000	2.032
$1/5\cdot2^{-3}$	$1.6\cdot10^{-02}$	0.02500	2.005	$6.0\cdot10^{-02}$	0.02500	2.008
$1/5\cdot2^{-4}$	$4.2\cdot10^{-03}$	0.01250	2.001	$1.5\cdot10^{-02}$	0.01250	2.002
$1/5\cdot2^{-5}$	$1.0\cdot10^{-03}$	0.00625	2.000	$3.7\cdot10^{-03}$	0.00625	2.000
$1/5\cdot2^{-6}$	$2.6\cdot10^{-04}$	0.00312	2.000	$9.4\cdot10^{-04}$	0.00312	2.000
$1/5\cdot2^{-7}$	$6.5\cdot10^{-05}$	0.00156	2.000	$2.3\cdot10^{-04}$	0.00156	2.000
$1/5\cdot2^{-8}$	$1.6\cdot10^{-05}$	0.00078	2.000	$5.8\cdot10^{-05}$	0.00078	2.000
$1/5\cdot2^{-9}$	$4.1\cdot10^{-06}$	0.00039	2.000	$1.4\cdot10^{-05}$	0.00039	2.000
$1/5\cdot2^{-10}$	$1.0\cdot10^{-06}$	0.00019	2.000	$3.6\cdot10^{-06}$	0.00019	2.000
$1/5\cdot2^{-11}$	$2.5\cdot10^{-07}$	0.00009	-23.029	$9.1\cdot10^{-07}$	0.00009	-20.382
$1/5\cdot2^{-12}$	2.2	0.99998	27.027	1.2	0.99993	24.379
$1/5\cdot2^{-13}$	$1.6\cdot10^{-08}$	0.00002		$5.7\cdot10^{-08}$	0.00002	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$2.5\cdot10^{+01}$	0.20000	2.657	$1.0\cdot10^{+02}$	0.20000	2.825
$1/5\cdot2^{-1}$	4.0	0.10000	2.171	$1.4\cdot10^{+01}$	0.10000	2.214
$1/5\cdot2^{-2}$	$8.9\cdot10^{-01}$	0.05000	2.043	3.2	0.05000	2.054
$1/5\cdot2^{-3}$	$2.1\cdot10^{-01}$	0.02500	2.010	$7.7\cdot10^{-01}$	0.02500	2.013
$1/5\cdot2^{-4}$	$5.3\cdot10^{-02}$	0.01250	2.002	$1.9\cdot10^{-01}$	0.01250	2.003
$1/5\cdot2^{-5}$	$1.3\cdot10^{-02}$	0.00625	2.000	$4.7\cdot10^{-02}$	0.00625	2.000
$1/5\cdot2^{-6}$	$3.3\cdot10^{-03}$	0.00312	2.000	$1.1\cdot10^{-02}$	0.00312	2.000
$1/5\cdot2^{-7}$	$8.3\cdot10^{-04}$	0.00156	2.000	$2.9\cdot10^{-03}$	0.00156	2.000
$1/5\cdot2^{-8}$	$2.0\cdot10^{-04}$	0.00078	2.000	$7.4\cdot10^{-04}$	0.00078	2.000
$1/5\cdot2^{-9}$	$5.2\cdot10^{-05}$	0.00039	2.000	$1.8\cdot10^{-04}$	0.00039	2.000
$1/5\cdot2^{-10}$	$1.3\cdot10^{-05}$	0.00019	2.000	$4.6\cdot10^{-05}$	0.00019	2.000
$1/5\cdot2^{-11}$	$3.2\cdot10^{-06}$	0.00009	-18.690	$1.1\cdot10^{-05}$	0.00009	-17.673
$1/5\cdot2^{-12}$	1.3	0.99984		2.4	0.99998	

Table 4.80: Explicit Euler, IDeC method on $[0,1]$ for 2.2a.

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-02}$	0.997	$5.3 \cdot 10^{-04}$	3.012
$1/5 \cdot 2^{-4}$	$5.7 \cdot 10^{-03}$	0.998	$6.5 \cdot 10^{-05}$	2.763
$1/5 \cdot 2^{-5}$	$2.9 \cdot 10^{-03}$	0.999	$9.7 \cdot 10^{-06}$	2.553
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-03}$	0.999	$1.6 \cdot 10^{-06}$	2.366
$1/5 \cdot 2^{-7}$	$7.2 \cdot 10^{-04}$	0.999	$3.2 \cdot 10^{-07}$	2.220
$1/5 \cdot 2^{-8}$	$3.6 \cdot 10^{-04}$	0.999	$6.8 \cdot 10^{-08}$	2.123
$1/5 \cdot 2^{-9}$	$1.8 \cdot 10^{-04}$	0.999	$1.5 \cdot 10^{-08}$	1.982
$1/5 \cdot 2^{-10}$	$9.0 \cdot 10^{-05}$	0.999	$4.0 \cdot 10^{-09}$	1.964
$1/5 \cdot 2^{-11}$	$4.5 \cdot 10^{-05}$	0.999	$1.0 \cdot 10^{-09}$	1.982
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-05}$	0.999	$2.5 \cdot 10^{-10}$	1.991
$1/5 \cdot 2^{-13}$	$1.1 \cdot 10^{-05}$	0.999	$6.5 \cdot 10^{-11}$	1.995
$1/5 \cdot 2^{-14}$	$5.6 \cdot 10^{-06}$	0.685	$1.6 \cdot 10^{-11}$	-34.642
$1/5 \cdot 2^{-15}$	$3.5 \cdot 10^{-06}$	1.314	$4.3 \cdot 10^{-01}$	38.635
$1/5 \cdot 2^{-16}$	$1.4 \cdot 10^{-06}$	0.999	$1.0 \cdot 10^{-12}$	1.999
$1/5 \cdot 2^{-17}$	$7.0 \cdot 10^{-07}$		$2.5 \cdot 10^{-13}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$1.0 \cdot 10^{-03}$	3.269	$4.9 \cdot 10^{-03}$	3.223
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-04}$	3.108	$5.2 \cdot 10^{-04}$	3.085
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-05}$	3.050	$6.2 \cdot 10^{-05}$	3.038
$1/5 \cdot 2^{-6}$	$1.5 \cdot 10^{-06}$	3.025	$7.5 \cdot 10^{-06}$	3.018
$1/5 \cdot 2^{-7}$	$1.8 \cdot 10^{-07}$	3.012	$9.3 \cdot 10^{-07}$	3.009
$1/5 \cdot 2^{-8}$	$2.3 \cdot 10^{-08}$	3.006	$1.1 \cdot 10^{-07}$	3.004
$1/5 \cdot 2^{-9}$	$2.9 \cdot 10^{-09}$	3.003	$1.4 \cdot 10^{-08}$	3.002
$1/5 \cdot 2^{-10}$	$3.6 \cdot 10^{-10}$	3.001	$1.8 \cdot 10^{-09}$	3.001
$1/5 \cdot 2^{-11}$	$4.5 \cdot 10^{-11}$	3.000	$2.2 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-12}$	$5.6 \cdot 10^{-12}$	3.000	$2.8 \cdot 10^{-11}$	3.000
$1/5 \cdot 2^{-13}$	$7.0 \cdot 10^{-13}$	3.000	$3.5 \cdot 10^{-12}$	3.000
$1/5 \cdot 2^{-14}$	$8.8 \cdot 10^{-14}$	-43.759	$4.4 \cdot 10^{-13}$	-39.043
$1/5 \cdot 2^{-15}$	1.3	49.755	$2.4 \cdot 10^{-01}$	45.039
$1/5 \cdot 2^{-16}$	$1.3 \cdot 10^{-15}$		$6.8 \cdot 10^{-15}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$1.6 \cdot 10^{-02}$	3.245	$6.0 \cdot 10^{-02}$	3.246
$1/5 \cdot 2^{-4}$	$1.7 \cdot 10^{-03}$	3.094	$6.3 \cdot 10^{-03}$	3.093
$1/5 \cdot 2^{-5}$	$2.0 \cdot 10^{-04}$	3.042	$7.4 \cdot 10^{-04}$	3.041
$1/5 \cdot 2^{-6}$	$2.5 \cdot 10^{-05}$	3.020	$9.0 \cdot 10^{-05}$	3.020
$1/5 \cdot 2^{-7}$	$3.1 \cdot 10^{-06}$	3.010	$1.1 \cdot 10^{-05}$	3.010
$1/5 \cdot 2^{-8}$	$3.8 \cdot 10^{-07}$	3.005	$1.3 \cdot 10^{-06}$	3.005
$1/5 \cdot 2^{-9}$	$4.8 \cdot 10^{-08}$	3.002	$1.7 \cdot 10^{-07}$	3.002
$1/5 \cdot 2^{-10}$	$6.0 \cdot 10^{-09}$	3.001	$2.1 \cdot 10^{-08}$	3.001
$1/5 \cdot 2^{-11}$	$7.4 \cdot 10^{-10}$	3.000	$2.6 \cdot 10^{-09}$	3.000
$1/5 \cdot 2^{-12}$	$9.3 \cdot 10^{-11}$	3.000	$3.3 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-13}$	$1.1 \cdot 10^{-11}$	3.000	$4.2 \cdot 10^{-11}$	3.000
$1/5 \cdot 2^{-14}$	$1.4 \cdot 10^{-12}$	-35.830	$5.2 \cdot 10^{-12}$	-38.017
$1/5 \cdot 2^{-15}$	$8.9 \cdot 10^{-02}$		1.4	

Table 4.81: Explicit Euler, IDeC method at $t = 1/8$ for 2.2a.

h	δ	p	δ	p
$1/5$	1.6	0.823	$4.3 \cdot 10^{-01}$	1.656
$1/5 \cdot 2^{-1}$	$9.0 \cdot 10^{-01}$	0.905	$1.3 \cdot 10^{-01}$	1.756
$1/5 \cdot 2^{-2}$	$4.8 \cdot 10^{-01}$	0.950	$4.0 \cdot 10^{-02}$	1.864
$1/5 \cdot 2^{-3}$	$2.5 \cdot 10^{-01}$	0.974	$1.1 \cdot 10^{-02}$	1.928
$1/5 \cdot 2^{-4}$	$1.2 \cdot 10^{-01}$	0.987	$2.9 \cdot 10^{-03}$	1.963
$1/5 \cdot 2^{-5}$	$6.4 \cdot 10^{-02}$	0.993	$7.5 \cdot 10^{-04}$	1.981
$1/5 \cdot 2^{-6}$	$3.2 \cdot 10^{-02}$	0.996	$1.9 \cdot 10^{-04}$	1.990
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-02}$	0.998	$4.8 \cdot 10^{-05}$	1.995
$1/5 \cdot 2^{-8}$	$8.1 \cdot 10^{-03}$	0.999	$1.2 \cdot 10^{-05}$	1.997
$1/5 \cdot 2^{-9}$	$4.0 \cdot 10^{-03}$	0.999	$3.0 \cdot 10^{-06}$	1.998
$1/5 \cdot 2^{-10}$	$2.0 \cdot 10^{-03}$	0.999	$7.5 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-03}$	0.572	$1.8 \cdot 10^{-07}$	-21.885
$1/5 \cdot 2^{-12}$	$6.8 \cdot 10^{-04}$	1.426	$7.3 \cdot 10^{-01}$	25.882
$1/5 \cdot 2^{-13}$	$2.5 \cdot 10^{-04}$	0.999	$1.1 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-14}$	$1.2 \cdot 10^{-04}$		$2.9 \cdot 10^{-09}$	

h	δ	p	δ	p
$1/5$	$2.7 \cdot 10^{-01}$	2.590	$7.0 \cdot 10^{-01}$	3.367
$1/5 \cdot 2^{-1}$	$4.4 \cdot 10^{-02}$	2.663	$6.8 \cdot 10^{-02}$	2.717
$1/5 \cdot 2^{-2}$	$7.0 \cdot 10^{-03}$	2.812	$1.0 \cdot 10^{-02}$	2.777
$1/5 \cdot 2^{-3}$	$1.0 \cdot 10^{-03}$	2.907	$1.5 \cdot 10^{-03}$	2.886
$1/5 \cdot 2^{-4}$	$1.3 \cdot 10^{-04}$	2.957	$2.0 \cdot 10^{-04}$	2.950
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-05}$	2.980	$2.6 \cdot 10^{-05}$	2.980
$1/5 \cdot 2^{-6}$	$2.1 \cdot 10^{-06}$	2.991	$3.3 \cdot 10^{-06}$	2.993
$1/5 \cdot 2^{-7}$	$2.7 \cdot 10^{-07}$	2.996	$4.2 \cdot 10^{-07}$	2.998
$1/5 \cdot 2^{-8}$	$3.4 \cdot 10^{-08}$	2.998	$5.2 \cdot 10^{-08}$	3.000
$1/5 \cdot 2^{-9}$	$4.3 \cdot 10^{-09}$	2.999	$6.5 \cdot 10^{-09}$	3.000
$1/5 \cdot 2^{-10}$	$5.4 \cdot 10^{-10}$	2.999	$8.2 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-11}$	$6.7 \cdot 10^{-11}$	-34.925	$1.0 \cdot 10^{-10}$	-31.917
$1/5 \cdot 2^{-12}$	2.2	40.921	$4.1 \cdot 10^{-01}$	37.914
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-12}$		$1.6 \cdot 10^{-12}$	

h	δ	p	δ	p
$1/5$	3.0	3.555	$1.2 \cdot 10^{+01}$	3.717
$1/5 \cdot 2^{-1}$	$2.5 \cdot 10^{-01}$	2.811	$9.5 \cdot 10^{-01}$	2.847
$1/5 \cdot 2^{-2}$	$3.6 \cdot 10^{-02}$	2.819	$1.3 \cdot 10^{-01}$	2.827
$1/5 \cdot 2^{-3}$	$5.1 \cdot 10^{-03}$	2.906	$1.8 \cdot 10^{-02}$	2.908
$1/5 \cdot 2^{-4}$	$6.9 \cdot 10^{-04}$	2.960	$2.4 \cdot 10^{-03}$	2.960
$1/5 \cdot 2^{-5}$	$8.8 \cdot 10^{-05}$	2.986	$3.1 \cdot 10^{-04}$	2.985
$1/5 \cdot 2^{-6}$	$1.1 \cdot 10^{-05}$	2.996	$4.0 \cdot 10^{-05}$	2.996
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-06}$	3.000	$5.0 \cdot 10^{-06}$	2.999
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-07}$	3.001	$6.3 \cdot 10^{-07}$	3.000
$1/5 \cdot 2^{-9}$	$2.1 \cdot 10^{-08}$	3.000	$7.8 \cdot 10^{-08}$	3.000
$1/5 \cdot 2^{-10}$	$2.7 \cdot 10^{-09}$	3.000	$9.8 \cdot 10^{-09}$	3.000
$1/5 \cdot 2^{-11}$	$3.4 \cdot 10^{-10}$	-28.709	$1.2 \cdot 10^{-09}$	-30.892
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-01}$		2.4	

Table 4.82: Explicit Euler, IDeC method at $t = 1$ for 2.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$3.8 \cdot 10^{-02}$	0.80000	1.196	$2.6 \cdot 10^{-02}$	0.20000	1.973
$1/5 \cdot 2^{-1}$	$1.6 \cdot 10^{-02}$	0.80000	1.085	$6.6 \cdot 10^{-03}$	0.10000	1.993
$1/5 \cdot 2^{-2}$	$7.9 \cdot 10^{-03}$	0.75000	1.040	$1.6 \cdot 10^{-03}$	0.05000	1.998
$1/5 \cdot 2^{-3}$	$3.8 \cdot 10^{-03}$	0.75000	1.019	$4.1 \cdot 10^{-04}$	0.02500	1.999
$1/5 \cdot 2^{-4}$	$1.9 \cdot 10^{-03}$	0.75000	1.009	$1.0 \cdot 10^{-04}$	0.01250	1.999
$1/5 \cdot 2^{-5}$	$9.4 \cdot 10^{-04}$	0.75000	1.004	$2.6 \cdot 10^{-05}$	0.00625	1.999
$1/5 \cdot 2^{-6}$	$4.7 \cdot 10^{-04}$	0.75312	1.002	$6.5 \cdot 10^{-06}$	0.00312	1.999
$1/5 \cdot 2^{-7}$	$2.3 \cdot 10^{-04}$	0.75156	1.001	$1.6 \cdot 10^{-06}$	0.00156	1.999
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-04}$	0.75156	1.000	$4.0 \cdot 10^{-07}$	0.00078	1.999
$1/5 \cdot 2^{-9}$	$5.8 \cdot 10^{-05}$	0.75117	1.000	$1.0 \cdot 10^{-07}$	0.00039	1.999
$1/5 \cdot 2^{-10}$	$2.9 \cdot 10^{-05}$	0.75097	1.000	$2.5 \cdot 10^{-08}$	0.00019	1.999
$1/5 \cdot 2^{-11}$	$1.4 \cdot 10^{-05}$	0.75097	1.000	$6.3 \cdot 10^{-09}$	0.00009	-24.700
$1/5 \cdot 2^{-12}$	$7.3 \cdot 10^{-06}$	0.75100	1.000	$1.7 \cdot 10^{-01}$	0.99984	28.697
$1/5 \cdot 2^{-13}$	$3.6 \cdot 10^{-06}$	0.75100	1.000	$3.9 \cdot 10^{-10}$	0.00002	1.999
$1/5 \cdot 2^{-14}$	$1.8 \cdot 10^{-06}$	0.75100		$9.9 \cdot 10^{-11}$	0.00001	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$7.6 \cdot 10^{-02}$	0.20000	1.731	$2.1 \cdot 10^{-01}$	0.20000	1.440
$1/5 \cdot 2^{-1}$	$2.3 \cdot 10^{-02}$	0.10000	1.940	$7.8 \cdot 10^{-02}$	0.10000	1.877
$1/5 \cdot 2^{-2}$	$6.0 \cdot 10^{-03}$	0.05000	1.985	$2.1 \cdot 10^{-02}$	0.05000	1.970
$1/5 \cdot 2^{-3}$	$1.5 \cdot 10^{-03}$	0.02500	1.996	$5.4 \cdot 10^{-03}$	0.02500	1.992
$1/5 \cdot 2^{-4}$	$3.8 \cdot 10^{-04}$	0.01250	1.999	$1.3 \cdot 10^{-03}$	0.01250	1.998
$1/5 \cdot 2^{-5}$	$9.5 \cdot 10^{-05}$	0.00625	1.999	$3.4 \cdot 10^{-04}$	0.00625	1.999
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-05}$	0.00312	1.999	$8.5 \cdot 10^{-05}$	0.00312	1.999
$1/5 \cdot 2^{-7}$	$5.9 \cdot 10^{-06}$	0.00156	1.999	$2.1 \cdot 10^{-05}$	0.00156	1.999
$1/5 \cdot 2^{-8}$	$1.4 \cdot 10^{-06}$	0.00078	1.999	$5.3 \cdot 10^{-06}$	0.00078	1.999
$1/5 \cdot 2^{-9}$	$3.7 \cdot 10^{-07}$	0.00039	1.999	$1.3 \cdot 10^{-06}$	0.00039	1.999
$1/5 \cdot 2^{-10}$	$9.3 \cdot 10^{-08}$	0.00019	1.999	$3.3 \cdot 10^{-07}$	0.00019	1.999
$1/5 \cdot 2^{-11}$	$2.3 \cdot 10^{-08}$	0.00009	-23.826	$8.3 \cdot 10^{-08}$	0.00009	-21.178
$1/5 \cdot 2^{-12}$	$3.4 \cdot 10^{-01}$	0.99998	27.823	$1.9 \cdot 10^{-01}$	0.99993	25.176
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-09}$	0.00002		$5.2 \cdot 10^{-09}$	0.00002	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$7.0 \cdot 10^{-01}$	0.40000	1.415	1.9	0.40000	1.156
$1/5 \cdot 2^{-1}$	$2.6 \cdot 10^{-01}$	0.10000	1.823	$8.9 \cdot 10^{-01}$	0.10000	1.754
$1/5 \cdot 2^{-2}$	$7.4 \cdot 10^{-02}$	0.05000	1.956	$2.6 \cdot 10^{-01}$	0.05000	1.942
$1/5 \cdot 2^{-3}$	$1.9 \cdot 10^{-02}$	0.02500	1.989	$6.8 \cdot 10^{-02}$	0.02500	1.985
$1/5 \cdot 2^{-4}$	$4.8 \cdot 10^{-03}$	0.01250	1.997	$1.7 \cdot 10^{-02}$	0.01250	1.996
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-03}$	0.00625	1.999	$4.3 \cdot 10^{-03}$	0.00625	1.999
$1/5 \cdot 2^{-6}$	$3.0 \cdot 10^{-04}$	0.00312	1.999	$1.0 \cdot 10^{-03}$	0.00312	1.999
$1/5 \cdot 2^{-7}$	$7.6 \cdot 10^{-05}$	0.00156	1.999	$2.7 \cdot 10^{-04}$	0.00156	1.999
$1/5 \cdot 2^{-8}$	$1.9 \cdot 10^{-05}$	0.00078	1.999	$6.7 \cdot 10^{-05}$	0.00078	1.999
$1/5 \cdot 2^{-9}$	$4.7 \cdot 10^{-06}$	0.00039	1.999	$1.6 \cdot 10^{-05}$	0.00039	1.999
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-06}$	0.00019	1.999	$4.2 \cdot 10^{-06}$	0.00019	1.999
$1/5 \cdot 2^{-11}$	$2.9 \cdot 10^{-07}$	0.00009	-19.486	$1.0 \cdot 10^{-06}$	0.00009	-18.469
$1/5 \cdot 2^{-12}$	$2.1 \cdot 10^{-01}$	0.99984		$3.8 \cdot 10^{-01}$	0.99998	

Table 4.83: Explicit Euler, IDeC method on $[0, 1]$ for 2.3a.

For problem 3.1a, it is interesting to note that $t^{\max} \approx 0.6$ for the second iterate in spite of the fact that an order reduction down to 2 is observed. The last iteration step suddenly improves the convergence order to a level of accuracy $p = 4$ which is surprising for the explicit Euler method, as for classical problems an improvement of the convergence order by 1 is generally observed in every step. However, the same effect is visible at a fixed point t. The results are listed in Tables 4.86 – 4.88.

For problem 3.1b, the maximal error on $[0, 1]$ and the error at a fixed point show the same behavior, an order reduction down to 2 occurs, but the last iteration step spontaneously increases the level of accuracy by 1. Table 4.89 shows the result for the maximal error, the order sequences at $t = 1/8$ and $t = 1$ are omitted.

The special properties of the solution of problem 3.1c were already discussed in §4.2.2. Thus, as for the box scheme, a higher convergence order than for the other singular test examples can be obtained by IDeC based on the explicit Euler method. Since polynomials of degree 5 were used, this is even the highest order which can be achieved in general. The fact that IDeC works for 3.1c in a fully classical manner is also reflected by the fact that for all iterates, $t^{\max} \approx 1$ holds. Naturally, the same properties are observed at a fixed point t. Therefore, only the results for $[0, 1]$ are given in Table 4.90.

Table 4.91 shows the results for 3.2a. The usual order reduction occurs, the improvement in the last step is again observed. Surprisingly, though, $t^{\max} \approx 1$ in every iteration step.

For the nonlinear problem 3.3a, the results on $[0, 1]$ and at a fixed point t are equal, therefore only Table 4.92 is displayed. For the third iteration step $t^{\max} \to 0$ as $h \to 0$.

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$1.0 \cdot 10^{-03}$	1.003	$4.8 \cdot 10^{-05}$	2.930
$1/5 \cdot 2^{-4}$	$5.1 \cdot 10^{-04}$	1.001	$6.3 \cdot 10^{-06}$	2.680
$1/5 \cdot 2^{-5}$	$2.5 \cdot 10^{-04}$	1.000	$9.9 \cdot 10^{-07}$	2.456
$1/5 \cdot 2^{-6}$	$1.2 \cdot 10^{-04}$	1.000	$1.8 \cdot 10^{-07}$	2.279
$1/5 \cdot 2^{-7}$	$6.4 \cdot 10^{-05}$	1.000	$3.7 \cdot 10^{-08}$	2.158
$1/5 \cdot 2^{-8}$	$3.2 \cdot 10^{-05}$	1.000	$8.3 \cdot 10^{-09}$	2.085
$1/5 \cdot 2^{-9}$	$1.6 \cdot 10^{-05}$	1.000	$1.9 \cdot 10^{-09}$	2.044
$1/5 \cdot 2^{-10}$	$8.0 \cdot 10^{-06}$	1.000	$4.7 \cdot 10^{-10}$	2.022
$1/5 \cdot 2^{-11}$	$4.0 \cdot 10^{-06}$	1.000	$1.1 \cdot 10^{-10}$	2.011
$1/5 \cdot 2^{-12}$	$2.0 \cdot 10^{-06}$	1.000	$2.9 \cdot 10^{-11}$	2.005
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-06}$	1.000	$7.2 \cdot 10^{-12}$	2.002
$1/5 \cdot 2^{-14}$	$5.0 \cdot 10^{-07}$	0.681	$1.8 \cdot 10^{-12}$	-36.097
$1/5 \cdot 2^{-15}$	$3.1 \cdot 10^{-07}$	1.318	$1.3 \cdot 10^{-01}$	40.096
$1/5 \cdot 2^{-16}$	$1.2 \cdot 10^{-07}$	1.000	$1.1 \cdot 10^{-13}$	2.000
$1/5 \cdot 2^{-17}$	$6.2 \cdot 10^{-08}$		$2.8 \cdot 10^{-14}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$9.4 \cdot 10^{-05}$	3.243	$4.4 \cdot 10^{-04}$	3.182
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-05}$	3.119	$4.8 \cdot 10^{-05}$	3.090
$1/5 \cdot 2^{-5}$	$1.1 \cdot 10^{-06}$	3.058	$5.7 \cdot 10^{-06}$	3.045
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-07}$	3.029	$6.9 \cdot 10^{-07}$	3.022
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-08}$	3.014	$8.5 \cdot 10^{-08}$	3.011
$1/5 \cdot 2^{-8}$	$2.0 \cdot 10^{-09}$	3.007	$1.0 \cdot 10^{-08}$	3.005
$1/5 \cdot 2^{-9}$	$2.6 \cdot 10^{-10}$	3.003	$1.3 \cdot 10^{-09}$	3.002
$1/5 \cdot 2^{-10}$	$3.2 \cdot 10^{-11}$	3.001	$1.6 \cdot 10^{-10}$	3.001
$1/5 \cdot 2^{-11}$	$4.0 \cdot 10^{-12}$	3.000	$2.0 \cdot 10^{-11}$	3.000
$1/5 \cdot 2^{-12}$	$5.0 \cdot 10^{-13}$	3.000	$2.5 \cdot 10^{-12}$	3.000
$1/5 \cdot 2^{-13}$	$6.3 \cdot 10^{-14}$	3.000	$3.2 \cdot 10^{-13}$	3.000
$1/5 \cdot 2^{-14}$	$7.9 \cdot 10^{-15}$	-45.521	$4.0 \cdot 10^{-14}$	-40.777
$1/5 \cdot 2^{-15}$	$3.9 \cdot 10^{-01}$	51.516	$7.5 \cdot 10^{-02}$	46.772
$1/5 \cdot 2^{-16}$	$1.2 \cdot 10^{-16}$		$6.2 \cdot 10^{-16}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$1.5 \cdot 10^{-03}$	3.196	$5.3 \cdot 10^{-03}$	3.194
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-04}$	3.097	$5.8 \cdot 10^{-04}$	3.095
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-05}$	3.049	$6.8 \cdot 10^{-05}$	3.048
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-06}$	3.024	$8.3 \cdot 10^{-06}$	3.024
$1/5 \cdot 2^{-7}$	$2.8 \cdot 10^{-07}$	3.012	$1.0 \cdot 10^{-06}$	3.012
$1/5 \cdot 2^{-8}$	$3.5 \cdot 10^{-08}$	3.006	$1.2 \cdot 10^{-07}$	3.006
$1/5 \cdot 2^{-9}$	$4.3 \cdot 10^{-09}$	3.003	$1.5 \cdot 10^{-08}$	3.003
$1/5 \cdot 2^{-10}$	$5.4 \cdot 10^{-10}$	3.001	$1.9 \cdot 10^{-09}$	3.001
$1/5 \cdot 2^{-11}$	$6.8 \cdot 10^{-11}$	3.000	$2.4 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-12}$	$8.5 \cdot 10^{-12}$	3.000	$3.0 \cdot 10^{-11}$	3.000
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-12}$	3.000	$3.8 \cdot 10^{-12}$	3.000
$1/5 \cdot 2^{-14}$	$1.3 \cdot 10^{-13}$	-37.564	$4.8 \cdot 10^{-13}$	-39.751
$1/5 \cdot 2^{-15}$	$2.7 \cdot 10^{-02}$		$4.4 \cdot 10^{-01}$	

Table 4.84: Explicit Euler, IDeC method at $t = 1/8$ for 2.3a.

h	δ	p	δ	p
$1/5$	$3.6\cdot10^{-02}$	1.215	$1.5\cdot10^{-02}$	2.242
$1/5\cdot2^{-1}$	$1.5\cdot10^{-02}$	1.106	$3.3\cdot10^{-03}$	2.251
$1/5\cdot2^{-2}$	$7.3\cdot10^{-03}$	1.052	$7.0\cdot10^{-04}$	2.128
$1/5\cdot2^{-3}$	$3.5\cdot10^{-03}$	1.026	$1.6\cdot10^{-04}$	2.056
$1/5\cdot2^{-4}$	$1.7\cdot10^{-03}$	1.013	$3.8\cdot10^{-05}$	2.026
$1/5\cdot2^{-5}$	$8.6\cdot10^{-04}$	1.006	$9.4\cdot10^{-06}$	2.010
$1/5\cdot2^{-6}$	$4.2\cdot10^{-04}$	1.003	$2.3\cdot10^{-06}$	2.001
$1/5\cdot2^{-7}$	$2.1\cdot10^{-04}$	1.001	$5.8\cdot10^{-07}$	2.000
$1/5\cdot2^{-8}$	$1.0\cdot10^{-04}$	1.000	$1.4\cdot10^{-07}$	2.000
$1/5\cdot2^{-9}$	$5.3\cdot10^{-05}$	1.000	$3.6\cdot10^{-08}$	2.000
$1/5\cdot2^{-10}$	$2.6\cdot10^{-05}$	1.000	$9.1\cdot10^{-09}$	2.000
$1/5\cdot2^{-11}$	$1.3\cdot10^{-05}$	0.533	$2.2\cdot10^{-09}$	-25.584
$1/5\cdot2^{-12}$	$9.2\cdot10^{-06}$	1.467	$1.1\cdot10^{-01}$	29.581
$1/5\cdot2^{-13}$	$3.3\cdot10^{-06}$	1.000	$1.4\cdot10^{-10}$	2.000
$1/5\cdot2^{-14}$	$1.6\cdot10^{-06}$		$3.5\cdot10^{-11}$	

h	δ	p	δ	p
$1/5$	$8.6\cdot10^{-03}$	3.235	$2.1\cdot10^{-02}$	1.965
$1/5\cdot2^{-1}$	$9.1\cdot10^{-04}$	3.327	$5.4\cdot10^{-03}$	3.084
$1/5\cdot2^{-2}$	$9.1\cdot10^{-05}$	3.340	$6.4\cdot10^{-04}$	3.283
$1/5\cdot2^{-3}$	$9.0\cdot10^{-06}$	3.144	$6.6\cdot10^{-05}$	3.172
$1/5\cdot2^{-4}$	$1.0\cdot10^{-06}$	3.039	$7.3\cdot10^{-06}$	3.080
$1/5\cdot2^{-5}$	$1.2\cdot10^{-07}$	3.000	$8.7\cdot10^{-07}$	3.032
$1/5\cdot2^{-6}$	$1.5\cdot10^{-08}$	2.991	$1.0\cdot10^{-07}$	3.011
$1/5\cdot2^{-7}$	$1.9\cdot10^{-09}$	2.990	$1.3\cdot10^{-08}$	3.002
$1/5\cdot2^{-8}$	$2.4\cdot10^{-10}$	2.993	$1.6\cdot10^{-09}$	3.000
$1/5\cdot2^{-9}$	$3.0\cdot10^{-11}$	2.995	$2.0\cdot10^{-10}$	2.999
$1/5\cdot2^{-10}$	$3.8\cdot10^{-12}$	2.997	$2.5\cdot10^{-11}$	2.999
$1/5\cdot2^{-11}$	$4.8\cdot10^{-13}$	-39.383	$3.2\cdot10^{-12}$	-34.251
$1/5\cdot2^{-12}$	$3.4\cdot10^{-01}$	45.376	$6.5\cdot10^{-02}$	40.246
$1/5\cdot2^{-13}$	$7.5\cdot10^{-15}$		$5.0\cdot10^{-14}$	

h	δ	p	δ	p
$1/5$	$5.0\cdot10^{-01}$	3.935	1.3	5.734
$1/5\cdot2^{-1}$	$3.2\cdot10^{-02}$	3.744	$2.6\cdot10^{-02}$	2.128
$1/5\cdot2^{-2}$	$2.4\cdot10^{-03}$	3.436	$6.0\cdot10^{-03}$	2.987
$1/5\cdot2^{-3}$	$2.2\cdot10^{-04}$	3.204	$7.5\cdot10^{-04}$	3.116
$1/5\cdot2^{-4}$	$2.4\cdot10^{-05}$	3.086	$8.7\cdot10^{-05}$	3.072
$1/5\cdot2^{-5}$	$2.8\cdot10^{-06}$	3.032	$1.0\cdot10^{-05}$	3.031
$1/5\cdot2^{-6}$	$3.5\cdot10^{-07}$	3.010	$1.2\cdot10^{-06}$	3.010
$1/5\cdot2^{-7}$	$4.3\cdot10^{-08}$	3.002	$1.5\cdot10^{-07}$	3.002
$1/5\cdot2^{-8}$	$5.4\cdot10^{-09}$	2.999	$1.9\cdot10^{-08}$	2.999
$1/5\cdot2^{-9}$	$6.8\cdot10^{-10}$	2.999	$2.4\cdot10^{-09}$	2.999
$1/5\cdot2^{-10}$	$8.5\cdot10^{-11}$	2.999	$3.0\cdot10^{-10}$	2.999
$1/5\cdot2^{-11}$	$1.0\cdot10^{-11}$	-31.037	$3.8\cdot10^{-11}$	-33.225
$1/5\cdot2^{-12}$	$2.3\cdot10^{-02}$		$3.8\cdot10^{-01}$	

Table 4.85: Explicit Euler, IDeC method at $t = 1$ for 2.3a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$9.3 \cdot 10^{-01}$	0.80000	0.914	$4.8 \cdot 10^{-01}$	0.40000	1.913
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-01}$	0.70000	0.965	$1.2 \cdot 10^{-01}$	0.20000	1.970
$1/5 \cdot 2^{-2}$	$2.5 \cdot 10^{-01}$	0.65000	0.984	$3.3 \cdot 10^{-02}$	0.10000	1.991
$1/5 \cdot 2^{-3}$	$1.2 \cdot 10^{-01}$	0.62500	0.992	$8.3 \cdot 10^{-03}$	0.05000	1.997
$1/5 \cdot 2^{-4}$	$6.4 \cdot 10^{-02}$	0.62500	0.996	$2.0 \cdot 10^{-03}$	0.02500	1.999
$1/5 \cdot 2^{-5}$	$3.2 \cdot 10^{-02}$	0.61250	0.998	$5.2 \cdot 10^{-04}$	0.01250	1.999
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-02}$	0.61250	0.999	$1.3 \cdot 10^{-04}$	0.00625	1.999
$1/5 \cdot 2^{-7}$	$8.0 \cdot 10^{-03}$	0.61250	0.999	$3.2 \cdot 10^{-05}$	0.00312	1.999
$1/5 \cdot 2^{-8}$	$4.0 \cdot 10^{-03}$	0.61250	0.999	$8.1 \cdot 10^{-06}$	0.00156	1.999
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-03}$	0.61250	0.999	$2.0 \cdot 10^{-06}$	0.00078	1.999
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-03}$	0.61230	0.999	$5.0 \cdot 10^{-07}$	0.00039	1.999
$1/5 \cdot 2^{-11}$	$5.0 \cdot 10^{-04}$	0.61220	0.999	$1.2 \cdot 10^{-07}$	0.00019	-19.343
$1/5 \cdot 2^{-12}$	$2.5 \cdot 10^{-04}$	0.61222	0.999	$8.4 \cdot 10^{-02}$	0.99984	23.341
$1/5 \cdot 2^{-13}$	$1.2 \cdot 10^{-04}$	0.61220	0.999	$7.9 \cdot 10^{-09}$	0.00004	1.999
$1/5 \cdot 2^{-14}$	$6.3 \cdot 10^{-05}$	0.61220		$1.9 \cdot 10^{-09}$	0.00002	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$8.2 \cdot 10^{-01}$	1.00000	2.354	$1.6 \cdot 10^{-01}$	1.00000	1.485
$1/5 \cdot 2^{-1}$	$1.6 \cdot 10^{-01}$	0.80000	2.181	$5.8 \cdot 10^{-02}$	0.40000	1.835
$1/5 \cdot 2^{-2}$	$3.5 \cdot 10^{-02}$	0.70000	2.093	$1.6 \cdot 10^{-02}$	0.20000	1.961
$1/5 \cdot 2^{-3}$	$8.3 \cdot 10^{-03}$	0.70000	2.048	$4.1 \cdot 10^{-03}$	0.10000	1.990
$1/5 \cdot 2^{-4}$	$2.0 \cdot 10^{-03}$	0.67500	2.024	$1.0 \cdot 10^{-03}$	0.05000	1.997
$1/5 \cdot 2^{-5}$	$4.9 \cdot 10^{-04}$	0.66875	2.012	$2.6 \cdot 10^{-04}$	0.02500	1.999
$1/5 \cdot 2^{-6}$	$1.2 \cdot 10^{-04}$	0.66562	2.006	$6.5 \cdot 10^{-05}$	0.01250	1.999
$1/5 \cdot 2^{-7}$	$3.0 \cdot 10^{-05}$	0.66406	2.003	$1.6 \cdot 10^{-05}$	0.00625	1.999
$1/5 \cdot 2^{-8}$	$7.6 \cdot 10^{-06}$	0.66328	2.001	$4.1 \cdot 10^{-06}$	0.00312	1.999
$1/5 \cdot 2^{-9}$	$1.9 \cdot 10^{-06}$	0.66250	2.000	$1.0 \cdot 10^{-06}$	0.00156	1.999
$1/5 \cdot 2^{-10}$	$4.7 \cdot 10^{-07}$	0.66230	2.000	$2.5 \cdot 10^{-07}$	0.00078	1.999
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-07}$	0.66220	-20.438	$6.4 \cdot 10^{-08}$	0.00039	-20.519
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-01}$	0.99998	24.436	$9.6 \cdot 10^{-02}$	0.99993	24.516
$1/5 \cdot 2^{-13}$	$7.4 \cdot 10^{-09}$	0.66208		$4.0 \cdot 10^{-09}$	0.00009	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$2.0 \cdot 10^{-01}$	0.40000	3.581	$1.3 \cdot 10^{-01}$	0.80000	4.622
$1/5 \cdot 2^{-1}$	$1.7 \cdot 10^{-02}$	0.50000	1.389	$5.5 \cdot 10^{-03}$	0.40000	4.393
$1/5 \cdot 2^{-2}$	$6.5 \cdot 10^{-03}$	0.25000	1.913	$2.6 \cdot 10^{-04}$	0.20000	4.133
$1/5 \cdot 2^{-3}$	$1.7 \cdot 10^{-03}$	0.12500	1.981	$1.5 \cdot 10^{-05}$	0.10000	4.036
$1/5 \cdot 2^{-4}$	$4.3 \cdot 10^{-04}$	0.06250	1.995	$9.1 \cdot 10^{-07}$	0.05000	4.009
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-04}$	0.03125	1.998	$5.6 \cdot 10^{-08}$	0.02500	4.002
$1/5 \cdot 2^{-6}$	$2.7 \cdot 10^{-05}$	0.01562	1.999	$3.5 \cdot 10^{-09}$	0.01250	4.000
$1/5 \cdot 2^{-7}$	$6.8 \cdot 10^{-06}$	0.00781	1.999	$2.2 \cdot 10^{-10}$	0.00625	4.000
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-06}$	0.00390	1.999	$1.3 \cdot 10^{-11}$	0.00312	4.000
$1/5 \cdot 2^{-9}$	$4.2 \cdot 10^{-07}$	0.00195	1.999	$8.6 \cdot 10^{-13}$	0.00156	4.000
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-07}$	0.00097	1.999	$5.4 \cdot 10^{-14}$	0.00078	4.000
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-08}$	0.00048	-21.924	$3.3 \cdot 10^{-15}$	0.00039	-45.666
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-01}$	0.99984		$1.8 \cdot 10^{-01}$	0.99998	

Table 4.86: Explicit Euler, IDeC method on $[0, 1]$ for 3.1a.

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$3.6 \cdot 10^{-02}$	0.895	$7.6 \cdot 10^{-03}$	2.012
$1/5 \cdot 2^{-4}$	$1.9 \cdot 10^{-02}$	0.950	$1.9 \cdot 10^{-03}$	2.006
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-02}$	0.975	$4.7 \cdot 10^{-04}$	2.003
$1/5 \cdot 2^{-6}$	$5.0 \cdot 10^{-03}$	0.988	$1.1 \cdot 10^{-04}$	2.001
$1/5 \cdot 2^{-7}$	$2.5 \cdot 10^{-03}$	0.994	$2.9 \cdot 10^{-05}$	2.000
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-03}$	0.997	$7.3 \cdot 10^{-06}$	2.000
$1/5 \cdot 2^{-9}$	$6.4 \cdot 10^{-04}$	0.998	$1.8 \cdot 10^{-06}$	2.000
$1/5 \cdot 2^{-10}$	$3.2 \cdot 10^{-04}$	0.999	$4.6 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-11}$	$1.6 \cdot 10^{-04}$	0.999	$1.1 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-12}$	$8.0 \cdot 10^{-05}$	0.999	$2.8 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-13}$	$4.0 \cdot 10^{-05}$	0.999	$7.1 \cdot 10^{-09}$	2.000
$1/5 \cdot 2^{-14}$	$2.0 \cdot 10^{-05}$	0.782	$1.7 \cdot 10^{-09}$	-27.093
$1/5 \cdot 2^{-15}$	$1.1 \cdot 10^{-05}$	1.217	$2.5 \cdot 10^{-01}$	31.090
$1/5 \cdot 2^{-16}$	$5.0 \cdot 10^{-06}$	0.999	$1.1 \cdot 10^{-10}$	2.000
$1/5 \cdot 2^{-17}$	$2.5 \cdot 10^{-06}$		$2.8 \cdot 10^{-11}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$7.9 \cdot 10^{-03}$	2.010	$3.4 \cdot 10^{-03}$	1.988
$1/5 \cdot 2^{-4}$	$1.9 \cdot 10^{-03}$	2.006	$8.7 \cdot 10^{-04}$	1.997
$1/5 \cdot 2^{-5}$	$4.9 \cdot 10^{-04}$	2.003	$2.1 \cdot 10^{-04}$	1.999
$1/5 \cdot 2^{-6}$	$1.2 \cdot 10^{-04}$	2.001	$5.4 \cdot 10^{-05}$	1.999
$1/5 \cdot 2^{-7}$	$3.0 \cdot 10^{-05}$	2.000	$1.3 \cdot 10^{-05}$	1.999
$1/5 \cdot 2^{-8}$	$7.6 \cdot 10^{-06}$	2.000	$3.4 \cdot 10^{-06}$	1.999
$1/5 \cdot 2^{-9}$	$1.9 \cdot 10^{-06}$	2.000	$8.5 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-10}$	$4.7 \cdot 10^{-07}$	2.000	$2.1 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-07}$	2.000	$5.3 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-12}$	$2.9 \cdot 10^{-08}$	2.000	$1.3 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-13}$	$7.4 \cdot 10^{-09}$	2.000	$3.3 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-14}$	$1.8 \cdot 10^{-09}$	-28.628	$8.3 \cdot 10^{-10}$	-27.383
$1/5 \cdot 2^{-15}$	$7.7 \cdot 10^{-01}$	32.625	$1.4 \cdot 10^{-01}$	31.380
$1/5 \cdot 2^{-16}$	$1.1 \cdot 10^{-10}$		$5.2 \cdot 10^{-11}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-3}$	$1.7 \cdot 10^{-03}$	1.981	$1.3 \cdot 10^{-05}$	4.044
$1/5 \cdot 2^{-4}$	$4.3 \cdot 10^{-04}$	1.995	$8.2 \cdot 10^{-07}$	4.009
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-04}$	1.998	$5.1 \cdot 10^{-08}$	4.000
$1/5 \cdot 2^{-6}$	$2.7 \cdot 10^{-05}$	1.999	$3.2 \cdot 10^{-09}$	3.998
$1/5 \cdot 2^{-7}$	$6.8 \cdot 10^{-06}$	1.999	$2.0 \cdot 10^{-10}$	3.999
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-06}$	1.999	$1.2 \cdot 10^{-11}$	3.999
$1/5 \cdot 2^{-9}$	$4.2 \cdot 10^{-07}$	1.999	$7.8 \cdot 10^{-13}$	3.999
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-07}$	1.999	$4.9 \cdot 10^{-14}$	3.999
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-08}$	1.999	$3.0 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-12}$	$6.7 \cdot 10^{-09}$	1.999	$1.9 \cdot 10^{-16}$	3.999
$1/5 \cdot 2^{-13}$	$1.6 \cdot 10^{-09}$	1.999	$1.1 \cdot 10^{-17}$	3.999
$1/5 \cdot 2^{-14}$	$4.1 \cdot 10^{-10}$	-26.899	$7.4 \cdot 10^{-19}$	-59.993
$1/5 \cdot 2^{-15}$	$5.2 \cdot 10^{-02}$		$8.5 \cdot 10^{-01}$	

Table 4.87: Explicit Euler, IDeC method at $t = 1/8$ for 3.1a.

h	δ	p	δ	p
$1/5$	$7.3 \cdot 10^{-01}$	0.915	$4.1 \cdot 10^{-01}$	2.006
$1/5 \cdot 2^{-1}$	$3.9 \cdot 10^{-01}$	0.940	$1.0 \cdot 10^{-01}$	2.034
$1/5 \cdot 2^{-2}$	$2.0 \cdot 10^{-01}$	0.971	$2.5 \cdot 10^{-02}$	2.019
$1/5 \cdot 2^{-3}$	$1.0 \cdot 10^{-01}$	0.985	$6.2 \cdot 10^{-03}$	2.010
$1/5 \cdot 2^{-4}$	$5.2 \cdot 10^{-02}$	0.992	$1.5 \cdot 10^{-03}$	2.005
$1/5 \cdot 2^{-5}$	$2.6 \cdot 10^{-02}$	0.996	$3.8 \cdot 10^{-04}$	2.002
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-02}$	0.998	$9.6 \cdot 10^{-05}$	2.001
$1/5 \cdot 2^{-7}$	$6.6 \cdot 10^{-03}$	0.999	$2.4 \cdot 10^{-05}$	2.000
$1/5 \cdot 2^{-8}$	$3.3 \cdot 10^{-03}$	0.999	$6.0 \cdot 10^{-06}$	2.000
$1/5 \cdot 2^{-9}$	$1.6 \cdot 10^{-03}$	0.999	$1.5 \cdot 10^{-06}$	2.000
$1/5 \cdot 2^{-10}$	$8.2 \cdot 10^{-04}$	0.999	$3.7 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-11}$	$4.1 \cdot 10^{-04}$	0.424	$9.4 \cdot 10^{-08}$	-19.191
$1/5 \cdot 2^{-12}$	$3.0 \cdot 10^{-04}$	1.575	$5.6 \cdot 10^{-02}$	23.189
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-04}$	0.999	$5.9 \cdot 10^{-09}$	2.000
$1/5 \cdot 2^{-14}$	$5.1 \cdot 10^{-05}$		$1.4 \cdot 10^{-09}$	

h	δ	p	δ	p
$1/5$	$8.2 \cdot 10^{-01}$	2.389	$1.6 \cdot 10^{-01}$	1.552
$1/5 \cdot 2^{-1}$	$1.5 \cdot 10^{-01}$	2.176	$5.5 \cdot 10^{-02}$	1.976
$1/5 \cdot 2^{-2}$	$3.4 \cdot 10^{-02}$	2.082	$1.4 \cdot 10^{-02}$	2.001
$1/5 \cdot 2^{-3}$	$8.2 \cdot 10^{-03}$	2.039	$3.5 \cdot 10^{-03}$	2.001
$1/5 \cdot 2^{-4}$	$2.0 \cdot 10^{-03}$	2.019	$8.7 \cdot 10^{-04}$	2.000
$1/5 \cdot 2^{-5}$	$4.9 \cdot 10^{-04}$	2.009	$2.1 \cdot 10^{-04}$	2.000
$1/5 \cdot 2^{-6}$	$1.2 \cdot 10^{-04}$	2.004	$5.4 \cdot 10^{-05}$	2.000
$1/5 \cdot 2^{-7}$	$3.0 \cdot 10^{-05}$	2.002	$1.3 \cdot 10^{-05}$	2.000
$1/5 \cdot 2^{-8}$	$7.6 \cdot 10^{-06}$	2.001	$3.4 \cdot 10^{-06}$	2.000
$1/5 \cdot 2^{-9}$	$1.9 \cdot 10^{-06}$	2.000	$8.5 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-10}$	$4.7 \cdot 10^{-07}$	2.000	$2.1 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-07}$	-20.437	$5.3 \cdot 10^{-08}$	-19.201
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-01}$	24.435	$3.2 \cdot 10^{-02}$	23.199
$1/5 \cdot 2^{-13}$	$7.4 \cdot 10^{-09}$		$3.3 \cdot 10^{-09}$	

h	δ	p	δ	p
$1/5$	$1.4 \cdot 10^{-01}$	3.147	$1.2 \cdot 10^{-01}$	4.830
$1/5 \cdot 2^{-1}$	$1.6 \cdot 10^{-02}$	1.375	$4.5 \cdot 10^{-03}$	4.403
$1/5 \cdot 2^{-2}$	$6.4 \cdot 10^{-03}$	1.907	$2.1 \cdot 10^{-04}$	4.091
$1/5 \cdot 2^{-3}$	$1.7 \cdot 10^{-03}$	1.980	$1.2 \cdot 10^{-05}$	3.993
$1/5 \cdot 2^{-4}$	$4.3 \cdot 10^{-04}$	1.995	$7.9 \cdot 10^{-07}$	3.980
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-04}$	1.998	$5.0 \cdot 10^{-08}$	3.985
$1/5 \cdot 2^{-6}$	$2.7 \cdot 10^{-05}$	1.999	$3.1 \cdot 10^{-09}$	3.991
$1/5 \cdot 2^{-7}$	$6.8 \cdot 10^{-06}$	1.999	$1.9 \cdot 10^{-10}$	3.995
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-06}$	1.999	$1.2 \cdot 10^{-11}$	3.997
$1/5 \cdot 2^{-9}$	$4.2 \cdot 10^{-07}$	1.999	$7.8 \cdot 10^{-13}$	3.998
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-07}$	1.999	$4.9 \cdot 10^{-14}$	3.999
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-08}$	-18.720	$3.0 \cdot 10^{-15}$	-45.803
$1/5 \cdot 2^{-12}$	$1.1 \cdot 10^{-02}$		$1.8 \cdot 10^{-01}$	

Table 4.88: Explicit Euler, IDeC method at $t = 1$ for 3.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	2.4	1.00000	0.934	$6.7 \cdot 10^{-01}$	1.00000	1.800
$1/5 \cdot 2^{-1}$	1.3	1.00000	0.968	$1.9 \cdot 10^{-01}$	1.00000	1.897
$1/5 \cdot 2^{-2}$	$6.6 \cdot 10^{-01}$	1.00000	0.984	$5.2 \cdot 10^{-02}$	1.00000	1.949
$1/5 \cdot 2^{-3}$	$3.3 \cdot 10^{-01}$	1.00000	0.992	$1.3 \cdot 10^{-02}$	1.00000	1.974
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-01}$	1.00000	0.996	$3.4 \cdot 10^{-03}$	1.00000	1.987
$1/5 \cdot 2^{-5}$	$8.4 \cdot 10^{-02}$	1.00000	0.998	$8.6 \cdot 10^{-04}$	1.00000	1.993
$1/5 \cdot 2^{-6}$	$4.2 \cdot 10^{-02}$	1.00000	0.999	$2.1 \cdot 10^{-04}$	1.00000	1.996
$1/5 \cdot 2^{-7}$	$2.1 \cdot 10^{-02}$	1.00000	0.999	$5.4 \cdot 10^{-05}$	1.00000	1.998
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-02}$	1.00000	0.999	$1.3 \cdot 10^{-05}$	1.00000	1.999
$1/5 \cdot 2^{-9}$	$5.3 \cdot 10^{-03}$	1.00000	0.999	$3.4 \cdot 10^{-06}$	1.00000	1.999
$1/5 \cdot 2^{-10}$	$2.6 \cdot 10^{-03}$	1.00000	0.999	$8.5 \cdot 10^{-07}$	1.00000	1.999
$1/5 \cdot 2^{-11}$	$1.3 \cdot 10^{-03}$	1.00000	1.000	$2.1 \cdot 10^{-07}$	1.00000	-22.865
$1/5 \cdot 2^{-12}$	$6.6 \cdot 10^{-04}$	0.99998	0.999	1.6	0.99984	26.863
$1/5 \cdot 2^{-13}$	$3.3 \cdot 10^{-04}$	1.00000	0.999	$1.3 \cdot 10^{-08}$	1.00000	1.999
$1/5 \cdot 2^{-14}$	$1.6 \cdot 10^{-04}$	1.00000		$3.3 \cdot 10^{-09}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.1 \cdot 10^{-01}$	1.00000	2.843	$3.7 \cdot 10^{-02}$	0.80000	1.927
$1/5 \cdot 2^{-1}$	$1.6 \cdot 10^{-02}$	1.00000	1.571	$9.9 \cdot 10^{-03}$	0.40000	1.727
$1/5 \cdot 2^{-2}$	$5.5 \cdot 10^{-03}$	0.25000	1.804	$2.9 \cdot 10^{-03}$	0.20000	1.838
$1/5 \cdot 2^{-3}$	$1.5 \cdot 10^{-03}$	0.12500	1.927	$8.3 \cdot 10^{-04}$	0.10000	1.917
$1/5 \cdot 2^{-4}$	$4.1 \cdot 10^{-04}$	0.06250	1.968	$2.2 \cdot 10^{-04}$	0.05000	1.958
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-04}$	0.03125	1.985	$5.6 \cdot 10^{-05}$	0.02500	1.979
$1/5 \cdot 2^{-6}$	$2.6 \cdot 10^{-05}$	0.01562	1.992	$1.4 \cdot 10^{-05}$	0.01250	1.989
$1/5 \cdot 2^{-7}$	$6.7 \cdot 10^{-06}$	0.00781	1.996	$3.6 \cdot 10^{-06}$	0.00625	1.994
$1/5 \cdot 2^{-8}$	$1.6 \cdot 10^{-06}$	0.00390	1.998	$9.1 \cdot 10^{-07}$	0.00312	1.997
$1/5 \cdot 2^{-9}$	$4.2 \cdot 10^{-07}$	0.00195	1.999	$2.2 \cdot 10^{-07}$	0.00156	1.998
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-07}$	0.00097	1.999	$5.7 \cdot 10^{-08}$	0.00078	1.999
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-08}$	0.00048	-26.877	$1.4 \cdot 10^{-08}$	0.00039	-26.957
$1/5 \cdot 2^{-12}$	3.2	0.99998	30.874	1.8	0.99993	30.953
$1/5 \cdot 2^{-13}$	$1.6 \cdot 10^{-09}$	0.00012		$8.9 \cdot 10^{-10}$	0.00009	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$2.0 \cdot 10^{-02}$	0.60000	2.465	$1.5 \cdot 10^{-02}$	1.00000	3.734
$1/5 \cdot 2^{-1}$	$3.6 \cdot 10^{-03}$	0.50000	1.590	$1.1 \cdot 10^{-03}$	0.50000	2.917
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-03}$	0.25000	1.818	$1.4 \cdot 10^{-04}$	0.25000	2.894
$1/5 \cdot 2^{-3}$	$3.4 \cdot 10^{-04}$	0.12500	1.907	$2.0 \cdot 10^{-05}$	0.12500	2.939
$1/5 \cdot 2^{-4}$	$9.1 \cdot 10^{-05}$	0.06250	1.953	$2.6 \cdot 10^{-06}$	0.06250	2.968
$1/5 \cdot 2^{-5}$	$2.3 \cdot 10^{-05}$	0.03125	1.976	$3.3 \cdot 10^{-07}$	0.03125	2.984
$1/5 \cdot 2^{-6}$	$6.0 \cdot 10^{-06}$	0.01562	1.988	$4.2 \cdot 10^{-08}$	0.01562	2.992
$1/5 \cdot 2^{-7}$	$1.5 \cdot 10^{-06}$	0.00781	1.994	$5.3 \cdot 10^{-09}$	0.00781	2.996
$1/5 \cdot 2^{-8}$	$3.7 \cdot 10^{-07}$	0.00390	1.997	$6.6 \cdot 10^{-10}$	0.00390	2.997
$1/5 \cdot 2^{-9}$	$9.5 \cdot 10^{-08}$	0.00195	1.998	$8.3 \cdot 10^{-11}$	0.00195	2.998
$1/5 \cdot 2^{-10}$	$2.3 \cdot 10^{-08}$	0.00097	1.999	$1.0 \cdot 10^{-11}$	0.00097	2.999
$1/5 \cdot 2^{-11}$	$5.9 \cdot 10^{-09}$	0.00048	-28.364	$1.3 \cdot 10^{-12}$	0.00048	-41.335
$1/5 \cdot 2^{-12}$	2.0	0.99984		3.6	0.99998	

Table 4.89: Explicit Euler, IDeC method on $[0, 1]$ for 3.1b.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	8.2	1.00000	0.854	4.0	1.00000	1.800
$1/5 \cdot 2^{-1}$	4.5	1.00000	0.931	1.1	1.00000	1.890
$1/5 \cdot 2^{-2}$	2.4	1.00000	0.966	$3.1 \cdot 10^{-01}$	1.00000	1.944
$1/5 \cdot 2^{-3}$	1.2	1.00000	0.983	$8.0 \cdot 10^{-02}$	1.00000	1.972
$1/5 \cdot 2^{-4}$	$6.2 \cdot 10^{-01}$	1.00000	0.991	$2.0 \cdot 10^{-02}$	1.00000	1.986
$1/5 \cdot 2^{-5}$	$3.1 \cdot 10^{-01}$	1.00000	0.995	$5.2 \cdot 10^{-03}$	1.00000	1.993
$1/5 \cdot 2^{-6}$	$1.5 \cdot 10^{-01}$	1.00000	0.997	$1.3 \cdot 10^{-03}$	1.00000	1.996
$1/5 \cdot 2^{-7}$	$7.8 \cdot 10^{-02}$	1.00000	0.998	$3.2 \cdot 10^{-04}$	1.00000	1.998
$1/5 \cdot 2^{-8}$	$3.9 \cdot 10^{-02}$	1.00000	0.999	$8.2 \cdot 10^{-05}$	1.00000	1.999
$1/5 \cdot 2^{-9}$	$1.9 \cdot 10^{-02}$	1.00000	0.999	$2.0 \cdot 10^{-05}$	1.00000	1.999
$1/5 \cdot 2^{-10}$	$9.8 \cdot 10^{-03}$	1.00000	0.999	$5.1 \cdot 10^{-06}$	1.00000	1.999
$1/5 \cdot 2^{-11}$	$4.9 \cdot 10^{-03}$	1.00000	1.000	$1.2 \cdot 10^{-06}$	1.00000	-21.278
$1/5 \cdot 2^{-12}$	$2.4 \cdot 10^{-03}$	0.99998	0.999	3.2	0.99984	25.276
$1/5 \cdot 2^{-13}$	$1.2 \cdot 10^{-03}$	1.00000	0.999	$8.0 \cdot 10^{-08}$	1.00000	1.999
$1/5 \cdot 2^{-14}$	$6.1 \cdot 10^{-04}$	1.00000		$2.0 \cdot 10^{-08}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	1.5	1.00000	2.637	$9.9 \cdot 10^{-01}$	1.00000	3.907
$1/5 \cdot 2^{-1}$	$2.4 \cdot 10^{-01}$	1.00000	2.828	$6.6 \cdot 10^{-02}$	1.00000	4.038
$1/5 \cdot 2^{-2}$	$3.4 \cdot 10^{-02}$	1.00000	2.920	$4.0 \cdot 10^{-03}$	1.00000	3.998
$1/5 \cdot 2^{-3}$	$4.5 \cdot 10^{-03}$	1.00000	2.962	$2.5 \cdot 10^{-04}$	1.00000	3.958
$1/5 \cdot 2^{-4}$	$5.8 \cdot 10^{-04}$	1.00000	2.981	$1.6 \cdot 10^{-05}$	1.00000	3.982
$1/5 \cdot 2^{-5}$	$7.4 \cdot 10^{-05}$	1.00000	2.991	$1.0 \cdot 10^{-06}$	1.00000	3.991
$1/5 \cdot 2^{-6}$	$9.3 \cdot 10^{-06}$	1.00000	2.995	$6.4 \cdot 10^{-08}$	1.00000	3.996
$1/5 \cdot 2^{-7}$	$1.1 \cdot 10^{-06}$	1.00000	2.997	$4.0 \cdot 10^{-09}$	1.00000	3.998
$1/5 \cdot 2^{-8}$	$1.4 \cdot 10^{-07}$	1.00000	2.998	$2.5 \cdot 10^{-10}$	1.00000	3.999
$1/5 \cdot 2^{-9}$	$1.8 \cdot 10^{-08}$	1.00000	2.999	$1.5 \cdot 10^{-11}$	1.00000	3.999
$1/5 \cdot 2^{-10}$	$2.3 \cdot 10^{-09}$	1.00000	2.999	$9.9 \cdot 10^{-13}$	1.00000	3.999
$1/5 \cdot 2^{-11}$	$2.8 \cdot 10^{-10}$	1.00000	-34.402	$6.1 \cdot 10^{-14}$	1.00000	-45.774
$1/5 \cdot 2^{-12}$	6.5	0.99998	40.398	3.7	0.99993	53.769
$1/5 \cdot 2^{-13}$	$4.4 \cdot 10^{-12}$	1.00000		$2.4 \cdot 10^{-16}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$8.4 \cdot 10^{-01}$	1.00000	5.380	$5.0 \cdot 10^{-01}$	1.00000	4.728
$1/5 \cdot 2^{-1}$	$2.0 \cdot 10^{-02}$	1.00000	4.997	$1.9 \cdot 10^{-02}$	1.00000	4.925
$1/5 \cdot 2^{-2}$	$6.3 \cdot 10^{-04}$	1.00000	4.977	$6.3 \cdot 10^{-04}$	1.00000	4.977
$1/5 \cdot 2^{-3}$	$2.0 \cdot 10^{-05}$	1.00000	4.992	$2.0 \cdot 10^{-05}$	1.00000	4.992
$1/5 \cdot 2^{-4}$	$6.3 \cdot 10^{-07}$	1.00000	4.994	$6.3 \cdot 10^{-07}$	1.00000	4.997
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-08}$	1.00000	4.989	$1.9 \cdot 10^{-08}$	1.00000	4.998
$1/5 \cdot 2^{-6}$	$6.2 \cdot 10^{-10}$	1.00000	4.994	$6.1 \cdot 10^{-10}$	1.00000	4.999
$1/5 \cdot 2^{-7}$	$1.9 \cdot 10^{-11}$	1.00000	4.996	$1.9 \cdot 10^{-11}$	1.00000	4.999
$1/5 \cdot 2^{-8}$	$6.1 \cdot 10^{-13}$	1.00000	4.998	$6.0 \cdot 10^{-13}$	1.00000	4.999
$1/5 \cdot 2^{-9}$	$1.9 \cdot 10^{-14}$	1.00000	4.999	$1.8 \cdot 10^{-14}$	1.00000	4.999
$1/5 \cdot 2^{-10}$	$5.9 \cdot 10^{-16}$	1.00000	4.999	$5.8 \cdot 10^{-16}$	1.00000	4.999
$1/5 \cdot 2^{-11}$	$1.8 \cdot 10^{-17}$	1.00000	-57.611	$1.8 \cdot 10^{-17}$	1.00000	-58.453
$1/5 \cdot 2^{-12}$	4.1	0.99984		7.2	0.99998	

Table 4.90: Explicit Euler, IDeC method on $[0, 1]$ for 3.1c.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	4.5	1.00000	0.574	2.6	1.00000	1.077
$1/5 \cdot 2^{-1}$	3.0	1.00000	0.717	1.2	1.00000	1.358
$1/5 \cdot 2^{-2}$	1.8	1.00000	0.827	$4.9 \cdot 10^{-01}$	1.00000	1.586
$1/5 \cdot 2^{-3}$	1.0	1.00000	0.901	$1.6 \cdot 10^{-01}$	1.00000	1.746
$1/5 \cdot 2^{-4}$	$5.5 \cdot 10^{-01}$	1.00000	0.946	$4.9 \cdot 10^{-02}$	1.00000	1.851
$1/5 \cdot 2^{-5}$	$2.8 \cdot 10^{-01}$	1.00000	0.971	$1.3 \cdot 10^{-02}$	1.00000	1.915
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-01}$	1.00000	0.985	$3.6 \cdot 10^{-03}$	1.00000	1.952
$1/5 \cdot 2^{-7}$	$7.4 \cdot 10^{-02}$	1.00000	0.992	$9.3 \cdot 10^{-04}$	1.00000	1.973
$1/5 \cdot 2^{-8}$	$3.7 \cdot 10^{-02}$	1.00000	0.996	$2.3 \cdot 10^{-04}$	1.00000	1.985
$1/5 \cdot 2^{-9}$	$1.8 \cdot 10^{-02}$	1.00000	0.998	$6.0 \cdot 10^{-05}$	1.00000	1.992
$1/5 \cdot 2^{-10}$	$9.3 \cdot 10^{-03}$	1.00000	0.999	$1.5 \cdot 10^{-05}$	1.00000	1.995
$1/5 \cdot 2^{-11}$	$4.6 \cdot 10^{-03}$	1.00000	0.999	$3.7 \cdot 10^{-06}$	1.00000	-18.716
$1/5 \cdot 2^{-12}$	$2.3 \cdot 10^{-03}$	0.99998	0.999	1.6	0.99984	22.710
$1/5 \cdot 2^{-13}$	$1.1 \cdot 10^{-03}$	1.00000	0.999	$2.3 \cdot 10^{-07}$	1.00000	1.999
$1/5 \cdot 2^{-14}$	$5.8 \cdot 10^{-04}$	1.00000		$5.9 \cdot 10^{-08}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	1.5	1.00000	1.873	$9.5 \cdot 10^{-01}$	1.00000	1.518
$1/5 \cdot 2^{-1}$	$4.2 \cdot 10^{-01}$	1.00000	2.546	$3.3 \cdot 10^{-01}$	1.00000	2.166
$1/5 \cdot 2^{-2}$	$7.3 \cdot 10^{-02}$	1.00000	5.720	$7.4 \cdot 10^{-02}$	1.00000	2.206
$1/5 \cdot 2^{-3}$	$1.3 \cdot 10^{-03}$	0.02500	-1.732	$1.6 \cdot 10^{-02}$	1.00000	1.952
$1/5 \cdot 2^{-4}$	$4.6 \cdot 10^{-03}$	1.00000	1.085	$4.1 \cdot 10^{-03}$	1.00000	1.789
$1/5 \cdot 2^{-5}$	$2.1 \cdot 10^{-03}$	1.00000	1.583	$1.2 \cdot 10^{-03}$	1.00000	1.790
$1/5 \cdot 2^{-6}$	$7.2 \cdot 10^{-04}$	1.00000	1.783	$3.4 \cdot 10^{-04}$	1.00000	1.847
$1/5 \cdot 2^{-7}$	$2.1 \cdot 10^{-04}$	1.00000	1.882	$9.6 \cdot 10^{-05}$	1.00000	1.901
$1/5 \cdot 2^{-8}$	$5.7 \cdot 10^{-05}$	1.00000	1.935	$2.5 \cdot 10^{-05}$	1.00000	1.939
$1/5 \cdot 2^{-9}$	$1.5 \cdot 10^{-05}$	1.00000	1.964	$6.7 \cdot 10^{-06}$	1.00000	1.964
$1/5 \cdot 2^{-10}$	$3.8 \cdot 10^{-06}$	1.00000	1.980	$1.7 \cdot 10^{-06}$	1.00000	1.979
$1/5 \cdot 2^{-11}$	$9.7 \cdot 10^{-07}$	1.00000	-21.676	$4.3 \cdot 10^{-07}$	1.00000	-22.021
$1/5 \cdot 2^{-12}$	3.2	0.99998	25.657	1.8	0.99993	26.000
$1/5 \cdot 2^{-13}$	$6.1 \cdot 10^{-08}$	1.00000		$2.7 \cdot 10^{-08}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	3.1	0.20000	3.647	$1.1 \cdot 10^{+01}$	0.20000	3.927
$1/5 \cdot 2^{-1}$	$2.5 \cdot 10^{-01}$	1.00000	1.677	$7.5 \cdot 10^{-01}$	1.00000	2.600
$1/5 \cdot 2^{-2}$	$7.9 \cdot 10^{-02}$	1.00000	2.121	$1.2 \cdot 10^{-01}$	1.00000	2.738
$1/5 \cdot 2^{-3}$	$1.8 \cdot 10^{-02}$	1.00000	2.229	$1.8 \cdot 10^{-02}$	1.00000	2.781
$1/5 \cdot 2^{-4}$	$3.9 \cdot 10^{-03}$	1.00000	2.169	$2.7 \cdot 10^{-03}$	1.00000	2.853
$1/5 \cdot 2^{-5}$	$8.6 \cdot 10^{-04}$	1.00000	2.062	$3.7 \cdot 10^{-04}$	1.00000	2.934
$1/5 \cdot 2^{-6}$	$2.0 \cdot 10^{-04}$	1.00000	1.993	$4.9 \cdot 10^{-05}$	1.00000	3.004
$1/5 \cdot 2^{-7}$	$5.2 \cdot 10^{-05}$	1.00000	1.969	$6.1 \cdot 10^{-06}$	1.00000	3.059
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-05}$	1.00000	1.968	$7.3 \cdot 10^{-07}$	1.00000	3.101
$1/5 \cdot 2^{-9}$	$3.4 \cdot 10^{-06}$	1.00000	1.976	$8.6 \cdot 10^{-08}$	1.00000	3.136
$1/5 \cdot 2^{-10}$	$8.6 \cdot 10^{-07}$	1.00000	1.983	$9.7 \cdot 10^{-09}$	1.00000	3.168
$1/5 \cdot 2^{-11}$	$2.1 \cdot 10^{-07}$	1.00000	-23.165	$1.0 \cdot 10^{-09}$	1.00000	-31.634
$1/5 \cdot 2^{-12}$	2.0	0.99984		3.6	0.99998	

Table 4.91: Explicit Euler, IDeC method on $[0, 1]$ for 3.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$2.1 \cdot 10^{-01}$	1.00000	1.213	$1.4 \cdot 10^{-01}$	0.80000	2.621
$1/5 \cdot 2^{-1}$	$9.4 \cdot 10^{-02}$	1.00000	1.154	$2.3 \cdot 10^{-02}$	1.00000	2.037
$1/5 \cdot 2^{-2}$	$4.2 \cdot 10^{-02}$	1.00000	1.081	$5.7 \cdot 10^{-03}$	1.00000	2.095
$1/5 \cdot 2^{-3}$	$1.9 \cdot 10^{-02}$	1.00000	1.041	$1.3 \cdot 10^{-03}$	1.00000	2.056
$1/5 \cdot 2^{-4}$	$9.7 \cdot 10^{-03}$	1.00000	1.020	$3.2 \cdot 10^{-04}$	1.00000	2.029
$1/5 \cdot 2^{-5}$	$4.7 \cdot 10^{-03}$	1.00000	1.010	$7.9 \cdot 10^{-05}$	1.00000	2.015
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-03}$	1.00000	1.005	$1.9 \cdot 10^{-05}$	1.00000	2.007
$1/5 \cdot 2^{-7}$	$1.1 \cdot 10^{-03}$	1.00000	1.002	$4.8 \cdot 10^{-06}$	1.00000	2.003
$1/5 \cdot 2^{-8}$	$5.9 \cdot 10^{-04}$	1.00000	1.001	$1.2 \cdot 10^{-06}$	1.00000	2.001
$1/5 \cdot 2^{-9}$	$2.9 \cdot 10^{-04}$	1.00000	1.000	$3.0 \cdot 10^{-07}$	1.00000	2.000
$1/5 \cdot 2^{-10}$	$1.4 \cdot 10^{-04}$	1.00000	1.000	$7.6 \cdot 10^{-08}$	1.00000	2.000
$1/5 \cdot 2^{-11}$	$7.3 \cdot 10^{-05}$	1.00000	1.000	$1.9 \cdot 10^{-08}$	1.00000	-22.826
$1/5 \cdot 2^{-12}$	$3.6 \cdot 10^{-05}$	0.99998	1.000	$1.4 \cdot 10^{-01}$	0.99984	26.824
$1/5 \cdot 2^{-13}$	$1.8 \cdot 10^{-05}$	1.00000	1.000	$1.1 \cdot 10^{-09}$	1.00000	2.000
$1/5 \cdot 2^{-14}$	$9.2 \cdot 10^{-06}$	1.00000		$2.9 \cdot 10^{-10}$	1.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$2.6 \cdot 10^{-01}$	1.00000	2.785	$2.2 \cdot 10^{-01}$	0.80000	4.015
$1/5 \cdot 2^{-1}$	$3.7 \cdot 10^{-02}$	0.80000	2.684	$1.3 \cdot 10^{-02}$	1.00000	2.386
$1/5 \cdot 2^{-2}$	$5.9 \cdot 10^{-03}$	0.75000	2.362	$2.6 \cdot 10^{-03}$	0.85000	2.316
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-03}$	0.72500	2.176	$5.3 \cdot 10^{-04}$	0.82500	2.175
$1/5 \cdot 2^{-4}$	$2.5 \cdot 10^{-04}$	0.75000	2.083	$1.1 \cdot 10^{-04}$	0.80000	2.008
$1/5 \cdot 2^{-5}$	$5.9 \cdot 10^{-05}$	0.76875	2.039	$2.9 \cdot 10^{-05}$	0.02500	1.995
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-05}$	0.77500	2.019	$7.3 \cdot 10^{-06}$	0.01250	1.998
$1/5 \cdot 2^{-7}$	$3.5 \cdot 10^{-06}$	0.77968	2.009	$1.8 \cdot 10^{-06}$	0.00625	1.999
$1/5 \cdot 2^{-8}$	$8.9 \cdot 10^{-07}$	0.78281	2.004	$4.5 \cdot 10^{-07}$	0.00312	1.999
$1/5 \cdot 2^{-9}$	$2.2 \cdot 10^{-07}$	0.78437	2.002	$1.1 \cdot 10^{-07}$	0.00156	1.999
$1/5 \cdot 2^{-10}$	$5.5 \cdot 10^{-08}$	0.78515	2.001	$2.8 \cdot 10^{-08}$	0.00078	1.999
$1/5 \cdot 2^{-11}$	$1.3 \cdot 10^{-08}$	0.78544	-24.282	$7.1 \cdot 10^{-09}$	0.00039	-24.428
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-01}$	0.99998	28.281	$1.6 \cdot 10^{-01}$	0.99993	28.426
$1/5 \cdot 2^{-13}$	$8.6 \cdot 10^{-10}$	0.78569		$4.4 \cdot 10^{-10}$	0.00009	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$4.4 \cdot 10^{-01}$	0.80000	6.671	$2.3 \cdot 10^{-01}$	1.00000	4.549
$1/5 \cdot 2^{-1}$	$4.3 \cdot 10^{-03}$	0.30000	2.132	$9.8 \cdot 10^{-03}$	0.60000	4.851
$1/5 \cdot 2^{-2}$	$1.0 \cdot 10^{-03}$	0.95000	1.999	$3.4 \cdot 10^{-04}$	0.10000	4.124
$1/5 \cdot 2^{-3}$	$2.5 \cdot 10^{-04}$	0.82500	2.113	$1.9 \cdot 10^{-05}$	0.80000	3.153
$1/5 \cdot 2^{-4}$	$5.7 \cdot 10^{-05}$	0.80000	2.082	$2.2 \cdot 10^{-06}$	0.80000	2.988
$1/5 \cdot 2^{-5}$	$1.3 \cdot 10^{-05}$	0.79375	2.047	$2.7 \cdot 10^{-07}$	0.81250	2.949
$1/5 \cdot 2^{-6}$	$3.3 \cdot 10^{-06}$	0.78750	2.025	$3.5 \cdot 10^{-08}$	0.81562	2.952
$1/5 \cdot 2^{-7}$	$8.1 \cdot 10^{-07}$	0.78750	2.012	$4.6 \cdot 10^{-09}$	0.81875	2.965
$1/5 \cdot 2^{-8}$	$2.0 \cdot 10^{-07}$	0.78671	2.006	$5.9 \cdot 10^{-10}$	0.81953	2.977
$1/5 \cdot 2^{-9}$	$5.0 \cdot 10^{-08}$	0.78593	2.003	$7.5 \cdot 10^{-11}$	0.81953	2.985
$1/5 \cdot 2^{-10}$	$1.2 \cdot 10^{-08}$	0.78593	2.001	$9.5 \cdot 10^{-12}$	0.81992	2.991
$1/5 \cdot 2^{-11}$	$3.1 \cdot 10^{-09}$	0.78583	-25.768	$1.1 \cdot 10^{-12}$	0.81992	-37.934
$1/5 \cdot 2^{-12}$	$1.7 \cdot 10^{-01}$	0.99984		$3.1 \cdot 10^{-01}$	0.99998	

Table 4.92: Explicit Euler, IDeC method on $[0, 1]$ for 3.3a.

As before, the asymptotic properties observed for the IDeC method are now linked to the existence of asymptotic error expansions by comparing them with the performance of extrapolation. Only the linear problems from §2.3 are considered.

It turns out that the order sequences for extrapolation are indeed the same as for IDeC. Nonetheless, the behavior of $t^{\max}$ is not the same for all problems.

Naturally, the spontaneous increase in the convergence order which was observed in the last step of the IDeC iteration for many problems, is not observed for extrapolation. Accordingly, the effect is obviously caused by some technical feature of IDeC and not by the structure of the asymptotic error expansions.

Tables 4.93–4.101 contain the results for extrapolation based on the explicit Euler method.

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.601 \cdot 10^{+01}$	0.60000	2.109	$0.179 \cdot 10^{+03}$
$1/5 \cdot 2^{-1}$	$0.139 \cdot 10^{+01}$	1.00000	0.980	$0.133 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.706 \cdot 10^{+00}$	1.00000	0.991	$0.138 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.355 \cdot 10^{+00}$	1.00000	0.996	$0.140 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.178 \cdot 10^{+00}$	1.00000	0.998	$0.141 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.892 \cdot 10^{-01}$	1.00000	0.999	$0.142 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.446 \cdot 10^{-01}$	1.00000	1.000	$0.142 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.223 \cdot 10^{-01}$	1.00000	1.000	$0.143 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.112 \cdot 10^{-01}$	1.00000	1.000	$0.143 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.558 \cdot 10^{-02}$	1.00000	1.000	$0.143 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.279 \cdot 10^{-02}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.807 \cdot 10^{+01}$	0.20000	2.255	$0.145 \cdot 10^{+04}$
$1/5 \cdot 2^{-2}$	$0.169 \cdot 10^{+01}$	0.10000	2.126	$0.987 \cdot 10^{+03}$
$1/5 \cdot 2^{-3}$	$0.387 \cdot 10^{+00}$	0.05000	2.063	$0.781 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.927 \cdot 10^{-01}$	0.02500	2.031	$0.680 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.227 \cdot 10^{-01}$	0.01250	2.016	$0.628 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.561 \cdot 10^{-02}$	0.00625	2.008	$0.601 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.139 \cdot 10^{-02}$	0.00313	2.004	$0.586 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.348 \cdot 10^{-03}$	0.00156	2.002	$0.578 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.868 \cdot 10^{-04}$	0.00078	2.001	$0.573 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.217 \cdot 10^{-04}$	0.00039		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.420 \cdot 10^{+01}$	0.20000	2.299	$0.412 \cdot 10^{+04}$
$1/5 \cdot 2^{-3}$	$0.854 \cdot 10^{+00}$	0.10000	2.153	$0.240 \cdot 10^{+04}$
$1/5 \cdot 2^{-4}$	$0.192 \cdot 10^{+00}$	0.05000	2.077	$0.172 \cdot 10^{+04}$
$1/5 \cdot 2^{-5}$	$0.455 \cdot 10^{-01}$	0.02500	2.039	$0.142 \cdot 10^{+04}$
$1/5 \cdot 2^{-6}$	$0.111 \cdot 10^{-01}$	0.01250	2.019	$0.127 \cdot 10^{+04}$
$1/5 \cdot 2^{-7}$	$0.273 \cdot 10^{-02}$	0.00625	2.010	$0.119 \cdot 10^{+04}$
$1/5 \cdot 2^{-8}$	$0.678 \cdot 10^{-03}$	0.00313	2.005	$0.115 \cdot 10^{+04}$
$1/5 \cdot 2^{-9}$	$0.169 \cdot 10^{-03}$	0.00156	2.002	$0.113 \cdot 10^{+04}$
$1/5 \cdot 2^{-10}$	$0.422 \cdot 10^{-04}$	0.00078		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.444 \cdot 10^{+00}$	0.40000	1.997	$0.703 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.111 \cdot 10^{+00}$	0.10000	1.854	$0.374 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.308 \cdot 10^{-01}$	0.05000	1.939	$0.577 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.802 \cdot 10^{-02}$	0.02500	1.972	$0.699 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.204 \cdot 10^{-02}$	0.01250	1.987	$0.768 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.516 \cdot 10^{-03}$	0.00625	1.993	$0.807 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.130 \cdot 10^{-03}$	0.00313	1.997	$0.828 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.325 \cdot 10^{-04}$	0.00156		

Table 4.93: Explicit Euler, basic sol. and extrapolation steps $1 - 3$ for 1.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.908 \cdot 10^{+00}$	0.20000	1.870	$0.184 \cdot 10^{+02}$
$1/5 \cdot 2^{-1}$	$0.248 \cdot 10^{+00}$	0.10000	1.935	$0.214 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.650 \cdot 10^{-01}$	0.05000	1.968	$0.236 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.166 \cdot 10^{-01}$	0.02500	1.984	$0.250 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.420 \cdot 10^{-02}$	0.01250	1.033	$0.388 \cdot 10^{+00}$
$1/5 \cdot 2^{-5}$	$0.205 \cdot 10^{-02}$	0.28125	1.003	$0.334 \cdot 10^{+00}$
$1/5 \cdot 2^{-6}$	$0.102 \cdot 10^{-02}$	0.28438	1.001	$0.330 \cdot 10^{+00}$
$1/5 \cdot 2^{-7}$	$0.512 \cdot 10^{-03}$	0.28438	1.001	$0.329 \cdot 10^{+00}$
$1/5 \cdot 2^{-8}$	$0.256 \cdot 10^{-03}$	0.28438	1.000	$0.328 \cdot 10^{+00}$
$1/5 \cdot 2^{-9}$	$0.128 \cdot 10^{-03}$	0.28477	1.000	$0.328 \cdot 10^{+00}$
$1/5 \cdot 2^{-10}$	$0.639 \cdot 10^{-04}$	0.28477		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.182 \cdot 10^{+01}$	0.20000	1.872	$0.135 \cdot 10^{+03}$
$1/5 \cdot 2^{-2}$	$0.497 \cdot 10^{+00}$	0.10000	1.935	$0.164 \cdot 10^{+03}$
$1/5 \cdot 2^{-3}$	$0.130 \cdot 10^{+00}$	0.05000	1.968	$0.185 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.332 \cdot 10^{-01}$	0.02500	1.984	$0.198 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.840 \cdot 10^{-02}$	0.01250	1.992	$0.206 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.211 \cdot 10^{-02}$	0.00625	1.996	$0.211 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.529 \cdot 10^{-03}$	0.00313	1.998	$0.214 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.133 \cdot 10^{-03}$	0.00156	1.999	$0.216 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.332 \cdot 10^{-04}$	0.00078	1.999	$0.216 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.829 \cdot 10^{-05}$	0.00039		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.124 \cdot 10^{+01}$	0.20000	1.885	$0.351 \cdot 10^{+03}$
$1/5 \cdot 2^{-3}$	$0.335 \cdot 10^{+00}$	0.10000	1.943	$0.435 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.872 \cdot 10^{-01}$	0.05000	1.972	$0.493 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.222 \cdot 10^{-01}$	0.02500	1.986	$0.530 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.561 \cdot 10^{-02}$	0.01250	1.993	$0.552 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.141 \cdot 10^{-02}$	0.00625	1.997	$0.564 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.353 \cdot 10^{-03}$	0.00313	1.998	$0.571 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.884 \cdot 10^{-04}$	0.00156	1.999	$0.575 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.221 \cdot 10^{-04}$	0.00078		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.181 \cdot 10^{+00}$	0.20000	0.910	$0.518 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.961 \cdot 10^{-01}$	0.10000	1.945	$0.484 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.250 \cdot 10^{-01}$	0.05000	1.973	$0.558 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.635 \cdot 10^{-02}$	0.02500	1.987	$0.603 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.160 \cdot 10^{-02}$	0.01250	1.993	$0.629 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.403 \cdot 10^{-03}$	0.00625	1.997	$0.644 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.101 \cdot 10^{-03}$	0.00313	1.998	$0.653 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.253 \cdot 10^{-04}$	0.00156		

Table 4.94: Explicit Euler, basic sol. and extrapolation steps $1 - 3$ for 1.2a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.636 \cdot 10^{+00}$	1.00000	1.865	$0.128 \cdot 10^{+02}$
$1/5 \cdot 2^{-1}$	$0.175 \cdot 10^{+00}$	0.10000	0.940	$0.152 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.910 \cdot 10^{-01}$	0.55000	0.598	$0.546 \cdot 10^{+00}$
$1/5 \cdot 2^{-3}$	$0.602 \cdot 10^{-01}$	0.52500	0.832	$0.130 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.338 \cdot 10^{-01}$	0.52500	0.923	$0.193 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.178 \cdot 10^{-01}$	0.52500	0.963	$0.236 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.914 \cdot 10^{-02}$	0.52500	0.982	$0.263 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.463 \cdot 10^{-02}$	0.52344	0.991	$0.279 \cdot 10^{+01}$
$1/5 \cdot 2^{-8}$	$0.233 \cdot 10^{-02}$	0.52422	0.996	$0.289 \cdot 10^{+01}$
$1/5 \cdot 2^{-9}$	$0.117 \cdot 10^{-02}$	0.52383	0.998	$0.294 \cdot 10^{+01}$
$1/5 \cdot 2^{-10}$	$0.585 \cdot 10^{-03}$	0.52363		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.787 \cdot 10^{+00}$	0.20000	1.853	$0.561 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.218 \cdot 10^{+00}$	0.10000	1.965	$0.784 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.558 \cdot 10^{-01}$	0.05000	1.991	$0.864 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.140 \cdot 10^{-01}$	0.02500	1.998	$0.890 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.351 \cdot 10^{-02}$	0.01250	1.999	$0.897 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.879 \cdot 10^{-03}$	0.00625	2.000	$0.899 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.220 \cdot 10^{-03}$	0.00313	2.000	$0.900 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.549 \cdot 10^{-04}$	0.00156	2.000	$0.900 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.137 \cdot 10^{-04}$	0.00078	2.000	$0.900 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.343 \cdot 10^{-05}$	0.00039		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.286 \cdot 10^{+00}$	0.20000	1.948	$0.981 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.741 \cdot 10^{-01}$	0.10000	1.987	$0.113 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.187 \cdot 10^{-01}$	0.05000	1.997	$0.118 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.468 \cdot 10^{-02}$	0.02500	1.999	$0.119 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.117 \cdot 10^{-02}$	0.01250	2.000	$0.120 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.293 \cdot 10^{-03}$	0.00625	2.000	$0.120 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.732 \cdot 10^{-04}$	0.00313	2.000	$0.120 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.183 \cdot 10^{-04}$	0.00156	2.000	$0.120 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.458 \cdot 10^{-05}$	0.00078		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.393 \cdot 10^{-01}$	0.20000	1.904	$0.441 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.105 \cdot 10^{-01}$	0.10000	1.977	$0.606 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.266 \cdot 10^{-02}$	0.05000	1.994	$0.663 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.669 \cdot 10^{-03}$	0.02500	1.999	$0.679 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.167 \cdot 10^{-03}$	0.01250	2.000	$0.684 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.418 \cdot 10^{-04}$	0.00625	2.000	$0.685 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.105 \cdot 10^{-04}$	0.00313	2.000	$0.686 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.262 \cdot 10^{-05}$	0.00156		

Table 4.95: Explicit Euler, basic sol. and extrapolation steps $1 - 3$ for 2.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.122 \cdot 10^{+01}$	1.00000	0.902	$0.519 \cdot 10^{+01}$
$1/5 \cdot 2^{-1}$	$0.651 \cdot 10^{+00}$	1.00000	0.949	$0.579 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.337 \cdot 10^{+00}$	1.00000	0.973	$0.623 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.172 \cdot 10^{+00}$	1.00000	0.986	$0.653 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.867 \cdot 10^{-01}$	1.00000	0.993	$0.672 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.435 \cdot 10^{-01}$	1.00000	0.996	$0.684 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.218 \cdot 10^{-01}$	1.00000	0.998	$0.691 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.109 \cdot 10^{-01}$	1.00000	0.999	$0.695 \cdot 10^{+01}$
$1/5 \cdot 2^{-8}$	$0.547 \cdot 10^{-02}$	1.00000	1.000	$0.697 \cdot 10^{+01}$
$1/5 \cdot 2^{-9}$	$0.273 \cdot 10^{-02}$	1.00000	1.000	$0.699 \cdot 10^{+01}$
$1/5 \cdot 2^{-10}$	$0.137 \cdot 10^{-02}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.151 \cdot 10^{+00}$	0.20000	1.983	$0.145 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.382 \cdot 10^{-01}$	0.10000	1.996	$0.151 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.959 \cdot 10^{-02}$	0.05000	1.999	$0.153 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.240 \cdot 10^{-02}$	0.02500	2.000	$0.153 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.600 \cdot 10^{-03}$	0.01250	2.000	$0.153 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.150 \cdot 10^{-03}$	0.00625	2.000	$0.154 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.375 \cdot 10^{-04}$	0.00313	2.000	$0.154 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.937 \cdot 10^{-05}$	0.00156	2.000	$0.154 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.234 \cdot 10^{-05}$	0.00078	2.000	$0.154 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.586 \cdot 10^{-06}$	0.00039		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.530 \cdot 10^{-01}$	0.20000	2.038	$0.238 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.129 \cdot 10^{-01}$	0.10000	2.010	$0.214 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.321 \cdot 10^{-02}$	0.05000	2.002	$0.207 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.800 \cdot 10^{-03}$	0.02500	2.001	$0.205 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.200 \cdot 10^{-03}$	0.01250	2.000	$0.205 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.500 \cdot 10^{-04}$	0.00625	2.000	$0.205 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.125 \cdot 10^{-04}$	0.00313	2.000	$0.205 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.312 \cdot 10^{-05}$	0.00156	2.000	$0.205 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.781 \cdot 10^{-06}$	0.00078		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.812 \cdot 10^{-02}$	0.20000	2.112	$0.196 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.188 \cdot 10^{-02}$	0.10000	2.030	$0.137 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.460 \cdot 10^{-03}$	0.05000	2.008	$0.122 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.114 \cdot 10^{-03}$	0.02500	2.002	$0.118 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.286 \cdot 10^{-04}$	0.01250	2.000	$0.117 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.714 \cdot 10^{-05}$	0.00625	2.000	$0.117 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.178 \cdot 10^{-05}$	0.00313	2.000	$0.117 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.446 \cdot 10^{-06}$	0.00156		

Table 4.96: Explicit Euler, basic sol. and extrapolation steps $1-3$ for 2.2a

5   The Numerical Solution of Singular Terminal Value Problems at $t = 1$

In this section, the numerical solution of singular terminal value problems of the form

$$z'(t) = \frac{M(t)}{t} z(t) + f(t, z(t)), \quad t \in (0, 1], \tag{5.1a}$$

$$z(1) = \beta, \tag{5.1b}$$

is discussed. In contrast to §4, $\beta \in \mathbb{R}^n$ can be chosen arbitrarily.

Again, the performance of IDeC is investigated when the one-step methods from §3.1 are used as basic methods. It turns out that in general no satisfactory improvement in the convergence orders is observed. Even for methods where the structure of the numerical approximations permits an error expansion of arbitrary length in principle and the exact solution z is sufficiently smooth, the inherent unsmoothness of solutions of (5.1) which depends on the eigenvalues of $M(0)$ (cf. §2.2), causes the error expansion to break down, because the variational equations do not have smooth solutions. We will make this point clearer in the sequel. Consequently, IDeC does not improve the level of accuracy beyond a certain point.

5.1   The Explicit Euler Method

Due to an obvious similarity in the structure, the investigations of the explicit Euler method for (5.1) are very similar to the analysis of the implicit Euler method for (4.1) which is given in [14]. Thus we will only give a sketch of the proof that the classical convergence order 1 holds for sufficiently smooth problems. However, the level of accuracy that can be obtained in the course of the IDeC iteration is limited by the length of the asymptotic expansion of the global error. This is demonstrated experimentally in §5.1.2 and for some examples verified theoretically in §5.1.3.

5.1.1   Convergence of the Basic Solution

We use induction to compute the solution z_h of the difference scheme for the solution of a linear problem (5.1) with constant coefficient matrix $M(t) \equiv M$,

$$\frac{z_j - z_{j-1}}{-h} = \frac{1}{t_{N-j+1}} M z_{j-1} + f(t_{N-j+1}), \quad j = 1, \ldots, N, \tag{5.2a}$$

$$z_0 = \beta. \tag{5.2b}$$

Consequently[13],

$$z_j = \prod_{l=0}^{j-1} \left(I_n - \frac{M}{N-l} \right) \beta - \sum_{l=0}^{j-2} \prod_{k=l}^{j-1} \left(I_n - \frac{M}{N-k} \right) h f_l - h f_{j-1}. \tag{5.3}$$

---

[13]We use the notation $f_j := f(t_{N-j})$.

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.135 \cdot 10^{+01}$	1.00000	1.255	$0.102 \cdot 10^{+02}$
$1/5 \cdot 2^{-1}$	$0.566 \cdot 10^{+00}$	1.00000	1.187	$0.870 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.249 \cdot 10^{+00}$	1.00000	1.109	$0.689 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.115 \cdot 10^{+00}$	1.00000	1.058	$0.572 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.553 \cdot 10^{-01}$	1.00000	0.948	$0.353 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.287 \cdot 10^{-01}$	0.61250	0.921	$0.308 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.151 \cdot 10^{-01}$	0.61250	0.956	$0.377 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.781 \cdot 10^{-02}$	0.61094	0.976	$0.428 \cdot 10^{+01}$
$1/5 \cdot 2^{-8}$	$0.397 \cdot 10^{-02}$	0.61172	0.987	$0.462 \cdot 10^{+01}$
$1/5 \cdot 2^{-9}$	$0.200 \cdot 10^{-02}$	0.61211	0.993	$0.484 \cdot 10^{+01}$
$1/5 \cdot 2^{-10}$	$0.101 \cdot 10^{-02}$	0.61211		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.292 \cdot 10^{+00}$	1.00000	0.832	$0.199 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.164 \cdot 10^{+00}$	1.00000	1.465	$0.132 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.595 \cdot 10^{-01}$	1.00000	1.644	$0.256 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.190 \cdot 10^{-01}$	1.00000	1.725	$0.365 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.575 \cdot 10^{-02}$	1.00000	1.772	$0.464 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.168 \cdot 10^{-02}$	1.00000	1.804	$0.558 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.482 \cdot 10^{-03}$	1.00000	1.828	$0.651 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.136 \cdot 10^{-03}$	1.00000	1.847	$0.742 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.378 \cdot 10^{-04}$	1.00000	1.861	$0.834 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.104 \cdot 10^{-04}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.129 \cdot 10^{+00}$	0.20000	1.908	$0.391 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.343 \cdot 10^{-01}$	0.10000	1.978	$0.505 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.871 \cdot 10^{-02}$	0.05000	1.994	$0.544 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.218 \cdot 10^{-02}$	0.02500	1.999	$0.555 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.547 \cdot 10^{-03}$	0.01250	2.000	$0.559 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.137 \cdot 10^{-03}$	0.00625	2.000	$0.560 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.342 \cdot 10^{-04}$	0.00313	2.000	$0.560 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.854 \cdot 10^{-05}$	0.00156	2.000	$0.560 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.214 \cdot 10^{-05}$	0.00078		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.132 \cdot 10^{-01}$	0.20000	2.167	$0.392 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.294 \cdot 10^{-02}$	0.20000	1.999	$0.187 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.735 \cdot 10^{-03}$	0.20000	1.992	$0.180 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.185 \cdot 10^{-03}$	0.22500	1.996	$0.185 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.464 \cdot 10^{-04}$	0.22500	1.999	$0.188 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.116 \cdot 10^{-04}$	0.23125	2.000	$0.189 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.290 \cdot 10^{-05}$	0.23438	2.000	$0.190 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.725 \cdot 10^{-06}$	0.23594		

Table 4.97: Explicit Euler, basic sol. and extrapolation steps $1 - 3$ for 3.1a

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.111 \cdot 10^{+01}$	1.00000	0.510	$0.252 \cdot 10^{+01}$
$1/5 \cdot 2^{-1}$	$0.778 \cdot 10^{+00}$	1.00000	0.793	$0.483 \cdot 10^{+01}$
$1/5 \cdot 2^{-2}$	$0.449 \cdot 10^{+00}$	1.00000	0.900	$0.665 \cdot 10^{+01}$
$1/5 \cdot 2^{-3}$	$0.240 \cdot 10^{+00}$	1.00000	0.949	$0.796 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.125 \cdot 10^{+00}$	1.00000	0.973	$0.885 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.635 \cdot 10^{-01}$	1.00000	0.986	$0.944 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.320 \cdot 10^{-01}$	1.00000	0.992	$0.981 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.161 \cdot 10^{-01}$	1.00000	0.996	$0.100 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.808 \cdot 10^{-02}$	1.00000	0.998	$0.102 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.404 \cdot 10^{-02}$	1.00000	0.999	$0.103 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.202 \cdot 10^{-02}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.448 \cdot 10^{+00}$	1.00000	1.903	$0.358 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.120 \cdot 10^{+00}$	1.00000	1.897	$0.352 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.322 \cdot 10^{-01}$	1.00000	1.888	$0.340 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.869 \cdot 10^{-02}$	1.00000	1.884	$0.335 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.235 \cdot 10^{-02}$	1.00000	1.885	$0.337 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.637 \cdot 10^{-03}$	1.00000	1.889	$0.344 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.172 \cdot 10^{-03}$	1.00000	1.894	$0.356 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.463 \cdot 10^{-04}$	1.00000	1.900	$0.371 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.124 \cdot 10^{-04}$	1.00000	1.906	$0.388 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.331 \cdot 10^{-05}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.469 \cdot 10^{-01}$	0.20000	2.283	$0.439 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.964 \cdot 10^{-02}$	0.10000	2.151	$0.269 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.217 \cdot 10^{-02}$	0.05000	2.078	$0.196 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.514 \cdot 10^{-03}$	0.02500	2.040	$0.161 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.125 \cdot 10^{-03}$	0.01250	2.020	$0.144 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.308 \cdot 10^{-04}$	0.00625	2.010	$0.135 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.765 \cdot 10^{-05}$	0.00313	2.005	$0.130 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.191 \cdot 10^{-05}$	0.00156	2.003	$0.127 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.476 \cdot 10^{-06}$	0.00078		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.190 \cdot 10^{-02}$	1.00000	1.743	$0.117 \cdot 10^{+01}$
$1/5 \cdot 2^{-4}$	$0.566 \cdot 10^{-03}$	1.00000	1.891	$0.225 \cdot 10^{+01}$
$1/5 \cdot 2^{-5}$	$0.153 \cdot 10^{-03}$	1.00000	1.946	$0.297 \cdot 10^{+01}$
$1/5 \cdot 2^{-6}$	$0.397 \cdot 10^{-04}$	1.00000	1.972	$0.345 \cdot 10^{+01}$
$1/5 \cdot 2^{-7}$	$0.101 \cdot 10^{-04}$	1.00000	1.986	$0.377 \cdot 10^{+01}$
$1/5 \cdot 2^{-8}$	$0.255 \cdot 10^{-05}$	1.00000	1.993	$0.397 \cdot 10^{+01}$
$1/5 \cdot 2^{-9}$	$0.641 \cdot 10^{-06}$	1.00000	1.996	$0.408 \cdot 10^{+01}$
$1/5 \cdot 2^{-10}$	$0.161 \cdot 10^{-06}$	1.00000		

Table 4.98: Explicit Euler, basic sol. and extrapolation steps $1 - 3$ for 3.1b

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-0}$	$0.827 \cdot 10^{+01}$	1.00000	0.853	$0.326 \cdot 10^{+02}$
$1/5 \cdot 2^{-1}$	$0.458 \cdot 10^{+01}$	1.00000	0.931	$0.391 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.240 \cdot 10^{+01}$	1.00000	0.967	$0.435 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.123 \cdot 10^{+01}$	1.00000	0.984	$0.463 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.622 \cdot 10^{+00}$	1.00000	0.992	$0.480 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.313 \cdot 10^{+00}$	1.00000	0.996	$0.490 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.157 \cdot 10^{+00}$	1.00000	0.998	$0.496 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.785 \cdot 10^{-01}$	1.00000	0.999	$0.499 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.393 \cdot 10^{-01}$	1.00000	0.999	$0.501 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.196 \cdot 10^{-01}$	1.00000	1.000	$0.502 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.982 \cdot 10^{-02}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-1}$	$0.641 \cdot 10^{+00}$	1.00000	1.801	$0.405 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.184 \cdot 10^{+00}$	1.00000	1.894	$0.535 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.495 \cdot 10^{-01}$	1.00000	1.947	$0.651 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.128 \cdot 10^{-01}$	1.00000	1.974	$0.732 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.327 \cdot 10^{-02}$	1.00000	1.987	$0.783 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.825 \cdot 10^{-03}$	1.00000	1.994	$0.813 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.207 \cdot 10^{-03}$	1.00000	1.997	$0.831 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.519 \cdot 10^{-04}$	1.00000	1.998	$0.840 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.130 \cdot 10^{-04}$	1.00000	1.924	$0.469 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.342 \cdot 10^{-05}$	1.00000		

h	δ	t^{max}	p	c
$1/5 \cdot 2^{-2}$	$0.316 \cdot 10^{-01}$	1.00000	2.753	$0.121 \cdot 10^{+03}$
$1/5 \cdot 2^{-3}$	$0.469 \cdot 10^{-02}$	1.00000	2.926	$0.229 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.618 \cdot 10^{-03}$	1.00000	2.975	$0.283 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.785 \cdot 10^{-04}$	1.00000	2.991	$0.307 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.988 \cdot 10^{-05}$	1.00000	2.996	$0.317 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.124 \cdot 10^{-05}$	1.00000	2.998	$0.321 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.155 \cdot 10^{-06}$	1.00000	2.999	$0.323 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.194 \cdot 10^{-07}$	1.00000	3.000	$0.324 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.242 \cdot 10^{-08}$	1.00000		

Table 4.99: Explicit Euler, basic sol. and extrapolation steps 1,2 for 3.1c

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.845 \cdot 10^{-03}$	1.00000	4.590	$0.191 \cdot 10^{+05}$
$1/5 \cdot 2^{-4}$	$0.351 \cdot 10^{-04}$	1.00000	4.510	$0.134 \cdot 10^{+05}$
$1/5 \cdot 2^{-5}$	$0.154 \cdot 10^{-05}$	1.00000	4.409	$0.805 \cdot 10^{+04}$
$1/5 \cdot 2^{-6}$	$0.724 \cdot 10^{-07}$	1.00000	4.305	$0.441 \cdot 10^{+04}$
$1/5 \cdot 2^{-7}$	$0.366 \cdot 10^{-08}$	1.00000	4.211	$0.240 \cdot 10^{+04}$
$1/5 \cdot 2^{-8}$	$0.198 \cdot 10^{-09}$	1.00000	4.136	$0.140 \cdot 10^{+04}$
$1/5 \cdot 2^{-9}$	$0.113 \cdot 10^{-10}$	1.00000	4.083	$0.925 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.665 \cdot 10^{-12}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-4}$	$0.189 \cdot 10^{-04}$	1.00000	4.764	$0.220 \cdot 10^{+05}$
$1/5 \cdot 2^{-5}$	$0.696 \cdot 10^{-06}$	1.00000	4.780	$0.239 \cdot 10^{+05}$
$1/5 \cdot 2^{-6}$	$0.253 \cdot 10^{-07}$	1.00000	4.784	$0.244 \cdot 10^{+05}$
$1/5 \cdot 2^{-7}$	$0.920 \cdot 10^{-09}$	1.00000	4.794	$0.260 \cdot 10^{+05}$
$1/5 \cdot 2^{-8}$	$0.332 \cdot 10^{-10}$	1.00000	4.809	$0.290 \cdot 10^{+05}$
$1/5 \cdot 2^{-9}$	$0.118 \cdot 10^{-11}$	1.00000	4.825	$0.329 \cdot 10^{+05}$
$1/5 \cdot 2^{-10}$	$0.418 \cdot 10^{-13}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-5}$	$0.465 \cdot 10^{-06}$	0.20000	5.233	$0.159 \cdot 10^{+06}$
$1/5 \cdot 2^{-6}$	$0.124 \cdot 10^{-07}$	0.10000	5.123	$0.840 \cdot 10^{+05}$
$1/5 \cdot 2^{-7}$	$0.355 \cdot 10^{-09}$	0.05000	5.063	$0.572 \cdot 10^{+05}$
$1/5 \cdot 2^{-8}$	$0.106 \cdot 10^{-10}$	0.02500	5.032	$0.458 \cdot 10^{+05}$
$1/5 \cdot 2^{-9}$	$0.324 \cdot 10^{-12}$	0.01250	5.016	$0.404 \cdot 10^{+05}$
$1/5 \cdot 2^{-10}$	$0.100 \cdot 10^{-13}$	0.00625		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-6}$	$0.204 \cdot 10^{-08}$	1.00000	4.749	$0.160 \cdot 10^{+04}$
$1/5 \cdot 2^{-7}$	$0.757 \cdot 10^{-10}$	1.00000	4.901	$0.428 \cdot 10^{+04}$
$1/5 \cdot 2^{-8}$	$0.253 \cdot 10^{-11}$	1.00000	4.953	$0.620 \cdot 10^{+04}$
$1/5 \cdot 2^{-9}$	$0.818 \cdot 10^{-13}$	1.00000	4.976	$0.746 \cdot 10^{+04}$
$1/5 \cdot 2^{-10}$	$0.260 \cdot 10^{-14}$	1.00000		

Table 4.100: Explicit Euler, extrapolation steps 3 – 6 for 3.1c

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-0}$	$0.334 \cdot 10^{+01}$	1.00000	0.480	$0.723 \cdot 10^{+01}$
$1/5 \cdot 2^{-1}$	$0.239 \cdot 10^{+01}$	1.00000	0.635	$0.103 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.154 \cdot 10^{+01}$	1.00000	0.754	$0.147 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.913 \cdot 10^{+00}$	1.00000	0.841	$0.203 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.510 \cdot 10^{+00}$	1.00000	0.901	$0.264 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.273 \cdot 10^{+00}$	1.00000	0.941	$0.323 \cdot 10^{+02}$
$1/5 \cdot 2^{-6}$	$0.142 \cdot 10^{+00}$	1.00000	0.965	$0.373 \cdot 10^{+02}$
$1/5 \cdot 2^{-7}$	$0.729 \cdot 10^{-01}$	1.00000	0.980	$0.410 \cdot 10^{+02}$
$1/5 \cdot 2^{-8}$	$0.369 \cdot 10^{-01}$	1.00000	0.989	$0.437 \cdot 10^{+02}$
$1/5 \cdot 2^{-9}$	$0.186 \cdot 10^{-01}$	1.00000	0.994	$0.454 \cdot 10^{+02}$
$1/5 \cdot 2^{-10}$	$0.935 \cdot 10^{-02}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-1}$	$0.145 \cdot 10^{+01}$	1.00000	1.073	$0.171 \cdot 10^{+02}$
$1/5 \cdot 2^{-2}$	$0.688 \cdot 10^{+00}$	1.00000	1.262	$0.301 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.287 \cdot 10^{+00}$	1.00000	1.427	$0.553 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.107 \cdot 10^{+00}$	1.00000	1.558	$0.986 \cdot 10^{+02}$
$1/5 \cdot 2^{-5}$	$0.362 \cdot 10^{-01}$	1.00000	1.658	$0.163 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.115 \cdot 10^{-01}$	1.00000	1.729	$0.246 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.346 \cdot 10^{-02}$	1.00000	1.779	$0.341 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.101 \cdot 10^{-02}$	1.00000	1.815	$0.439 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.287 \cdot 10^{-03}$	1.00000	1.840	$0.537 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.801 \cdot 10^{-04}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-2}$	$0.435 \cdot 10^{+00}$	1.00000	1.505	$0.395 \cdot 10^{+02}$
$1/5 \cdot 2^{-3}$	$0.153 \cdot 10^{+00}$	1.00000	1.715	$0.855 \cdot 10^{+02}$
$1/5 \cdot 2^{-4}$	$0.466 \cdot 10^{-01}$	1.00000	1.873	$0.171 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.127 \cdot 10^{-01}$	1.00000	1.977	$0.290 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.323 \cdot 10^{-02}$	1.00000	2.031	$0.397 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.791 \cdot 10^{-03}$	1.00000	2.051	$0.451 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.191 \cdot 10^{-03}$	1.00000	2.051	$0.450 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.461 \cdot 10^{-04}$	1.00000	2.042	$0.419 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.112 \cdot 10^{-04}$	1.00000		

h	δ	$t^{\max}$	p	c
$1/5 \cdot 2^{-3}$	$0.113 \cdot 10^{+00}$	1.00000	1.844	$0.102 \cdot 10^{+03}$
$1/5 \cdot 2^{-4}$	$0.314 \cdot 10^{-01}$	1.00000	1.995	$0.197 \cdot 10^{+03}$
$1/5 \cdot 2^{-5}$	$0.788 \cdot 10^{-02}$	1.00000	2.070	$0.288 \cdot 10^{+03}$
$1/5 \cdot 2^{-6}$	$0.188 \cdot 10^{-02}$	1.00000	2.086	$0.316 \cdot 10^{+03}$
$1/5 \cdot 2^{-7}$	$0.442 \cdot 10^{-03}$	1.00000	2.072	$0.289 \cdot 10^{+03}$
$1/5 \cdot 2^{-8}$	$0.105 \cdot 10^{-03}$	1.00000	2.051	$0.247 \cdot 10^{+03}$
$1/5 \cdot 2^{-9}$	$0.254 \cdot 10^{-04}$	1.00000	2.032	$0.214 \cdot 10^{+03}$
$1/5 \cdot 2^{-10}$	$0.621 \cdot 10^{-05}$	1.00000		

Table 4.101: Explicit Euler, basic sol. and extrapolation steps $1 - 3$ for 3.2a

If we consider the scalar case with eigenvalue with positive real part, that is $M = \lambda = \sigma + i\kappa \in \mathbb{R} \times i\mathbb{R}$, we assume that $\sigma > 1$, because this condition is sufficient for $z \in C^1[0,1]$. Following the line of argument in [14] we can derive a bound for the solution

$$
\begin{aligned}
|z_j| &\leq c_1 \left(\frac{t_{N-j+1}}{t_{N+1}}\right)^\sigma |\beta| + c_2 \sum_{l=0}^{j-2} \left(\frac{t_{N-j+1}}{t_{N-l+1}}\right)^\sigma h|f_l| + h|f_{j-1}| \\
&\leq c_3 t_{N-j+1}^\sigma |\beta| + c_4 t_{N-j+1}^\sigma \sum_{l=0}^{j-1} t_{N-l+1}^{-\sigma} h\|f_h\|_h \\
&\leq c_3 t_{N-j+1}^\sigma |\beta| + c_5 t_{N-j+1}^\sigma |t_{N+2}^{-\sigma+1} - t_{N-j+1}^{-\sigma+1}| \|f_h\|_h \\
&\leq c_3 t_{N-j+1} |\beta| + c_6 |1 - t_{N-j+1}| \|f_h\|_h \\
&\leq \text{const}(|\beta| + \|f_h\|_h).
\end{aligned}
$$

Using this estimate, we can prove a result similar to Lemma 5.2.2 in [14].

If we consider the method on an interval $[0, \delta]$, we obtain the estimate

$$
|z_j| \leq \text{const}(|\beta| + \delta \|f_h\|_h).
$$

This bound is going to be used in the proof for the case of variable coefficient matrix $M(t)$.

To prove convergence for the general vector case, we need an estimate for (scalar) equations of the form

$$
\begin{aligned}
\frac{z_j - z_{j-1}}{-h} &= \frac{\lambda}{t_{N-j+1}} z_{j-1} + \frac{1}{t_{N-j+1}} w_{j-1} + f(t_{N-j+1}), \\
z_0 &= \beta,
\end{aligned}
$$

where

$$
|w_j| \leq \text{const}(|\gamma| + \delta \|g_h\|_h)
$$

on the interval $[0, \delta]$. This bound is readily obtained,

$$
\begin{aligned}
|z_j| &\leq \text{const} \left(\frac{t_{N-j+1}}{\delta}\right)^\sigma |\beta| + \text{const}\, t_{N-j+1}^\sigma \sum_{l=0}^{j-1} t_{N-l+1}^{-\sigma} h\|f_h\|_h \\
&\quad + \text{const}\, t_{N-j+1}^\sigma \sum_{l=0}^{j-1} t_{N-l+1}^{-\sigma-1} h(|\gamma| + \delta\|g_h\|_h) \\
&\quad + h|f_{j-1}| + \text{const}\, h t_{N-j+1}^{-1}(|\gamma| + \delta\|g_h\|_h) \\
&\leq \text{const}\, (\max\{|\beta|, |\gamma|\} + \delta \max\{\|f_h\|_h, \|g_h\|_h\}).
\end{aligned}
$$

Note that for regular problems we can obtain a similar result,

$$
|z_j| \leq |\beta| + \text{const}\, |t_{N+2} - t_{N-j+1}| \|f_h\|_h \leq |\beta| + \text{const}\, \delta \|f_h\|_h.
$$

Since no nontrivial Jordan blocks may appear for eigenvalue $\lambda = 0$ in the Jordan canonical form of M, the three estimates given above are sufficient to

prove convergence of the explicit Euler method applied to (5.1), following the arguments in [14].

For variable coefficient matrix $M(t) = M + tC(t)$, we first compute the explicit Euler solution of the regular problem on the interval $[\delta, 1]$ starting at $z_0 = z(1) = \beta$. From classical theory we conclude that

$$z_{\tilde{N}} = z(\delta) + c \cdot h. \tag{5.4}$$

To show that the solution of the scheme can be extended to $[0, \delta]$, we use a contraction argument on the Banach space of the grid vectors

$$\{(y_{\tilde{N}}, \ldots, y_N) : y_{\tilde{N}} = z(\delta) + c \cdot h\}$$

on $[0, \delta]$.

As in [14] we define a mapping $y_h = G_h(x_h)$ if y_h is the solution of

$$\frac{y_j - y_{j-1}}{-h} = \frac{1}{t_{N-j+1}} M y_{j-1} + C(t_{N-j+1}) x_{j-1} + f(t_{N-j+1}),$$
$$y_{\tilde{N}} = z(\delta) + c \cdot h.$$

It follows from the estimates derived earlier that the solution $v_h = G_h(x_h) - G_h(y_h)$ of

$$\frac{v_j - v_{j-1}}{-h} = \frac{1}{t_{N-j+1}} M v_{j-1} + C_{j-1}(x_{j-1} - y_{j-1}),$$
$$v_{\tilde{N}} = 0$$

can be bounded by

$$\|v_h\|_h \leq \text{const}\, \delta \|\tilde{x}_h - \tilde{y}_h\|_h,$$

where $\tilde{x}_h$ is the vector x_h without the component $x_{\tilde{N}}$. Thus G_h is a contraction and there is a unique solution to the difference equation on $[0, \delta]$. The convergence can now be concluded as in [14].

This proves the convergence of the explicit Euler method for general linear problems (5.1). In the nonlinear case, the same reasoning as in [14] yields the result. Consequently, we have proven the following theorem.

Theorem 5.1 *Consider the explicit Euler method for the singular terminal value problem (5.1). If $M \in C^2[0, 1]$ and f is continuously differentiable, the numerical solution z_h satisfies*

$$\|z_h - R_h(z)\|_h = O(h).$$

Proof: The smoothness assumptions on M and f imply that $z \in C^2[0, 1]$, and hence consistency of order 1 can be concluded. The stability of the scheme follows from the considerations given above.

5.1.2   Iterated Defect Correction

In this section, we discuss the application of IDeC based on the explicit Euler method to (5.1), where a polynomial degree $m = 5$ is used in the definition of the method.

The extremely simple structure of example 4.1a (the solution is a linear polynomial) permits no nontrivial conclusions about its numerical solution. Even the application of the basic method yields the exact solution (only roundoff errors are present), see Table 5.1. Thus, this example is going to be omitted from the investigation of the other methods.

Example 4.1b shows a more interesting behavior. The successive improvement of the convergence orders of the IDeC iterates is observed up to $O(h^5)$ when Zadunaisky polynomials of degree 5 are used. Surprisingly, however, this theoretical limit can even be exceeded for this example, see Table 5.3. This is no contradiction to the theory, however, since this only states sufficient conditions for the improvement of the convergence.

At $t = 0$ an extraordinary behavior is observed. The numerical solution at this point is equal to the exact value up to roundoff error. This surprising fact can be explained by explicitly calculating the numerical solution at this point. Using (5.3) for example 4.1a with general σ, we find

$$
\begin{aligned}
z_j &= \prod_{l=0}^{j-1} \left(I - \frac{M}{N-l} \right) \cdot \beta \\
&= \prod_{l=0}^{j-1} \begin{pmatrix} 1 & -\frac{1}{N-l} \\ 0 & 1 - \frac{\sigma}{N-l} \end{pmatrix} \cdot \begin{pmatrix} -1 \\ \sigma \end{pmatrix}.
\end{aligned}
$$

For $l = N - \sigma$, the corresponding factor in the above product is a singular matrix. Since the factors commute, it is sufficient to investigate the effect on the terminal value β,

$$
\begin{pmatrix} 1 & -\frac{1}{\sigma} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ \sigma \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} = z(0) \in \ker M.
$$

Thus, after the application of the factor for $l = N - \sigma$, all further values satisfy $z_j = z(0)$, $j = N - \sigma + 1, \ldots, N$. Therefore, a very special structure of problem 4.1a causes the effect that the problem is solved exactly at $t = 0$ by the explicit Euler method. For $\sigma = 1$ and $\sigma = 3$, this behavior is also reflected in all further IDeC iterates, see Table 5.4. This is not the case for $\sigma = 10$, however, where a convergence order of 10 is observed in Table 5.6. It is suggestive to note that the exact solution is a polynomial of degree σ.

For example 4.1d, IDeC does not work satisfactorily, see Tables 5.7 and 5.8. An order reduction down to $O(h^2)$ is observed on the whole interval as well as at $t = 0$. For the IDeC iterates where the order reductions occur, $t^{\max} \to 0$ holds. This behavior is already well-known from §4. Note that for $\sigma = 1$, the solution of a terminal value problem of the form (5.1) satisfies $z \in C[0, 1]$ in general. This fact causes the solutions of the variational equations to become

unsmooth although the exact solution of the original problem is smooth. This will be explained in more detail in §5.1.3.

A similar order reduction is observed for 4.1e. Here, the highest attainable convergence order is 4 due to Tables 5.9 and 5.10. This coincides with $\sigma = 3$.

Example 4.1f shows a rather irregular behavior in Table 5.11 and Table 5.12. For the first iterates, the maximal error on $[0, 1]$ is assumed near $t = 1$ and the classical improvement of the convergence order takes place. For iterates 4 and 5, however, the absolute value of the error is largest at $t = 0$, but the convergence order at this point is equal to 6. Consequently, for very high precision the error at $t = 0$ drops below that at points $t \approx 1$, and convergence order 5 is restored with $t^{\max} \approx 1$. The analogous effect is not observed earlier in the third iterate because in this case the convergence order gradually drops to the classical value 4.

The problems 4.2a and 5.2a with variable coefficient matrices show order reductions down to 3 and 4, respectively. Similar effects can be observed on $[0, 1]$ and at $t = 0$ in Tables 5.13 – 5.16. For 5.2a, a convergence order of 4 is obtained in the third iteration step in spite of the fact that $t^{\max} \to 0$, because this is the order of all IDeC iterates at $t = 0$ for this example.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$3.0 \cdot 10^{-33}$	0.20000	-4.000	$6.1 \cdot 10^{-32}$	0.80000	1.000
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-32}$	0.30000	1.000	$3.0 \cdot 10^{-32}$	0.40000	-1.263
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-32}$	0.20000	-3.169	$7.3 \cdot 10^{-32}$	0.65000	-1.584
$1/5 \cdot 2^{-3}$	$2.2 \cdot 10^{-31}$	0.07500	0.847	$2.2 \cdot 10^{-31}$	0.25000	-0.530
$1/5 \cdot 2^{-4}$	$1.2 \cdot 10^{-31}$	-0.0000	-2.925	$3.2 \cdot 10^{-31}$	-0.0000	-1.758
$1/5 \cdot 2^{-5}$	$9.3 \cdot 10^{-31}$	-0.0000	1.000	$1.0 \cdot 10^{-30}$	0.08125	-0.720
$1/5 \cdot 2^{-6}$	$4.6 \cdot 10^{-31}$	0.00000	-2.990	$1.7 \cdot 10^{-30}$	0.01250	0.493
$1/5 \cdot 2^{-7}$	$3.7 \cdot 10^{-30}$	0.00000	1.000	$1.2 \cdot 10^{-30}$	0.27187	-1.772
$1/5 \cdot 2^{-8}$	$1.8 \cdot 10^{-30}$	0.00078	-2.995	$4.3 \cdot 10^{-30}$	-0.0000	-1.740
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-29}$	0.00117	1.000	$1.4 \cdot 10^{-29}$	0.00507	-0.906
$1/5 \cdot 2^{-10}$	$7.4 \cdot 10^{-30}$	0.00078	-3.000	$2.7 \cdot 10^{-29}$	0.00078	0.585
$1/5 \cdot 2^{-11}$	$5.9 \cdot 10^{-29}$	0.00029	0.999	$1.8 \cdot 10^{-29}$	0.25136	-1.963
$1/5 \cdot 2^{-12}$	$2.9 \cdot 10^{-29}$	-0.0000	-2.999	$7.0 \cdot 10^{-29}$	-0.0000	-1.697
$1/5 \cdot 2^{-13}$	$2.3 \cdot 10^{-28}$	-0.0000	1.000	$2.2 \cdot 10^{-28}$	0.00031	-0.924
$1/5 \cdot 2^{-14}$	$1.1 \cdot 10^{-28}$	0.00000	-2.999	$4.3 \cdot 10^{-28}$	0.00004	0.598
$1/5 \cdot 2^{-15}$	$9.5 \cdot 10^{-28}$	0.00000		$2.8 \cdot 10^{-28}$	0.25008	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.4 \cdot 10^{-31}$	0.60000	1.263	$1.2 \cdot 10^{-31}$	0.60000	-0.263
$1/5 \cdot 2^{-1}$	$6.1 \cdot 10^{-32}$	0.40000	-1.584	$1.4 \cdot 10^{-31}$	0.30000	0.125
$1/5 \cdot 2^{-2}$	$1.8 \cdot 10^{-31}$	0.20000	-0.415	$1.3 \cdot 10^{-31}$	0.30000	-0.540
$1/5 \cdot 2^{-3}$	$2.4 \cdot 10^{-31}$	0.52500	0.621	$1.9 \cdot 10^{-31}$	0.70000	-1.169
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-31}$	0.50000	-2.914	$4.4 \cdot 10^{-31}$	-0.0000	-2.009
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-30}$	0.02500	-0.085	$1.7 \cdot 10^{-30}$	0.15625	1.688
$1/5 \cdot 2^{-6}$	$1.2 \cdot 10^{-30}$	0.09062	-2.250	$5.5 \cdot 10^{-31}$	0.06250	-2.438
$1/5 \cdot 2^{-7}$	$6.1 \cdot 10^{-30}$	0.03281	1.801	$3.0 \cdot 10^{-30}$	0.00000	0.078
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-30}$	0.09062	-3.417	$2.8 \cdot 10^{-30}$	0.30000	-0.968
$1/5 \cdot 2^{-9}$	$1.8 \cdot 10^{-29}$	0.00156	1.584	$5.5 \cdot 10^{-30}$	0.43945	0.085
$1/5 \cdot 2^{-10}$	$6.2 \cdot 10^{-30}$	0.21054	-3.462	$5.2 \cdot 10^{-30}$	0.03027	-3.950
$1/5 \cdot 2^{-11}$	$6.8 \cdot 10^{-29}$	0.00976	1.770	$8.1 \cdot 10^{-29}$	0.00097	1.248
$1/5 \cdot 2^{-12}$	$2.0 \cdot 10^{-29}$	0.04506	-3.949	$3.4 \cdot 10^{-29}$	0.45371	-1.660
$1/5 \cdot 2^{-13}$	$3.1 \cdot 10^{-28}$	0.00009	1.497	$1.0 \cdot 10^{-28}$	0.44270	0.899
$1/5 \cdot 2^{-14}$	$1.1 \cdot 10^{-28}$	0.00241		$5.7 \cdot 10^{-29}$	0.63653	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$4.9 \cdot 10^{-32}$	-0.0000	-0.807	$1.2 \cdot 10^{-31}$	0.80000	-0.137
$1/5 \cdot 2^{-1}$	$8.6 \cdot 10^{-32}$	0.90000	-0.362	$1.3 \cdot 10^{-31}$	0.80000	-0.862
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-31}$	0.10000	-2.078	$2.4 \cdot 10^{-31}$	0.20000	-0.263
$1/5 \cdot 2^{-3}$	$4.6 \cdot 10^{-31}$	0.17500	-0.144	$2.9 \cdot 10^{-31}$	0.05000	-1.273
$1/5 \cdot 2^{-4}$	$5.1 \cdot 10^{-31}$	0.01250	-0.893	$7.1 \cdot 10^{-31}$	-0.0000	-0.949
$1/5 \cdot 2^{-5}$	$9.6 \cdot 10^{-31}$	0.00625	0.427	$1.3 \cdot 10^{-30}$	0.16250	0.485
$1/5 \cdot 2^{-6}$	$7.1 \cdot 10^{-31}$	0.08125	-2.849	$9.8 \cdot 10^{-31}$	0.39062	-2.074
$1/5 \cdot 2^{-7}$	$5.1 \cdot 10^{-30}$	0.05937	2.199	$4.1 \cdot 10^{-30}$	0.00312	1.298
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-30}$	0.49921	-4.217	$1.6 \cdot 10^{-30}$	0.16484	-2.468
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-29}$	0.00117	2.190	$9.3 \cdot 10^{-30}$	0.00507	-0.288
$1/5 \cdot 2^{-10}$	$4.5 \cdot 10^{-30}$	0.31054	-3.631	$1.1 \cdot 10^{-29}$	0.52070	-1.844
$1/5 \cdot 2^{-11}$	$5.6 \cdot 10^{-29}$	0.16630	1.916	$4.0 \cdot 10^{-29}$	0.00537	0.660
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-29}$	0.57114	-3.955	$2.5 \cdot 10^{-29}$	0.45922	-2.174
$1/5 \cdot 2^{-13}$	$2.3 \cdot 10^{-28}$	0.00053		$1.1 \cdot 10^{-28}$	0.00166	

Table 5.1: Explicit Euler, IDeC method on $[0, 1]$ for 4.1a.

h	δ	p	δ	p
$1/5$	$6.1 \cdot 10^{-33}$	-3.000	$4.9 \cdot 10^{-32}$	1.000
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-32}$	1.000	$2.4 \cdot 10^{-32}$	-1.000
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-32}$	-3.000	$4.9 \cdot 10^{-32}$	-1.807
$1/5 \cdot 2^{-3}$	$1.9 \cdot 10^{-31}$	0.0000	$1.7 \cdot 10^{-31}$	-0.514
$1/5 \cdot 2^{-4}$	$1.9 \cdot 10^{-31}$	-2.000	$2.4 \cdot 10^{-31}$	-1.847
$1/5 \cdot 2^{-5}$	$7.8 \cdot 10^{-31}$	0.0000	$8.8 \cdot 10^{-31}$	-1.239
$1/5 \cdot 2^{-6}$	$7.8 \cdot 10^{-31}$	-2.000	$2.0 \cdot 10^{-30}$	1.912
$1/5 \cdot 2^{-7}$	$3.1 \cdot 10^{-30}$	0.0000	$5.5 \cdot 10^{-31}$	-2.445
$1/5 \cdot 2^{-8}$	$3.1 \cdot 10^{-30}$	-2.000	$3.0 \cdot 10^{-30}$	-2.005
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-29}$	0.0000	$1.2 \cdot 10^{-29}$	-1.409
$1/5 \cdot 2^{-10}$	$1.2 \cdot 10^{-29}$	-2.000	$3.2 \cdot 10^{-29}$	1.860
$1/5 \cdot 2^{-11}$	$5.0 \cdot 10^{-29}$	0.0000	$8.9 \cdot 10^{-30}$	-2.483
$1/5 \cdot 2^{-12}$	$5.0 \cdot 10^{-29}$	-1.999	$4.9 \cdot 10^{-29}$	-1.955
$1/5 \cdot 2^{-13}$	$2.0 \cdot 10^{-28}$	0.0000	$1.9 \cdot 10^{-28}$	-1.421
$1/5 \cdot 2^{-14}$	$2.0 \cdot 10^{-28}$	-2.000	$5.1 \cdot 10^{-28}$	1.860
$1/5 \cdot 2^{-15}$	$8.0 \cdot 10^{-28}$		$1.4 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5$	$7.3 \cdot 10^{-32}$	0.584	$7.3 \cdot 10^{-32}$	0.0000
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-32}$	-1.584	$7.3 \cdot 10^{-32}$	0.584
$1/5 \cdot 2^{-2}$	$1.4 \cdot 10^{-31}$	0.0000	$4.9 \cdot 10^{-32}$	-0.584
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-31}$	0.907	$7.3 \cdot 10^{-32}$	-2.321
$1/5 \cdot 2^{-4}$	$7.8 \cdot 10^{-32}$	-3.714	$3.6 \cdot 10^{-31}$	-1.951
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-30}$	0.637	$1.4 \cdot 10^{-30}$	1.157
$1/5 \cdot 2^{-6}$	$6.6 \cdot 10^{-31}$	-2.986	$6.4 \cdot 10^{-31}$	-1.928
$1/5 \cdot 2^{-7}$	$5.2 \cdot 10^{-30}$	2.024	$2.4 \cdot 10^{-30}$	0.912
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-30}$	-3.664	$1.2 \cdot 10^{-30}$	-1.821
$1/5 \cdot 2^{-9}$	$1.6 \cdot 10^{-29}$	1.661	$4.5 \cdot 10^{-30}$	-0.180
$1/5 \cdot 2^{-10}$	$5.1 \cdot 10^{-30}$	-3.515	$5.1 \cdot 10^{-30}$	-3.795
$1/5 \cdot 2^{-11}$	$5.9 \cdot 10^{-29}$	1.515	$7.2 \cdot 10^{-29}$	1.795
$1/5 \cdot 2^{-12}$	$2.0 \cdot 10^{-29}$	-3.729	$2.0 \cdot 10^{-29}$	-0.777
$1/5 \cdot 2^{-13}$	$2.7 \cdot 10^{-28}$	1.729	$3.5 \cdot 10^{-29}$	-1.222
$1/5 \cdot 2^{-14}$	$8.3 \cdot 10^{-29}$		$8.3 \cdot 10^{-29}$	

h	δ	p	δ	p
$1/5$	$4.9 \cdot 10^{-32}$	-0.584	$9.8 \cdot 10^{-32}$	1.000
$1/5 \cdot 2^{-1}$	$7.3 \cdot 10^{-32}$	-0.415	$4.9 \cdot 10^{-32}$	-2.000
$1/5 \cdot 2^{-2}$	$9.8 \cdot 10^{-32}$	-0.807	$1.9 \cdot 10^{-31}$	-0.169
$1/5 \cdot 2^{-3}$	$1.7 \cdot 10^{-31}$	-1.716	$2.2 \cdot 10^{-31}$	-1.530
$1/5 \cdot 2^{-4}$	$5.6 \cdot 10^{-31}$	-0.476	$6.4 \cdot 10^{-31}$	-0.157
$1/5 \cdot 2^{-5}$	$7.8 \cdot 10^{-31}$	1.274	$7.1 \cdot 10^{-31}$	1.133
$1/5 \cdot 2^{-6}$	$3.2 \cdot 10^{-31}$	-3.550	$3.2 \cdot 10^{-31}$	-3.455
$1/5 \cdot 2^{-7}$	$3.8 \cdot 10^{-30}$	1.126	$3.5 \cdot 10^{-30}$	1.462
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-30}$	-3.410	$1.2 \cdot 10^{-30}$	-2.391
$1/5 \cdot 2^{-9}$	$1.8 \cdot 10^{-29}$	1.840	$6.8 \cdot 10^{-30}$	0.388
$1/5 \cdot 2^{-10}$	$5.1 \cdot 10^{-30}$	-3.094	$5.1 \cdot 10^{-30}$	-2.599
$1/5 \cdot 2^{-11}$	$4.4 \cdot 10^{-29}$	1.094	$3.1 \cdot 10^{-29}$	0.599
$1/5 \cdot 2^{-12}$	$2.0 \cdot 10^{-29}$	-3.246	$2.0 \cdot 10^{-29}$	-1.961
$1/5 \cdot 2^{-13}$	$1.9 \cdot 10^{-28}$		$8.0 \cdot 10^{-29}$	

Table 5.2: Explicit Euler, IDeC method at $t = 0$ for 4.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$3.4 \cdot 10^{-01}$	0.60000	1.176	$2.1 \cdot 10^{-01}$	0.80000	2.638
$1/5 \cdot 2^{-1}$	$1.5 \cdot 10^{-01}$	0.70000	1.112	$3.4 \cdot 10^{-02}$	0.70000	2.285
$1/5 \cdot 2^{-2}$	$7.1 \cdot 10^{-02}$	0.65000	1.046	$7.0 \cdot 10^{-03}$	0.70000	2.130
$1/5 \cdot 2^{-3}$	$3.4 \cdot 10^{-02}$	0.67500	1.024	$1.6 \cdot 10^{-03}$	0.70000	2.062
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-02}$	0.66250	1.011	$3.8 \cdot 10^{-04}$	0.71250	2.030
$1/5 \cdot 2^{-5}$	$8.4 \cdot 10^{-03}$	0.66875	1.006	$9.4 \cdot 10^{-05}$	0.70625	2.015
$1/5 \cdot 2^{-6}$	$4.1 \cdot 10^{-03}$	0.66562	1.003	$2.3 \cdot 10^{-05}$	0.70625	2.007
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-03}$	0.66718	1.001	$5.7 \cdot 10^{-06}$	0.70625	2.003
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-03}$	0.66640	1.000	$1.4 \cdot 10^{-06}$	0.70546	2.001
$1/5 \cdot 2^{-9}$	$5.2 \cdot 10^{-04}$	0.66679	1.000	$3.6 \cdot 10^{-07}$	0.70546	2.000
$1/5 \cdot 2^{-10}$	$2.6 \cdot 10^{-04}$	0.66660	1.000	$9.0 \cdot 10^{-08}$	0.70546	2.000
$1/5 \cdot 2^{-11}$	$1.3 \cdot 10^{-04}$	0.66669	1.000	$2.2 \cdot 10^{-08}$	0.70546	2.000
$1/5 \cdot 2^{-12}$	$6.5 \cdot 10^{-05}$	0.66665	0.999	$5.6 \cdot 10^{-09}$	0.70551	2.000
$1/5 \cdot 2^{-13}$	$3.2 \cdot 10^{-05}$	0.66667	1.000	$1.4 \cdot 10^{-09}$	0.70551	2.000
$1/5 \cdot 2^{-14}$	$1.6 \cdot 10^{-05}$	0.66666	1.000	$3.5 \cdot 10^{-10}$	0.70550	2.000
$1/5 \cdot 2^{-15}$	$8.1 \cdot 10^{-06}$	0.66666		$8.8 \cdot 10^{-11}$	0.70550	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.6 \cdot 10^{-01}$	0.80000	4.110	$1.2 \cdot 10^{-01}$	0.80000	5.553
$1/5 \cdot 2^{-1}$	$9.4 \cdot 10^{-03}$	0.70000	3.468	$2.7 \cdot 10^{-03}$	0.70000	4.649
$1/5 \cdot 2^{-2}$	$8.5 \cdot 10^{-04}$	0.70000	3.210	$1.0 \cdot 10^{-04}$	0.70000	4.290
$1/5 \cdot 2^{-3}$	$9.2 \cdot 10^{-05}$	0.72500	3.104	$5.6 \cdot 10^{-06}$	0.72500	4.144
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-05}$	0.71250	3.050	$3.1 \cdot 10^{-07}$	0.72500	4.070
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-06}$	0.71875	3.025	$1.8 \cdot 10^{-08}$	0.71875	4.034
$1/5 \cdot 2^{-6}$	$1.5 \cdot 10^{-07}$	0.71875	3.012	$1.1 \cdot 10^{-09}$	0.72187	4.017
$1/5 \cdot 2^{-7}$	$1.9 \cdot 10^{-08}$	0.71718	3.006	$7.1 \cdot 10^{-11}$	0.72031	4.008
$1/5 \cdot 2^{-8}$	$2.4 \cdot 10^{-09}$	0.71718	3.003	$4.4 \cdot 10^{-12}$	0.72031	4.004
$1/5 \cdot 2^{-9}$	$3.0 \cdot 10^{-10}$	0.71718	3.001	$2.7 \cdot 10^{-13}$	0.72031	4.002
$1/5 \cdot 2^{-10}$	$3.8 \cdot 10^{-11}$	0.71718	3.000	$1.7 \cdot 10^{-14}$	0.72031	4.001
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-12}$	0.71718	3.000	$1.0 \cdot 10^{-15}$	0.72031	4.000
$1/5 \cdot 2^{-12}$	$5.9 \cdot 10^{-13}$	0.71713	3.000	$6.7 \cdot 10^{-17}$	0.72031	4.000
$1/5 \cdot 2^{-13}$	$7.4 \cdot 10^{-14}$	0.71713	3.000	$4.1 \cdot 10^{-18}$	0.72033	4.000
$1/5 \cdot 2^{-14}$	$9.3 \cdot 10^{-15}$	0.71712		$2.6 \cdot 10^{-19}$	0.72032	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.0 \cdot 10^{-01}$	0.80000	6.985	$8.3 \cdot 10^{-02}$	0.80000	8.415
$1/5 \cdot 2^{-1}$	$8.1 \cdot 10^{-04}$	0.70000	5.825	$2.4 \cdot 10^{-04}$	0.70000	7.002
$1/5 \cdot 2^{-2}$	$1.4 \cdot 10^{-05}$	0.75000	5.372	$1.9 \cdot 10^{-06}$	0.75000	6.454
$1/5 \cdot 2^{-3}$	$3.4 \cdot 10^{-07}$	0.72500	5.183	$2.1 \cdot 10^{-08}$	0.72500	6.222
$1/5 \cdot 2^{-4}$	$9.5 \cdot 10^{-09}$	0.72500	5.089	$2.9 \cdot 10^{-10}$	0.72500	6.109
$1/5 \cdot 2^{-5}$	$2.8 \cdot 10^{-10}$	0.71875	5.044	$4.2 \cdot 10^{-12}$	0.72500	6.053
$1/5 \cdot 2^{-6}$	$8.5 \cdot 10^{-12}$	0.72187	5.022	$6.3 \cdot 10^{-14}$	0.72187	6.026
$1/5 \cdot 2^{-7}$	$2.6 \cdot 10^{-13}$	0.72187	5.011	$9.7 \cdot 10^{-16}$	0.72187	6.013
$1/5 \cdot 2^{-8}$	$8.1 \cdot 10^{-15}$	0.72109	5.005	$1.5 \cdot 10^{-17}$	0.72187	6.006
$1/5 \cdot 2^{-9}$	$2.5 \cdot 10^{-16}$	0.72109	5.002	$2.3 \cdot 10^{-19}$	0.72148	6.003
$1/5 \cdot 2^{-10}$	$7.9 \cdot 10^{-18}$	0.72128	5.001	$3.6 \cdot 10^{-21}$	0.72148	6.001
$1/5 \cdot 2^{-11}$	$2.4 \cdot 10^{-19}$	0.72119	5.000	$5.6 \cdot 10^{-23}$	0.72138	6.001
$1/5 \cdot 2^{-12}$	$7.7 \cdot 10^{-21}$	0.72119	5.000	$8.8 \cdot 10^{-25}$	0.72143	5.994
$1/5 \cdot 2^{-13}$	$2.4 \cdot 10^{-22}$	0.72119		$1.3 \cdot 10^{-26}$	0.72155	

Table 5.3: Explicit Euler, IDeC method on $[0, 1]$ for 4.1b.

h	δ	p	δ	p
$1/5$	0.0		$2.4 \cdot 10^{-32}$	-1.584
$1/5 \cdot 2^{-1}$	$2.4 \cdot 10^{-32}$	0.0000	$7.3 \cdot 10^{-32}$	0.584
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-32}$	0.0000	$4.9 \cdot 10^{-32}$	-3.247
$1/5 \cdot 2^{-3}$	$2.4 \cdot 10^{-32}$	0.0000	$4.6 \cdot 10^{-31}$	-0.211
$1/5 \cdot 2^{-4}$	$2.4 \cdot 10^{-32}$	9.544	$5.4 \cdot 10^{-31}$	-0.241
$1/5 \cdot 2^{-5}$	$3.3 \cdot 10^{-35}$	-10.544	$6.4 \cdot 10^{-31}$	-1.299
$1/5 \cdot 2^{-6}$	$4.9 \cdot 10^{-32}$	-2.321	$1.5 \cdot 10^{-30}$	-0.845
$1/5 \cdot 2^{-7}$	$2.4 \cdot 10^{-31}$	0.514	$2.8 \cdot 10^{-30}$	-0.889
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-31}$	-1.000	$5.2 \cdot 10^{-30}$	-1.053
$1/5 \cdot 2^{-9}$	$3.4 \cdot 10^{-31}$	0.0000	$1.0 \cdot 10^{-29}$	-0.946
$1/5 \cdot 2^{-10}$	$3.4 \cdot 10^{-31}$	-0.440	$2.1 \cdot 10^{-29}$	-0.638
$1/5 \cdot 2^{-11}$	$4.6 \cdot 10^{-31}$	0.788	$3.2 \cdot 10^{-29}$	-1.119
$1/5 \cdot 2^{-12}$	$2.7 \cdot 10^{-31}$	1.874	$7.1 \cdot 10^{-29}$	-1.055
$1/5 \cdot 2^{-13}$	$7.3 \cdot 10^{-32}$	-6.309	$1.4 \cdot 10^{-28}$	-1.062
$1/5 \cdot 2^{-14}$	$5.8 \cdot 10^{-30}$	-0.819	$3.0 \cdot 10^{-28}$	-1.043
$1/5 \cdot 2^{-15}$	$1.0 \cdot 10^{-29}$		$6.3 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5$	$3.3 \cdot 10^{-32}$	-0.540	$2.7 \cdot 10^{-31}$	1.169
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-32}$	-2.321	$1.2 \cdot 10^{-31}$	-0.263
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-31}$	-0.378	$1.4 \cdot 10^{-31}$	-2.369
$1/5 \cdot 2^{-3}$	$3.2 \cdot 10^{-31}$	1.378	$7.6 \cdot 10^{-31}$	1.253
$1/5 \cdot 2^{-4}$	$1.2 \cdot 10^{-31}$	1.321	$3.2 \cdot 10^{-31}$	-0.387
$1/5 \cdot 2^{-5}$	$4.9 \cdot 10^{-32}$	-4.977	$4.1 \cdot 10^{-31}$	-2.197
$1/5 \cdot 2^{-6}$	$1.5 \cdot 10^{-30}$	0.094	$1.9 \cdot 10^{-30}$	0.285
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-30}$	0.127	$1.5 \cdot 10^{-30}$	0.385
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-30}$	0.0000	$1.2 \cdot 10^{-30}$	0.756
$1/5 \cdot 2^{-9}$	$1.3 \cdot 10^{-30}$	1.847	$7.1 \cdot 10^{-31}$	-3.468
$1/5 \cdot 2^{-10}$	$3.6 \cdot 10^{-31}$	-4.355	$7.9 \cdot 10^{-30}$	-0.528
$1/5 \cdot 2^{-11}$	$7.5 \cdot 10^{-30}$	-0.041	$1.1 \cdot 10^{-29}$	2.412
$1/5 \cdot 2^{-12}$	$7.7 \cdot 10^{-30}$	0.869	$2.1 \cdot 10^{-30}$	-1.505
$1/5 \cdot 2^{-13}$	$4.2 \cdot 10^{-30}$	-0.873	$6.0 \cdot 10^{-30}$	2.456
$1/5 \cdot 2^{-14}$	$7.8 \cdot 10^{-30}$		$1.1 \cdot 10^{-30}$	

h	δ	p	δ	p
$1/5$	$4.9 \cdot 10^{-32}$	-0.536	$6.7 \cdot 10^{-31}$	1.459
$1/5 \cdot 2^{-1}$	$7.1 \cdot 10^{-32}$	1.536	$2.4 \cdot 10^{-31}$	0.321
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-32}$	-3.169	$1.9 \cdot 10^{-31}$	-0.807
$1/5 \cdot 2^{-3}$	$2.2 \cdot 10^{-31}$	0.362	$3.4 \cdot 10^{-31}$	-0.584
$1/5 \cdot 2^{-4}$	$1.7 \cdot 10^{-31}$	-1.514	$5.1 \cdot 10^{-31}$	0.807
$1/5 \cdot 2^{-5}$	$4.9 \cdot 10^{-31}$	-2.185	$2.9 \cdot 10^{-31}$	-1.841
$1/5 \cdot 2^{-6}$	$2.2 \cdot 10^{-30}$	1.700	$1.0 \cdot 10^{-30}$	-0.948
$1/5 \cdot 2^{-7}$	$6.9 \cdot 10^{-31}$	0.485	$2.0 \cdot 10^{-30}$	2.205
$1/5 \cdot 2^{-8}$	$4.9 \cdot 10^{-31}$	-1.263	$4.4 \cdot 10^{-31}$	-1.852
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-30}$	-1.073	$1.6 \cdot 10^{-30}$	1.700
$1/5 \cdot 2^{-10}$	$2.4 \cdot 10^{-30}$	-1.956	$4.9 \cdot 10^{-31}$	-4.804
$1/5 \cdot 2^{-11}$	$9.6 \cdot 10^{-30}$	0.376	$1.3 \cdot 10^{-29}$	-0.390
$1/5 \cdot 2^{-12}$	$7.4 \cdot 10^{-30}$	0.188	$1.8 \cdot 10^{-29}$	0.138
$1/5 \cdot 2^{-13}$	$6.5 \cdot 10^{-30}$		$1.6 \cdot 10^{-29}$	

Table 5.4: Explicit Euler, IDeC method at $t = 0$ for 4.1b.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.2 \cdot 10^{+03}$	-0.0000	8.497	$9.6 \cdot 10^{+04}$	-0.0000	9.728
$1/5 \cdot 2^{-1}$	3.4	0.90000	1.640	$1.1 \cdot 10^{+02}$	-0.0000	7.373
$1/5 \cdot 2^{-2}$	1.1	0.90000	1.196	$6.8 \cdot 10^{-01}$	0.95000	2.608
$1/5 \cdot 2^{-3}$	$4.8 \cdot 10^{-01}$	0.90000	1.084	$1.1 \cdot 10^{-01}$	0.92500	2.273
$1/5 \cdot 2^{-4}$	$2.3 \cdot 10^{-01}$	0.90000	1.039	$2.3 \cdot 10^{-02}$	0.92500	2.125
$1/5 \cdot 2^{-5}$	$1.1 \cdot 10^{-01}$	0.90000	1.019	$5.3 \cdot 10^{-03}$	0.92500	2.058
$1/5 \cdot 2^{-6}$	$5.5 \cdot 10^{-02}$	0.90000	1.009	$1.2 \cdot 10^{-03}$	0.92187	2.029
$1/5 \cdot 2^{-7}$	$2.7 \cdot 10^{-02}$	0.90000	1.004	$3.1 \cdot 10^{-04}$	0.92187	2.014
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-02}$	0.90000	1.002	$7.7 \cdot 10^{-05}$	0.92109	2.007
$1/5 \cdot 2^{-9}$	$6.8 \cdot 10^{-03}$	0.90000	1.001	$1.9 \cdot 10^{-05}$	0.92070	2.003
$1/5 \cdot 2^{-10}$	$3.4 \cdot 10^{-03}$	0.90000	1.000	$4.8 \cdot 10^{-06}$	0.92089	2.001
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-03}$	0.90000	1.000	$1.2 \cdot 10^{-06}$	0.92080	2.001
$1/5 \cdot 2^{-12}$	$8.5 \cdot 10^{-04}$	0.90000	1.000	$3.0 \cdot 10^{-07}$	0.92080	2.000
$1/5 \cdot 2^{-13}$	$4.2 \cdot 10^{-04}$	0.90000	1.000	$7.5 \cdot 10^{-08}$	0.92077	2.000
$1/5 \cdot 2^{-14}$	$2.1 \cdot 10^{-04}$	0.90000	1.000	$1.8 \cdot 10^{-08}$	0.92078	2.000
$1/5 \cdot 2^{-15}$	$1.0 \cdot 10^{-04}$	0.90000		$4.7 \cdot 10^{-09}$	0.92078	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$7.4 \cdot 10^{+06}$	-0.0000	9.667	$5.7 \cdot 10^{+08}$	-0.0000	9.664
$1/5 \cdot 2^{-1}$	$9.1 \cdot 10^{+03}$	-0.0000	13.782	$7.0 \cdot 10^{+05}$	-0.0000	17.897
$1/5 \cdot 2^{-2}$	$6.4 \cdot 10^{-01}$	0.95000	4.133	2.8	0.00000	7.690
$1/5 \cdot 2^{-3}$	$3.6 \cdot 10^{-02}$	0.92500	3.480	$1.3 \cdot 10^{-02}$	0.92500	4.676
$1/5 \cdot 2^{-4}$	$3.3 \cdot 10^{-03}$	0.92500	3.216	$5.4 \cdot 10^{-04}$	0.93750	4.322
$1/5 \cdot 2^{-5}$	$3.5 \cdot 10^{-04}$	0.93125	3.106	$2.7 \cdot 10^{-05}$	0.93125	4.153
$1/5 \cdot 2^{-6}$	$4.1 \cdot 10^{-05}$	0.92812	3.052	$1.5 \cdot 10^{-06}$	0.93125	4.075
$1/5 \cdot 2^{-7}$	$4.9 \cdot 10^{-06}$	0.92812	3.025	$9.0 \cdot 10^{-08}$	0.93281	4.037
$1/5 \cdot 2^{-8}$	$6.1 \cdot 10^{-07}$	0.92812	3.012	$5.5 \cdot 10^{-09}$	0.93203	4.018
$1/5 \cdot 2^{-9}$	$7.5 \cdot 10^{-08}$	0.92851	3.006	$3.4 \cdot 10^{-10}$	0.93203	4.009
$1/5 \cdot 2^{-10}$	$9.4 \cdot 10^{-09}$	0.92832	3.003	$2.1 \cdot 10^{-11}$	0.93203	4.004
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-09}$	0.92832	3.001	$1.3 \cdot 10^{-12}$	0.93203	4.002
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-10}$	0.92832	3.000	$8.2 \cdot 10^{-14}$	0.93198	4.000
$1/5 \cdot 2^{-13}$	$1.8 \cdot 10^{-11}$	0.92832	3.000	$5.1 \cdot 10^{-15}$	0.93200	4.000
$1/5 \cdot 2^{-14}$	$2.2 \cdot 10^{-12}$	0.92832		$3.2 \cdot 10^{-16}$	0.93199	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$4.3 \cdot 10^{+10}$	-0.0000	9.664	$3.3 \cdot 10^{+12}$	-0.0000	9.664
$1/5 \cdot 2^{-1}$	$5.3 \cdot 10^{+07}$	-0.0000	17.897	$4.1 \cdot 10^{+09}$	-0.0000	17.897
$1/5 \cdot 2^{-2}$	$2.2 \cdot 10^{+02}$	0.00000	12.207	$1.6 \cdot 10^{+04}$	0.00000	12.207
$1/5 \cdot 2^{-3}$	$4.6 \cdot 10^{-02}$	0.00000	8.899	3.5	0.00000	10.938
$1/5 \cdot 2^{-4}$	$9.7 \cdot 10^{-05}$	0.93750	5.426	$1.8 \cdot 10^{-03}$	-0.0000	10.437
$1/5 \cdot 2^{-5}$	$2.2 \cdot 10^{-06}$	0.93750	5.203	$1.3 \cdot 10^{-06}$	-0.0000	9.699
$1/5 \cdot 2^{-6}$	$6.1 \cdot 10^{-08}$	0.93437	5.099	$1.5 \cdot 10^{-09}$	0.95625	5.436
$1/5 \cdot 2^{-7}$	$1.8 \cdot 10^{-09}$	0.93593	5.047	$3.6 \cdot 10^{-11}$	0.78906	4.891
$1/5 \cdot 2^{-8}$	$5.4 \cdot 10^{-11}$	0.93359	5.024	$1.2 \cdot 10^{-12}$	0.82421	4.898
$1/5 \cdot 2^{-9}$	$1.6 \cdot 10^{-12}$	0.93359	5.012	$4.1 \cdot 10^{-14}$	0.84765	4.936
$1/5 \cdot 2^{-10}$	$5.1 \cdot 10^{-14}$	0.93359	5.006	$1.3 \cdot 10^{-15}$	0.85937	4.965
$1/5 \cdot 2^{-11}$	$1.6 \cdot 10^{-15}$	0.93359	5.003	$4.3 \cdot 10^{-17}$	0.86523	4.982
$1/5 \cdot 2^{-12}$	$5.0 \cdot 10^{-17}$	0.93383	5.001	$1.3 \cdot 10^{-18}$	0.86791	4.990
$1/5 \cdot 2^{-13}$	$1.5 \cdot 10^{-18}$	0.93371		$4.2 \cdot 10^{-20}$	0.86926	

Table 5.5: Explicit Euler, IDeC method on $[0,1]$ for 4.1c.

h	δ	p	δ	p
$1/5$	$1.2 \cdot 10^{+03}$		$9.6 \cdot 10^{+04}$	9.728
$1/5 \cdot 2^{-1}$	0.0		$1.1 \cdot 10^{+02}$	17.495
$1/5 \cdot 2^{-2}$	$2.2 \cdot 10^{-36}$	-13.415	$6.1 \cdot 10^{-04}$	12.163
$1/5 \cdot 2^{-3}$	$2.4 \cdot 10^{-32}$	-1.000	$1.3 \cdot 10^{-07}$	10.923
$1/5 \cdot 2^{-4}$	$4.9 \cdot 10^{-32}$	0.0000	$6.9 \cdot 10^{-11}$	10.431
$1/5 \cdot 2^{-5}$	$4.9 \cdot 10^{-32}$	-1.000	$5.0 \cdot 10^{-14}$	10.209
$1/5 \cdot 2^{-6}$	$9.8 \cdot 10^{-32}$	2.000	$4.2 \cdot 10^{-17}$	10.102
$1/5 \cdot 2^{-7}$	$2.4 \cdot 10^{-32}$	-1.000	$3.8 \cdot 10^{-20}$	10.051
$1/5 \cdot 2^{-8}$	$4.9 \cdot 10^{-32}$	-2.584	$3.6 \cdot 10^{-23}$	10.025
$1/5 \cdot 2^{-9}$	$2.9 \cdot 10^{-31}$	1.584	$3.4 \cdot 10^{-26}$	10.012
$1/5 \cdot 2^{-10}$	$9.8 \cdot 10^{-32}$	1.000	$3.3 \cdot 10^{-29}$	-0.283
$1/5 \cdot 2^{-11}$	$4.9 \cdot 10^{-32}$	-4.248	$4.1 \cdot 10^{-29}$	-0.952
$1/5 \cdot 2^{-12}$	$9.3 \cdot 10^{-31}$	1.440	$7.9 \cdot 10^{-29}$	-0.981
$1/5 \cdot 2^{-13}$	$3.4 \cdot 10^{-31}$	-2.099	$1.5 \cdot 10^{-28}$	-1.015
$1/5 \cdot 2^{-14}$	$1.4 \cdot 10^{-30}$	0.352	$3.1 \cdot 10^{-28}$	-0.969
$1/5 \cdot 2^{-15}$	$1.1 \cdot 10^{-30}$		$6.2 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5$	$7.4 \cdot 10^{+06}$	9.667	$5.7 \cdot 10^{+08}$	9.664
$1/5 \cdot 2^{-1}$	$9.1 \cdot 10^{+03}$	17.911	$7.0 \cdot 10^{+05}$	17.897
$1/5 \cdot 2^{-2}$	$3.7 \cdot 10^{-02}$	12.220	2.8	12.206
$1/5 \cdot 2^{-3}$	$7.7 \cdot 10^{-06}$	10.943	$6.0 \cdot 10^{-04}$	10.937
$1/5 \cdot 2^{-4}$	$3.9 \cdot 10^{-09}$	10.439	$3.1 \cdot 10^{-07}$	10.437
$1/5 \cdot 2^{-5}$	$2.8 \cdot 10^{-12}$	10.212	$2.2 \cdot 10^{-10}$	10.211
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-15}$	10.104	$1.8 \cdot 10^{-13}$	10.104
$1/5 \cdot 2^{-7}$	$2.1 \cdot 10^{-18}$	10.052	$1.7 \cdot 10^{-16}$	10.051
$1/5 \cdot 2^{-8}$	$2.0 \cdot 10^{-21}$	10.025	$1.6 \cdot 10^{-19}$	10.025
$1/5 \cdot 2^{-9}$	$1.9 \cdot 10^{-24}$	10.012	$1.5 \cdot 10^{-22}$	10.012
$1/5 \cdot 2^{-10}$	$1.8 \cdot 10^{-27}$	7.329	$1.4 \cdot 10^{-25}$	10.006
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-29}$	-0.460	$1.4 \cdot 10^{-28}$	2.695
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-29}$	2.543	$2.2 \cdot 10^{-29}$	3.548
$1/5 \cdot 2^{-13}$	$2.7 \cdot 10^{-30}$	-2.677	$1.9 \cdot 10^{-30}$	-2.899
$1/5 \cdot 2^{-14}$	$1.7 \cdot 10^{-29}$		$1.4 \cdot 10^{-29}$	

h	δ	p	δ	p
$1/5$	$4.3 \cdot 10^{+10}$	9.664	$3.3 \cdot 10^{+12}$	9.664
$1/5 \cdot 2^{-1}$	$5.3 \cdot 10^{+07}$	17.897	$4.1 \cdot 10^{+09}$	17.897
$1/5 \cdot 2^{-2}$	$2.2 \cdot 10^{+02}$	12.207	$1.6 \cdot 10^{+04}$	12.207
$1/5 \cdot 2^{-3}$	$4.6 \cdot 10^{-02}$	10.938	3.5	10.938
$1/5 \cdot 2^{-4}$	$2.3 \cdot 10^{-05}$	10.437	$1.8 \cdot 10^{-03}$	10.437
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-08}$	10.211	$1.3 \cdot 10^{-06}$	10.211
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-11}$	10.104	$1.1 \cdot 10^{-09}$	10.104
$1/5 \cdot 2^{-7}$	$1.3 \cdot 10^{-14}$	10.051	$1.0 \cdot 10^{-12}$	10.051
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-17}$	10.025	$9.5 \cdot 10^{-16}$	10.025
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-20}$	10.012	$9.1 \cdot 10^{-19}$	10.012
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-23}$	10.006	$8.8 \cdot 10^{-22}$	10.006
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-26}$	9.225	$8.5 \cdot 10^{-25}$	10.003
$1/5 \cdot 2^{-12}$	$1.8 \cdot 10^{-29}$	0.457	$8.3 \cdot 10^{-28}$	4.459
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-29}$		$3.7 \cdot 10^{-29}$	

Table 5.6: Explicit Euler, IDeC method at $t = 0$ for 4.1c.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	1.9	-0.0000	1.001	$7.3 \cdot 10^{-01}$	0.40000	2.228
$1/5 \cdot 2^{-1}$	$9.6 \cdot 10^{-01}$	-0.0000	1.010	$1.5 \cdot 10^{-01}$	0.50000	2.129
$1/5 \cdot 2^{-2}$	$4.8 \cdot 10^{-01}$	0.00000	1.007	$3.5 \cdot 10^{-02}$	0.50000	2.064
$1/5 \cdot 2^{-3}$	$2.3 \cdot 10^{-01}$	0.00000	1.004	$8.6 \cdot 10^{-03}$	0.50000	2.031
$1/5 \cdot 2^{-4}$	$1.1 \cdot 10^{-01}$	-0.0000	1.002	$2.1 \cdot 10^{-03}$	0.51250	2.015
$1/5 \cdot 2^{-5}$	$5.9 \cdot 10^{-02}$	-0.0000	1.001	$5.2 \cdot 10^{-04}$	0.50625	2.007
$1/5 \cdot 2^{-6}$	$2.9 \cdot 10^{-02}$	0.00000	1.000	$1.2 \cdot 10^{-04}$	0.50625	2.003
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-02}$	0.00000	1.000	$3.2 \cdot 10^{-05}$	0.50781	2.001
$1/5 \cdot 2^{-8}$	$7.4 \cdot 10^{-03}$	-0.0000	1.000	$8.0 \cdot 10^{-06}$	0.50781	2.000
$1/5 \cdot 2^{-9}$	$3.7 \cdot 10^{-03}$	-0.0000	1.000	$2.0 \cdot 10^{-06}$	0.50820	2.000
$1/5 \cdot 2^{-10}$	$1.8 \cdot 10^{-03}$	0.00000	1.000	$5.0 \cdot 10^{-07}$	0.50800	2.000
$1/5 \cdot 2^{-11}$	$9.2 \cdot 10^{-04}$	0.00000	1.000	$1.2 \cdot 10^{-07}$	0.50810	2.000
$1/5 \cdot 2^{-12}$	$4.6 \cdot 10^{-04}$	-0.0000	0.999	$3.1 \cdot 10^{-08}$	0.50805	1.999
$1/5 \cdot 2^{-13}$	$2.3 \cdot 10^{-04}$	-0.0000	1.000	$7.8 \cdot 10^{-09}$	0.50808	2.000
$1/5 \cdot 2^{-14}$	$1.1 \cdot 10^{-04}$	0.00000	1.000	$1.9 \cdot 10^{-09}$	0.50808	2.000
$1/5 \cdot 2^{-15}$	$5.8 \cdot 10^{-05}$	0.00000		$4.9 \cdot 10^{-10}$	0.50808	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$2.3 \cdot 10^{-01}$	0.40000	3.263	$5.1 \cdot 10^{-02}$	0.20000	4.067
$1/5 \cdot 2^{-1}$	$2.4 \cdot 10^{-02}$	0.50000	3.145	$3.0 \cdot 10^{-03}$	0.40000	1.789
$1/5 \cdot 2^{-2}$	$2.8 \cdot 10^{-03}$	0.45000	3.077	$8.9 \cdot 10^{-04}$	0.20000	2.102
$1/5 \cdot 2^{-3}$	$3.3 \cdot 10^{-04}$	0.45000	2.805	$2.0 \cdot 10^{-04}$	0.07500	2.086
$1/5 \cdot 2^{-4}$	$4.7 \cdot 10^{-05}$	0.05000	2.123	$4.9 \cdot 10^{-05}$	0.03750	2.049
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-05}$	0.02500	2.116	$1.1 \cdot 10^{-05}$	0.01875	2.025
$1/5 \cdot 2^{-6}$	$2.5 \cdot 10^{-06}$	0.01250	2.077	$2.9 \cdot 10^{-06}$	0.00937	2.012
$1/5 \cdot 2^{-7}$	$5.9 \cdot 10^{-07}$	0.00625	2.045	$7.2 \cdot 10^{-07}$	0.00468	2.006
$1/5 \cdot 2^{-8}$	$1.4 \cdot 10^{-07}$	0.00312	2.024	$1.7 \cdot 10^{-07}$	0.00234	2.003
$1/5 \cdot 2^{-9}$	$3.5 \cdot 10^{-08}$	0.00156	2.012	$4.4 \cdot 10^{-08}$	0.00117	2.001
$1/5 \cdot 2^{-10}$	$8.8 \cdot 10^{-09}$	0.00078	2.006	$1.1 \cdot 10^{-08}$	0.00058	2.000
$1/5 \cdot 2^{-11}$	$2.1 \cdot 10^{-09}$	0.00039	2.003	$2.7 \cdot 10^{-09}$	0.00029	2.000
$1/5 \cdot 2^{-12}$	$5.4 \cdot 10^{-10}$	0.00019	2.001	$6.9 \cdot 10^{-10}$	0.00014	2.000
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-10}$	0.00009	2.000	$1.7 \cdot 10^{-10}$	0.00007	2.000
$1/5 \cdot 2^{-14}$	$3.4 \cdot 10^{-11}$	0.00004		$4.3 \cdot 10^{-11}$	0.00003	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$3.3 \cdot 10^{-02}$	0.60000	2.733	$2.9 \cdot 10^{-02}$	0.60000	3.172
$1/5 \cdot 2^{-1}$	$5.0 \cdot 10^{-03}$	0.30000	2.444	$3.3 \cdot 10^{-03}$	0.30000	2.715
$1/5 \cdot 2^{-2}$	$9.3 \cdot 10^{-04}$	0.15000	2.264	$5.0 \cdot 10^{-04}$	0.10000	2.339
$1/5 \cdot 2^{-3}$	$1.9 \cdot 10^{-04}$	0.07500	2.140	$9.9 \cdot 10^{-05}$	0.05000	2.178
$1/5 \cdot 2^{-4}$	$4.3 \cdot 10^{-05}$	0.03750	2.071	$2.2 \cdot 10^{-05}$	0.02500	2.092
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-05}$	0.01875	2.036	$5.1 \cdot 10^{-06}$	0.01250	2.046
$1/5 \cdot 2^{-6}$	$2.5 \cdot 10^{-06}$	0.00937	2.018	$1.2 \cdot 10^{-06}$	0.00625	2.023
$1/5 \cdot 2^{-7}$	$6.2 \cdot 10^{-07}$	0.00468	2.009	$3.0 \cdot 10^{-07}$	0.00312	2.011
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-07}$	0.00234	2.004	$7.6 \cdot 10^{-08}$	0.00156	2.005
$1/5 \cdot 2^{-9}$	$3.8 \cdot 10^{-08}$	0.00117	2.002	$1.8 \cdot 10^{-08}$	0.00078	2.002
$1/5 \cdot 2^{-10}$	$9.7 \cdot 10^{-09}$	0.00058	2.001	$4.7 \cdot 10^{-09}$	0.00039	2.001
$1/5 \cdot 2^{-11}$	$2.4 \cdot 10^{-09}$	0.00029	2.000	$1.1 \cdot 10^{-09}$	0.00019	2.000
$1/5 \cdot 2^{-12}$	$6.0 \cdot 10^{-10}$	0.00014	2.000	$2.9 \cdot 10^{-10}$	0.00009	2.000
$1/5 \cdot 2^{-13}$	$1.5 \cdot 10^{-10}$	0.00007		$7.3 \cdot 10^{-11}$	0.00004	

Table 5.7: Explicit Euler, IDeC method on $[0, 1]$ for 4.1d.

h	δ	p	δ	p
$1/5$	1.9	1.001	$3.9 \cdot 10^{-01}$	1.820
$1/5 \cdot 2^{-1}$	$9.6 \cdot 10^{-01}$	1.010	$1.1 \cdot 10^{-01}$	1.953
$1/5 \cdot 2^{-2}$	$4.8 \cdot 10^{-01}$	1.007	$2.9 \cdot 10^{-02}$	1.984
$1/5 \cdot 2^{-3}$	$2.3 \cdot 10^{-01}$	1.004	$7.3 \cdot 10^{-03}$	1.993
$1/5 \cdot 2^{-4}$	$1.1 \cdot 10^{-01}$	1.002	$1.8 \cdot 10^{-03}$	1.997
$1/5 \cdot 2^{-5}$	$5.9 \cdot 10^{-02}$	1.001	$4.6 \cdot 10^{-04}$	1.998
$1/5 \cdot 2^{-6}$	$2.9 \cdot 10^{-02}$	1.000	$1.1 \cdot 10^{-04}$	1.999
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-02}$	1.000	$2.8 \cdot 10^{-05}$	1.999
$1/5 \cdot 2^{-8}$	$7.4 \cdot 10^{-03}$	1.000	$7.2 \cdot 10^{-06}$	1.999
$1/5 \cdot 2^{-9}$	$3.7 \cdot 10^{-03}$	1.000	$1.8 \cdot 10^{-06}$	1.999
$1/5 \cdot 2^{-10}$	$1.8 \cdot 10^{-03}$	1.000	$4.5 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-11}$	$9.2 \cdot 10^{-04}$	1.000	$1.1 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-12}$	$4.6 \cdot 10^{-04}$	0.999	$2.8 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-13}$	$2.3 \cdot 10^{-04}$	1.000	$7.0 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-14}$	$1.1 \cdot 10^{-04}$	1.000	$1.7 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-15}$	$5.8 \cdot 10^{-05}$		$4.4 \cdot 10^{-10}$	

h	δ	p	δ	p
$1/5$	$1.2 \cdot 10^{-01}$	3.694	$2.6 \cdot 10^{-02}$	5.222
$1/5 \cdot 2^{-1}$	$9.3 \cdot 10^{-03}$	3.235	$7.1 \cdot 10^{-04}$	1.568
$1/5 \cdot 2^{-2}$	$9.9 \cdot 10^{-04}$	2.874	$2.4 \cdot 10^{-04}$	2.076
$1/5 \cdot 2^{-3}$	$1.3 \cdot 10^{-04}$	2.913	$5.7 \cdot 10^{-05}$	2.063
$1/5 \cdot 2^{-4}$	$1.7 \cdot 10^{-05}$	2.897	$1.3 \cdot 10^{-05}$	2.033
$1/5 \cdot 2^{-5}$	$2.4 \cdot 10^{-06}$	2.838	$3.3 \cdot 10^{-06}$	2.016
$1/5 \cdot 2^{-6}$	$3.3 \cdot 10^{-07}$	2.732	$8.2 \cdot 10^{-07}$	2.007
$1/5 \cdot 2^{-7}$	$5.0 \cdot 10^{-08}$	2.582	$2.0 \cdot 10^{-07}$	2.003
$1/5 \cdot 2^{-8}$	$8.4 \cdot 10^{-09}$	2.414	$5.1 \cdot 10^{-08}$	2.001
$1/5 \cdot 2^{-9}$	$1.5 \cdot 10^{-09}$	2.262	$1.2 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-10}$	$3.3 \cdot 10^{-10}$	2.151	$3.2 \cdot 10^{-09}$	2.000
$1/5 \cdot 2^{-11}$	$7.4 \cdot 10^{-11}$	2.082	$8.0 \cdot 10^{-10}$	2.000
$1/5 \cdot 2^{-12}$	$1.7 \cdot 10^{-11}$	2.042	$2.0 \cdot 10^{-10}$	2.000
$1/5 \cdot 2^{-13}$	$4.2 \cdot 10^{-12}$	2.022	$5.0 \cdot 10^{-11}$	2.000
$1/5 \cdot 2^{-14}$	$1.0 \cdot 10^{-12}$		$1.2 \cdot 10^{-11}$	

h	δ	p	δ	p
$1/5$	$2.3 \cdot 10^{-02}$	4.652	$5.5 \cdot 10^{-03}$	3.272
$1/5 \cdot 2^{-1}$	$9.4 \cdot 10^{-04}$	2.332	$5.7 \cdot 10^{-04}$	3.165
$1/5 \cdot 2^{-2}$	$1.8 \cdot 10^{-04}$	1.958	$6.4 \cdot 10^{-05}$	2.891
$1/5 \cdot 2^{-3}$	$4.8 \cdot 10^{-05}$	1.967	$8.6 \cdot 10^{-06}$	2.658
$1/5 \cdot 2^{-4}$	$1.2 \cdot 10^{-05}$	1.983	$1.3 \cdot 10^{-06}$	2.462
$1/5 \cdot 2^{-5}$	$3.1 \cdot 10^{-06}$	1.992	$2.4 \cdot 10^{-07}$	2.297
$1/5 \cdot 2^{-6}$	$7.8 \cdot 10^{-07}$	1.996	$5.0 \cdot 10^{-08}$	2.174
$1/5 \cdot 2^{-7}$	$1.9 \cdot 10^{-07}$	1.998	$1.1 \cdot 10^{-08}$	2.095
$1/5 \cdot 2^{-8}$	$4.9 \cdot 10^{-08}$	1.999	$2.6 \cdot 10^{-09}$	2.050
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-08}$	1.999	$6.3 \cdot 10^{-10}$	2.025
$1/5 \cdot 2^{-10}$	$3.0 \cdot 10^{-09}$	1.999	$1.5 \cdot 10^{-10}$	2.013
$1/5 \cdot 2^{-11}$	$7.7 \cdot 10^{-10}$	1.999	$3.8 \cdot 10^{-11}$	2.006
$1/5 \cdot 2^{-12}$	$1.9 \cdot 10^{-10}$	1.999	$9.5 \cdot 10^{-12}$	2.003
$1/5 \cdot 2^{-13}$	$4.8 \cdot 10^{-11}$		$2.3 \cdot 10^{-12}$	

Table 5.8: Explicit Euler, IDeC method at $t = 0$ for 4.1d.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	1.6	0.80000	1.239	1.2	0.80000	2.777
$1/5 \cdot 2^{-1}$	$6.8 \cdot 10^{-01}$	0.70000	1.124	$1.8 \cdot 10^{-01}$	0.70000	2.280
$1/5 \cdot 2^{-2}$	$3.1 \cdot 10^{-01}$	0.70000	1.057	$3.8 \cdot 10^{-02}$	0.75000	2.146
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-01}$	0.70000	1.027	$8.7 \cdot 10^{-03}$	0.75000	2.069
$1/5 \cdot 2^{-4}$	$7.3 \cdot 10^{-02}$	0.70000	1.013	$2.0 \cdot 10^{-03}$	0.73750	2.034
$1/5 \cdot 2^{-5}$	$3.6 \cdot 10^{-02}$	0.70625	1.006	$5.1 \cdot 10^{-04}$	0.74375	2.016
$1/5 \cdot 2^{-6}$	$1.8 \cdot 10^{-02}$	0.70312	1.003	$1.2 \cdot 10^{-04}$	0.74062	2.008
$1/5 \cdot 2^{-7}$	$9.0 \cdot 10^{-03}$	0.70312	1.001	$3.1 \cdot 10^{-05}$	0.74062	2.004
$1/5 \cdot 2^{-8}$	$4.5 \cdot 10^{-03}$	0.70312	1.000	$7.8 \cdot 10^{-06}$	0.73984	2.002
$1/5 \cdot 2^{-9}$	$2.2 \cdot 10^{-03}$	0.70273	1.000	$1.9 \cdot 10^{-06}$	0.74023	2.001
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-03}$	0.70292	1.000	$4.8 \cdot 10^{-07}$	0.74003	2.000
$1/5 \cdot 2^{-11}$	$5.6 \cdot 10^{-04}$	0.70292	1.000	$1.2 \cdot 10^{-07}$	0.74003	2.000
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-04}$	0.70288	0.999	$3.0 \cdot 10^{-08}$	0.74008	2.000
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-04}$	0.70288	1.000	$7.6 \cdot 10^{-09}$	0.74006	2.000
$1/5 \cdot 2^{-14}$	$7.0 \cdot 10^{-05}$	0.70286	1.000	$1.9 \cdot 10^{-09}$	0.74006	2.000
$1/5 \cdot 2^{-15}$	$3.5 \cdot 10^{-05}$	0.70286		$4.7 \cdot 10^{-10}$	0.74005	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	1.1	0.80000	4.105	$7.7 \cdot 10^{-01}$	0.60000	5.099
$1/5 \cdot 2^{-1}$	$6.4 \cdot 10^{-02}$	0.80000	3.462	$2.2 \cdot 10^{-02}$	0.70000	4.629
$1/5 \cdot 2^{-2}$	$5.8 \cdot 10^{-03}$	0.75000	3.232	$9.1 \cdot 10^{-04}$	0.75000	4.308
$1/5 \cdot 2^{-3}$	$6.2 \cdot 10^{-04}$	0.75000	3.111	$4.6 \cdot 10^{-05}$	0.75000	4.148
$1/5 \cdot 2^{-4}$	$7.2 \cdot 10^{-05}$	0.75000	3.054	$2.5 \cdot 10^{-06}$	0.75000	4.072
$1/5 \cdot 2^{-5}$	$8.7 \cdot 10^{-06}$	0.75000	3.027	$1.5 \cdot 10^{-07}$	0.74375	4.036
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{-06}$	0.74687	3.013	$9.4 \cdot 10^{-09}$	0.74375	4.017
$1/5 \cdot 2^{-7}$	$1.3 \cdot 10^{-07}$	0.74687	3.006	$5.8 \cdot 10^{-10}$	0.74531	4.008
$1/5 \cdot 2^{-8}$	$1.6 \cdot 10^{-08}$	0.74765	3.003	$3.6 \cdot 10^{-11}$	0.74531	4.004
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-09}$	0.74726	3.001	$2.2 \cdot 10^{-12}$	0.74531	4.002
$1/5 \cdot 2^{-10}$	$2.5 \cdot 10^{-10}$	0.74746	3.000	$1.4 \cdot 10^{-13}$	0.74511	4.001
$1/5 \cdot 2^{-11}$	$3.2 \cdot 10^{-11}$	0.74736	3.000	$8.7 \cdot 10^{-15}$	0.74511	4.000
$1/5 \cdot 2^{-12}$	$4.0 \cdot 10^{-12}$	0.74736	3.000	$5.4 \cdot 10^{-16}$	0.74516	4.000
$1/5 \cdot 2^{-13}$	$5.0 \cdot 10^{-13}$	0.74738	3.000	$3.4 \cdot 10^{-17}$	0.74516	4.000
$1/5 \cdot 2^{-14}$	$6.2 \cdot 10^{-14}$	0.74738		$2.1 \cdot 10^{-18}$	0.74515	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$4.8 \cdot 10^{-01}$	0.60000	4.385	$4.6 \cdot 10^{-01}$	-0.0000	3.764
$1/5 \cdot 2^{-1}$	$2.3 \cdot 10^{-02}$	0.40000	4.297	$3.4 \cdot 10^{-02}$	0.40000	4.461
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-03}$	0.20000	4.229	$1.5 \cdot 10^{-03}$	0.20000	4.245
$1/5 \cdot 2^{-3}$	$6.2 \cdot 10^{-05}$	0.10000	4.112	$8.2 \cdot 10^{-05}$	0.10000	4.121
$1/5 \cdot 2^{-4}$	$3.6 \cdot 10^{-06}$	0.05000	4.053	$4.7 \cdot 10^{-06}$	0.05000	4.060
$1/5 \cdot 2^{-5}$	$2.1 \cdot 10^{-07}$	0.02500	4.025	$2.8 \cdot 10^{-07}$	0.02500	4.030
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-08}$	0.01250	4.012	$1.7 \cdot 10^{-08}$	0.01250	4.015
$1/5 \cdot 2^{-7}$	$8.2 \cdot 10^{-10}$	0.00625	4.006	$1.0 \cdot 10^{-09}$	0.00625	4.007
$1/5 \cdot 2^{-8}$	$5.1 \cdot 10^{-11}$	0.00312	4.003	$6.6 \cdot 10^{-11}$	0.00312	4.003
$1/5 \cdot 2^{-9}$	$3.2 \cdot 10^{-12}$	0.00156	4.001	$4.1 \cdot 10^{-12}$	0.00156	4.001
$1/5 \cdot 2^{-10}$	$2.0 \cdot 10^{-13}$	0.00078	4.000	$2.5 \cdot 10^{-13}$	0.00078	4.000
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-14}$	0.00039	4.000	$1.6 \cdot 10^{-14}$	0.00039	4.000
$1/5 \cdot 2^{-12}$	$7.8 \cdot 10^{-16}$	0.00019	3.999	$1.0 \cdot 10^{-15}$	0.00019	4.000
$1/5 \cdot 2^{-13}$	$4.8 \cdot 10^{-17}$	0.00009		$6.3 \cdot 10^{-17}$	0.00009	

Table 5.9: Explicit Euler, IDeC method on $[0, 1]$ for 4.1e.

h	δ	p	δ	p
$1/5$	$8.5 \cdot 10^{-02}$	0.797	$1.9 \cdot 10^{-01}$	3.752
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-02}$	1.045	$1.4 \cdot 10^{-02}$	1.885
$1/5 \cdot 2^{-2}$	$2.3 \cdot 10^{-02}$	1.035	$3.8 \cdot 10^{-03}$	2.003
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-02}$	1.019	$9.7 \cdot 10^{-04}$	2.011
$1/5 \cdot 2^{-4}$	$5.7 \cdot 10^{-03}$	1.009	$2.4 \cdot 10^{-04}$	2.007
$1/5 \cdot 2^{-5}$	$2.8 \cdot 10^{-03}$	1.005	$5.9 \cdot 10^{-05}$	2.004
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-03}$	1.002	$1.4 \cdot 10^{-05}$	2.002
$1/5 \cdot 2^{-7}$	$7.0 \cdot 10^{-04}$	1.001	$3.7 \cdot 10^{-06}$	2.001
$1/5 \cdot 2^{-8}$	$3.5 \cdot 10^{-04}$	1.000	$9.3 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-9}$	$1.7 \cdot 10^{-04}$	1.000	$2.3 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-10}$	$8.8 \cdot 10^{-05}$	1.000	$5.8 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-11}$	$4.4 \cdot 10^{-05}$	1.000	$1.4 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-05}$	0.999	$3.6 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-13}$	$1.1 \cdot 10^{-05}$	1.000	$9.0 \cdot 10^{-10}$	2.000
$1/5 \cdot 2^{-14}$	$5.5 \cdot 10^{-06}$	1.000	$2.2 \cdot 10^{-10}$	2.000
$1/5 \cdot 2^{-15}$	$2.7 \cdot 10^{-06}$		$5.6 \cdot 10^{-11}$	

h	δ	p	δ	p
$1/5$	$2.7 \cdot 10^{-01}$	5.068	$1.4 \cdot 10^{-01}$	5.598
$1/5 \cdot 2^{-1}$	$8.2 \cdot 10^{-03}$	4.558	$2.9 \cdot 10^{-03}$	5.159
$1/5 \cdot 2^{-2}$	$3.4 \cdot 10^{-04}$	3.439	$8.2 \cdot 10^{-05}$	4.850
$1/5 \cdot 2^{-3}$	$3.2 \cdot 10^{-05}$	2.881	$2.8 \cdot 10^{-06}$	4.596
$1/5 \cdot 2^{-4}$	$4.3 \cdot 10^{-06}$	2.953	$1.1 \cdot 10^{-07}$	4.385
$1/5 \cdot 2^{-5}$	$5.6 \cdot 10^{-07}$	2.979	$5.6 \cdot 10^{-09}$	4.228
$1/5 \cdot 2^{-6}$	$7.1 \cdot 10^{-08}$	2.990	$3.0 \cdot 10^{-10}$	4.126
$1/5 \cdot 2^{-7}$	$9.0 \cdot 10^{-09}$	2.995	$1.7 \cdot 10^{-11}$	4.066
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-09}$	2.997	$1.0 \cdot 10^{-12}$	4.034
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-10}$	2.998	$6.2 \cdot 10^{-14}$	4.017
$1/5 \cdot 2^{-10}$	$1.7 \cdot 10^{-11}$	2.999	$3.8 \cdot 10^{-15}$	4.008
$1/5 \cdot 2^{-11}$	$2.2 \cdot 10^{-12}$	2.999	$2.4 \cdot 10^{-16}$	4.004
$1/5 \cdot 2^{-12}$	$2.7 \cdot 10^{-13}$	2.999	$1.4 \cdot 10^{-17}$	4.002
$1/5 \cdot 2^{-13}$	$3.4 \cdot 10^{-14}$	2.999	$9.3 \cdot 10^{-19}$	4.001
$1/5 \cdot 2^{-14}$	$4.3 \cdot 10^{-15}$		$5.8 \cdot 10^{-20}$	

h	δ	p	δ	p
$1/5$	$1.1 \cdot 10^{-01}$	4.102	$4.6 \cdot 10^{-01}$	4.804
$1/5 \cdot 2^{-1}$	$6.7 \cdot 10^{-03}$	4.163	$1.6 \cdot 10^{-02}$	4.408
$1/5 \cdot 2^{-2}$	$3.7 \cdot 10^{-04}$	4.101	$7.8 \cdot 10^{-04}$	4.205
$1/5 \cdot 2^{-3}$	$2.1 \cdot 10^{-05}$	4.054	$4.2 \cdot 10^{-05}$	4.103
$1/5 \cdot 2^{-4}$	$1.3 \cdot 10^{-06}$	4.028	$2.4 \cdot 10^{-06}$	4.051
$1/5 \cdot 2^{-5}$	$8.0 \cdot 10^{-08}$	4.014	$1.4 \cdot 10^{-07}$	4.025
$1/5 \cdot 2^{-6}$	$4.9 \cdot 10^{-09}$	4.007	$9.1 \cdot 10^{-09}$	4.012
$1/5 \cdot 2^{-7}$	$3.0 \cdot 10^{-10}$	4.003	$5.6 \cdot 10^{-10}$	4.006
$1/5 \cdot 2^{-8}$	$1.9 \cdot 10^{-11}$	4.001	$3.5 \cdot 10^{-11}$	4.003
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-12}$	4.000	$2.2 \cdot 10^{-12}$	4.001
$1/5 \cdot 2^{-10}$	$7.5 \cdot 10^{-14}$	4.000	$1.3 \cdot 10^{-13}$	4.000
$1/5 \cdot 2^{-11}$	$4.6 \cdot 10^{-15}$	4.000	$8.5 \cdot 10^{-15}$	4.000
$1/5 \cdot 2^{-12}$	$2.9 \cdot 10^{-16}$	3.999	$5.3 \cdot 10^{-16}$	3.999
$1/5 \cdot 2^{-13}$	$1.8 \cdot 10^{-17}$		$3.3 \cdot 10^{-17}$	

Table 5.10: Explicit Euler, IDeC method at $t = 0$ for 4.1e.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.3\cdot10^{+03}$	-0.0000	8.481	$1.0\cdot10^{+05}$	-0.0000	9.779
$1/5\cdot2^{-1}$	3.8	0.90000	1.627	$1.1\cdot10^{+02}$	-0.0000	7.359
$1/5\cdot2^{-2}$	1.2	0.90000	1.192	$7.2\cdot10^{-01}$	0.95000	2.603
$1/5\cdot2^{-3}$	$5.4\cdot10^{-01}$	0.90000	1.083	$1.1\cdot10^{-01}$	0.92500	2.272
$1/5\cdot2^{-4}$	$2.5\cdot10^{-01}$	0.90000	1.039	$2.4\cdot10^{-02}$	0.92500	2.124
$1/5\cdot2^{-5}$	$1.2\cdot10^{-01}$	0.90000	1.018	$5.6\cdot10^{-03}$	0.91875	2.058
$1/5\cdot2^{-6}$	$6.1\cdot10^{-02}$	0.89687	1.009	$1.3\cdot10^{-03}$	0.92187	2.028
$1/5\cdot2^{-7}$	$3.0\cdot10^{-02}$	0.89687	1.004	$3.3\cdot10^{-04}$	0.92031	2.014
$1/5\cdot2^{-8}$	$1.5\cdot10^{-02}$	0.89687	1.002	$8.2\cdot10^{-05}$	0.92031	2.007
$1/5\cdot2^{-9}$	$7.5\cdot10^{-03}$	0.89648	1.001	$2.0\cdot10^{-05}$	0.91992	2.003
$1/5\cdot2^{-10}$	$3.7\cdot10^{-03}$	0.89648	1.000	$5.1\cdot10^{-06}$	0.92011	2.001
$1/5\cdot2^{-11}$	$1.8\cdot10^{-03}$	0.89638	1.000	$1.2\cdot10^{-06}$	0.92001	2.000
$1/5\cdot2^{-12}$	$9.4\cdot10^{-04}$	0.89643	1.000	$3.2\cdot10^{-07}$	0.92001	2.000
$1/5\cdot2^{-13}$	$4.7\cdot10^{-04}$	0.89641	1.000	$8.0\cdot10^{-08}$	0.92001	2.000
$1/5\cdot2^{-14}$	$2.3\cdot10^{-04}$	0.89641	1.000	$2.0\cdot10^{-08}$	0.92001	2.000
$1/5\cdot2^{-15}$	$1.1\cdot10^{-04}$	0.89641		$5.0\cdot10^{-09}$	0.92002	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$8.0\cdot10^{+06}$	-0.0000	9.718	$6.1\cdot10^{+08}$	-0.0000	9.715
$1/5\cdot2^{-1}$	$9.5\cdot10^{+03}$	-0.0000	13.779	$7.3\cdot10^{+05}$	-0.0000	18.084
$1/5\cdot2^{-2}$	$6.8\cdot10^{-01}$	0.95000	4.133	2.6	0.00000	7.506
$1/5\cdot2^{-3}$	$3.8\cdot10^{-02}$	0.92500	3.480	$1.4\cdot10^{-02}$	0.92500	4.677
$1/5\cdot2^{-4}$	$3.4\cdot10^{-03}$	0.92500	3.217	$5.6\cdot10^{-04}$	0.93750	4.322
$1/5\cdot2^{-5}$	$3.7\cdot10^{-04}$	0.93125	3.105	$2.8\cdot10^{-05}$	0.93125	4.153
$1/5\cdot2^{-6}$	$4.3\cdot10^{-05}$	0.92812	3.052	$1.6\cdot10^{-06}$	0.93125	4.075
$1/5\cdot2^{-7}$	$5.2\cdot10^{-06}$	0.92812	3.025	$9.4\cdot10^{-08}$	0.93281	4.037
$1/5\cdot2^{-8}$	$6.4\cdot10^{-07}$	0.92812	3.012	$5.7\cdot10^{-09}$	0.93203	4.018
$1/5\cdot2^{-9}$	$7.9\cdot10^{-08}$	0.92812	3.006	$3.5\cdot10^{-10}$	0.93203	4.009
$1/5\cdot2^{-10}$	$9.9\cdot10^{-09}$	0.92812	3.003	$2.2\cdot10^{-11}$	0.93183	4.004
$1/5\cdot2^{-11}$	$1.2\cdot10^{-09}$	0.92802	3.001	$1.3\cdot10^{-12}$	0.93183	4.002
$1/5\cdot2^{-12}$	$1.5\cdot10^{-10}$	0.92802	3.000	$8.6\cdot10^{-14}$	0.93188	4.000
$1/5\cdot2^{-13}$	$1.9\cdot10^{-11}$	0.92805	3.000	$5.3\cdot10^{-15}$	0.93186	4.000
$1/5\cdot2^{-14}$	$2.4\cdot10^{-12}$	0.92803		$3.3\cdot10^{-16}$	0.93186	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$4.7\cdot10^{+10}$	-0.0000	9.715	$3.6\cdot10^{+12}$	-0.0000	9.715
$1/5\cdot2^{-1}$	$5.6\cdot10^{+07}$	-0.0000	18.084	$4.3\cdot10^{+09}$	-0.0000	18.084
$1/5\cdot2^{-2}$	$2.0\cdot10^{+02}$	0.00000	9.340	$1.5\cdot10^{+04}$	0.00000	9.340
$1/5\cdot2^{-3}$	$3.1\cdot10^{-01}$	0.00000	5.812	$2.4\cdot10^{+01}$	0.00000	5.812
$1/5\cdot2^{-4}$	$5.5\cdot10^{-03}$	-0.0000	6.000	$4.2\cdot10^{-01}$	-0.0000	6.000
$1/5\cdot2^{-5}$	$8.7\cdot10^{-05}$	-0.0000	6.003	$6.6\cdot10^{-03}$	-0.0000	6.003
$1/5\cdot2^{-6}$	$1.3\cdot10^{-06}$	0.00000	6.001	$1.0\cdot10^{-04}$	0.00000	6.001
$1/5\cdot2^{-7}$	$2.1\cdot10^{-08}$	0.00000	6.000	$1.6\cdot10^{-06}$	0.00000	6.000
$1/5\cdot2^{-8}$	$3.3\cdot10^{-10}$	-0.0000	6.000	$2.5\cdot10^{-08}$	-0.0000	6.000
$1/5\cdot2^{-9}$	$5.1\cdot10^{-12}$	-0.0000	6.000	$3.9\cdot10^{-10}$	-0.0000	6.000
$1/5\cdot2^{-10}$	$8.0\cdot10^{-14}$	0.00000	5.589	$6.2\cdot10^{-12}$	0.00000	6.000
$1/5\cdot2^{-11}$	$1.6\cdot10^{-15}$	0.93359	5.003	$9.6\cdot10^{-14}$	0.00000	6.000
$1/5\cdot2^{-12}$	$5.2\cdot10^{-17}$	0.93359	5.001	$1.5\cdot10^{-15}$	-0.0000	5.999
$1/5\cdot2^{-13}$	$1.6\cdot10^{-18}$	0.93371		$2.3\cdot10^{-17}$	-0.0000	

Table 5.11: Explicit Euler, IDeC method on $[0,1]$ for 4.1f.

h	δ	p	δ	p
$1/5$	$1.3 \cdot 10^{+03}$	12.253	$1.0 \cdot 10^{+05}$	9.779
$1/5 \cdot 2^{-1}$	$2.8 \cdot 10^{-01}$	1.021	$1.1 \cdot 10^{+02}$	14.164
$1/5 \cdot 2^{-2}$	$1.3 \cdot 10^{-01}$	1.011	$6.5 \cdot 10^{-03}$	2.030
$1/5 \cdot 2^{-3}$	$6.8 \cdot 10^{-02}$	1.005	$1.5 \cdot 10^{-03}$	2.008
$1/5 \cdot 2^{-4}$	$3.4 \cdot 10^{-02}$	1.002	$3.9 \cdot 10^{-04}$	2.004
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-02}$	1.001	$9.8 \cdot 10^{-05}$	2.002
$1/5 \cdot 2^{-6}$	$8.5 \cdot 10^{-03}$	1.000	$2.4 \cdot 10^{-05}$	2.001
$1/5 \cdot 2^{-7}$	$4.2 \cdot 10^{-03}$	1.000	$6.1 \cdot 10^{-06}$	2.000
$1/5 \cdot 2^{-8}$	$2.1 \cdot 10^{-03}$	1.000	$1.5 \cdot 10^{-06}$	2.000
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-03}$	1.000	$3.8 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-10}$	$5.3 \cdot 10^{-04}$	1.000	$9.6 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-04}$	1.000	$2.4 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-04}$	0.999	$6.0 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-13}$	$6.6 \cdot 10^{-05}$	1.000	$1.5 \cdot 10^{-09}$	2.000
$1/5 \cdot 2^{-14}$	$3.3 \cdot 10^{-05}$	1.000	$3.7 \cdot 10^{-10}$	2.000
$1/5 \cdot 2^{-15}$	$1.6 \cdot 10^{-05}$		$9.3 \cdot 10^{-11}$	

h	δ	p	δ	p
$1/5$	$8.0 \cdot 10^{+06}$	9.718	$6.1 \cdot 10^{+08}$	9.715
$1/5 \cdot 2^{-1}$	$9.5 \cdot 10^{+03}$	18.100	$7.3 \cdot 10^{+05}$	18.084
$1/5 \cdot 2^{-2}$	$3.4 \cdot 10^{-02}$	9.318	2.6	9.340
$1/5 \cdot 2^{-3}$	$5.3 \cdot 10^{-05}$	3.910	$4.0 \cdot 10^{-03}$	5.812
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-06}$	2.972	$7.2 \cdot 10^{-05}$	6.000
$1/5 \cdot 2^{-5}$	$4.5 \cdot 10^{-07}$	2.998	$1.1 \cdot 10^{-06}$	6.003
$1/5 \cdot 2^{-6}$	$5.6 \cdot 10^{-08}$	3.000	$1.7 \cdot 10^{-08}$	6.001
$1/5 \cdot 2^{-7}$	$7.0 \cdot 10^{-09}$	3.000	$2.7 \cdot 10^{-10}$	6.000
$1/5 \cdot 2^{-8}$	$8.8 \cdot 10^{-10}$	3.000	$4.3 \cdot 10^{-12}$	6.000
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-10}$	3.000	$6.7 \cdot 10^{-14}$	5.408
$1/5 \cdot 2^{-10}$	$1.3 \cdot 10^{-11}$	3.000	$1.5 \cdot 10^{-15}$	3.929
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-12}$	3.000	$1.0 \cdot 10^{-16}$	3.983
$1/5 \cdot 2^{-12}$	$2.1 \cdot 10^{-13}$	2.999	$6.5 \cdot 10^{-18}$	3.995
$1/5 \cdot 2^{-13}$	$2.6 \cdot 10^{-14}$	3.000	$4.1 \cdot 10^{-19}$	3.998
$1/5 \cdot 2^{-14}$	$3.3 \cdot 10^{-15}$		$2.5 \cdot 10^{-20}$	

h	δ	p	δ	p
$1/5$	$4.7 \cdot 10^{+10}$	9.715	$3.6 \cdot 10^{+12}$	9.715
$1/5 \cdot 2^{-1}$	$5.6 \cdot 10^{+07}$	18.084	$4.3 \cdot 10^{+09}$	18.084
$1/5 \cdot 2^{-2}$	$2.0 \cdot 10^{+02}$	9.340	$1.5 \cdot 10^{+04}$	9.340
$1/5 \cdot 2^{-3}$	$3.1 \cdot 10^{-01}$	5.812	$2.4 \cdot 10^{+01}$	5.812
$1/5 \cdot 2^{-4}$	$5.5 \cdot 10^{-03}$	6.000	$4.2 \cdot 10^{-01}$	6.000
$1/5 \cdot 2^{-5}$	$8.7 \cdot 10^{-05}$	6.003	$6.6 \cdot 10^{-03}$	6.003
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-06}$	6.001	$1.0 \cdot 10^{-04}$	6.001
$1/5 \cdot 2^{-7}$	$2.1 \cdot 10^{-08}$	6.000	$1.6 \cdot 10^{-06}$	6.000
$1/5 \cdot 2^{-8}$	$3.3 \cdot 10^{-10}$	6.000	$2.5 \cdot 10^{-08}$	6.000
$1/5 \cdot 2^{-9}$	$5.1 \cdot 10^{-12}$	6.000	$3.9 \cdot 10^{-10}$	6.000
$1/5 \cdot 2^{-10}$	$8.0 \cdot 10^{-14}$	6.000	$6.2 \cdot 10^{-12}$	6.000
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-15}$	6.000	$9.6 \cdot 10^{-14}$	6.000
$1/5 \cdot 2^{-12}$	$1.9 \cdot 10^{-17}$	5.999	$1.5 \cdot 10^{-15}$	5.999
$1/5 \cdot 2^{-13}$	$3.0 \cdot 10^{-19}$		$2.3 \cdot 10^{-17}$	

Table 5.12: Explicit Euler, IDeC method at $t = 0$ for 4.1f.

5.1.3 Asymptotic Error Expansions

In this section, some of the results from §5.1.2 will be explained by explicitly computing the asymptotic error expansions for test problems with a simple structure (cf. §4.2.3). Only for problems with constant coefficient matrix M we were able to compute the solutions of the variational equations using MAPLE.

It is obvious from the methods' definitions that the general form of the variational equations for the explicit Euler rule for terminal value problems (5.1), where the solution advances from right to left, is the same as for the implicit Euler for problems posed at $t = 0$ and integration from left to right. Clearly, the boundary condition is posed as $e_*(1) = 0$.

For problem 4.1a, the task of computing the error expansion is obviously obsolete since the problem is solved exactly.

For 4.1b we obtain[14]

$$
\begin{aligned}
e_1(t) &= \left(-3t^2 + 3t^3, -9t^2 + 9t^3\right), \\
e_2(t) &= \left(2t - 9t^2 + 7t^3, 6t - 27t^2 + 21t^3\right), \\
e_3(t) &= \left(6t - 21t^2 + 15t^3, 18t - 63t^2 + 45t^3\right), \\
e_4(t) &= \left(14t - 45t^2 + 31t^3, 42t - 135t^2 + 189t^3\right), \\
e_5(t) &= \left(30t - 93t^2 + 63t^3, 90t - 279t^2 + 189t^3\right), \\
e_6(t) &= \left(62t - 189t^2 + 127t^3, 186t - 567t^2 + 381t^3\right).
\end{aligned}
$$

Accordingly, IDeC shows the classical order sequence $O(h), \ldots, O(h^5)$. The fact that even a convergence order $O(h^6)$ is observed is no contradiction to the theory.

For example 4.1c, the error expansion also proves sufficiently long to guarantee the successive improvement of the accuracy of the IDeC iterates up to $O(h^5)$. Namely,

$$
\begin{aligned}
e_1(t) &= (-45t^9 + 45t^{10}, -450t^9 + 450t^{10}), \\
e_2(t) &= (870t^8 - 2025t^9 + 1155t^{10}, 8700t^8 - 20250t^9 + 11550t^{10}), \\
e_3(t) &= (-9450t^7 + 39150t^8 - 51975t^9 + 22275t^{10}, \ldots \\
&\quad -94500t^7 + 391500t^8 - 519750t^9 + 222750t^{10}), \\
e_4(t) &= (63273t^6 - 425250t^7 + 1004850t^8 - 1002375t^9 \\
&\quad + 359502t^{10}, 632730t^6 - 4252500t^7 \\
&\quad + 10048500t^8 - 10023750t^9 + 3595020t^{10}), \\
e_5(t) &= (-269325t^5 + 2847285t^6 - 10914750t^7 + 19379250t^8 \\
&\quad - 16177590t^9 + 5135130t^{10}, -2693250t^5 + 28472850t^6 \\
&\quad - 109147500t^7 + 193792500t^8 - 161775900t^9 + 51351300t^{10}), \\
e_6(t) &= (723680t^4 - 12119625t^5 + 73080315t^6 - 210498750t^7 \\
&\quad + 312766740t^8 - 231080850t^9 + 67128490t^{10}, 7236800t^4
\end{aligned}
$$

---

[14]Note that no higher order terms are omitted from the solutions of the variational equations.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.5\cdot10^{+02}$	-0.0000	1.980	$6.3\cdot10^{+02}$	0.20000	2.518
$1/5\cdot2^{-1}$	$3.8\cdot10^{+01}$	0.20000	1.976	$1.1\cdot10^{+02}$	-0.0000	3.276
$1/5\cdot2^{-2}$	9.8	0.30000	1.545	$1.1\cdot10^{+01}$	0.25000	2.841
$1/5\cdot2^{-3}$	3.3	0.62500	1.185	1.6	0.30000	2.440
$1/5\cdot2^{-4}$	1.4	0.62500	1.092	$2.9\cdot10^{-01}$	0.31250	2.225
$1/5\cdot2^{-5}$	$6.9\cdot10^{-01}$	0.63125	1.045	$6.3\cdot10^{-02}$	0.31875	2.088
$1/5\cdot2^{-6}$	$3.3\cdot10^{-01}$	0.63437	1.022	$1.4\cdot10^{-02}$	0.57187	2.044
$1/5\cdot2^{-7}$	$1.6\cdot10^{-01}$	0.63437	1.011	$3.5\cdot10^{-03}$	0.57343	2.021
$1/5\cdot2^{-8}$	$8.1\cdot10^{-02}$	0.63593	1.005	$8.8\cdot10^{-04}$	0.57500	2.010
$1/5\cdot2^{-9}$	$4.0\cdot10^{-02}$	0.63593	1.002	$2.1\cdot10^{-04}$	0.57578	2.005
$1/5\cdot2^{-10}$	$2.0\cdot10^{-02}$	0.63593	1.001	$5.4\cdot10^{-05}$	0.57597	2.002
$1/5\cdot2^{-11}$	$1.0\cdot10^{-02}$	0.63603	1.000	$1.3\cdot10^{-05}$	0.57617	2.001
$1/5\cdot2^{-12}$	$5.0\cdot10^{-03}$	0.63603	1.000	$3.4\cdot10^{-06}$	0.57622	2.000
$1/5\cdot2^{-13}$	$2.5\cdot10^{-03}$	0.63603	1.000	$8.5\cdot10^{-07}$	0.57622	2.000
$1/5\cdot2^{-14}$	$1.2\cdot10^{-03}$	0.63604	1.000	$2.1\cdot10^{-07}$	0.57624	2.000
$1/5\cdot2^{-15}$	$6.3\cdot10^{-04}$	0.63604		$5.3\cdot10^{-08}$	0.57625	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$5.2\cdot10^{+03}$	-0.0000	4.338	$1.5\cdot10^{+04}$	-0.0000	6.121
$1/5\cdot2^{-1}$	$2.5\cdot10^{+02}$	-0.0000	4.695	$2.2\cdot10^{+02}$	0.20000	4.237
$1/5\cdot2^{-2}$	9.9	0.15000	3.847	$1.2\cdot10^{+01}$	0.00000	5.229
$1/5\cdot2^{-3}$	$6.9\cdot10^{-01}$	0.22500	3.515	$3.2\cdot10^{-01}$	0.35000	4.592
$1/5\cdot2^{-4}$	$6.0\cdot10^{-02}$	0.47500	3.246	$1.3\cdot10^{-02}$	0.40000	4.314
$1/5\cdot2^{-5}$	$6.3\cdot10^{-03}$	0.49375	3.125	$6.6\cdot10^{-04}$	0.41875	3.846
$1/5\cdot2^{-6}$	$7.3\cdot10^{-04}$	0.50312	3.062	$4.6\cdot10^{-05}$	0.01250	3.009
$1/5\cdot2^{-7}$	$8.7\cdot10^{-05}$	0.50468	3.031	$5.7\cdot10^{-06}$	0.00625	2.990
$1/5\cdot2^{-8}$	$1.0\cdot10^{-05}$	0.50703	3.015	$7.2\cdot10^{-07}$	0.00312	2.994
$1/5\cdot2^{-9}$	$1.3\cdot10^{-06}$	0.50781	3.007	$9.1\cdot10^{-08}$	0.00156	2.997
$1/5\cdot2^{-10}$	$1.6\cdot10^{-07}$	0.50800	3.003	$1.1\cdot10^{-08}$	0.00078	2.999
$1/5\cdot2^{-11}$	$2.0\cdot10^{-08}$	0.50830	3.002	$1.4\cdot10^{-09}$	0.00039	2.999
$1/5\cdot2^{-12}$	$2.5\cdot10^{-09}$	0.50839	3.000	$1.7\cdot10^{-10}$	0.00019	2.999
$1/5\cdot2^{-13}$	$3.2\cdot10^{-10}$	0.50844	3.000	$2.2\cdot10^{-11}$	0.00009	2.999
$1/5\cdot2^{-14}$	$4.0\cdot10^{-11}$	0.50848		$2.7\cdot10^{-12}$	0.00004	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.2\cdot10^{+05}$	-0.0000	6.782	$3.5\cdot10^{+05}$	-0.0000	6.786
$1/5\cdot2^{-1}$	$1.0\cdot10^{+03}$	-0.0000	6.945	$3.1\cdot10^{+03}$	-0.0000	7.523
$1/5\cdot2^{-2}$	8.8	0.15000	5.920	$1.7\cdot10^{+01}$	0.00000	8.059
$1/5\cdot2^{-3}$	$1.4\cdot10^{-01}$	0.27500	4.779	$6.4\cdot10^{-02}$	0.20000	3.905
$1/5\cdot2^{-4}$	$5.3\cdot10^{-03}$	0.05000	3.480	$4.3\cdot10^{-03}$	0.03750	3.455
$1/5\cdot2^{-5}$	$4.7\cdot10^{-04}$	0.02500	3.132	$3.9\cdot10^{-04}$	0.01875	3.132
$1/5\cdot2^{-6}$	$5.4\cdot10^{-05}$	0.01250	3.031	$4.5\cdot10^{-05}$	0.00937	3.037
$1/5\cdot2^{-7}$	$6.6\cdot10^{-06}$	0.00625	3.008	$5.4\cdot10^{-06}$	0.00468	3.011
$1/5\cdot2^{-8}$	$8.3\cdot10^{-07}$	0.00312	3.002	$6.8\cdot10^{-07}$	0.00234	3.003
$1/5\cdot2^{-9}$	$1.0\cdot10^{-07}$	0.00156	3.000	$8.4\cdot10^{-08}$	0.00117	3.001
$1/5\cdot2^{-10}$	$1.2\cdot10^{-08}$	0.00078	3.000	$1.0\cdot10^{-08}$	0.00058	3.000
$1/5\cdot2^{-11}$	$1.6\cdot10^{-09}$	0.00039	3.000	$1.3\cdot10^{-09}$	0.00029	3.000
$1/5\cdot2^{-12}$	$2.0\cdot10^{-10}$	0.00019	2.999	$1.6\cdot10^{-10}$	0.00014	2.999
$1/5\cdot2^{-13}$	$2.5\cdot10^{-11}$	0.00009		$2.0\cdot10^{-11}$	0.00007	

Table 5.13: Explicit Euler, IDeC method on $[0,1]$ for 4.2a.

h	δ	p	δ	p
$1/5$	$1.5 \cdot 10^{+02}$	2.958	$5.7 \cdot 10^{+02}$	2.368
$1/5 \cdot 2^{-1}$	$1.9 \cdot 10^{+01}$	3.836	$1.1 \cdot 10^{+02}$	5.014
$1/5 \cdot 2^{-2}$	1.3	3.958	3.4	4.673
$1/5 \cdot 2^{-3}$	$8.8 \cdot 10^{-02}$	0.647	$1.3 \cdot 10^{-01}$	4.240
$1/5 \cdot 2^{-4}$	$5.6 \cdot 10^{-02}$	0.925	$7.1 \cdot 10^{-03}$	2.315
$1/5 \cdot 2^{-5}$	$2.9 \cdot 10^{-02}$	0.974	$1.4 \cdot 10^{-03}$	2.065
$1/5 \cdot 2^{-6}$	$1.5 \cdot 10^{-02}$	0.990	$3.4 \cdot 10^{-04}$	2.025
$1/5 \cdot 2^{-7}$	$7.5 \cdot 10^{-03}$	0.995	$8.4 \cdot 10^{-05}$	2.009
$1/5 \cdot 2^{-8}$	$3.8 \cdot 10^{-03}$	0.997	$2.0 \cdot 10^{-05}$	2.004
$1/5 \cdot 2^{-9}$	$1.9 \cdot 10^{-03}$	0.999	$5.2 \cdot 10^{-06}$	2.001
$1/5 \cdot 2^{-10}$	$9.5 \cdot 10^{-04}$	0.999	$1.2 \cdot 10^{-06}$	2.000
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-04}$	0.999	$3.2 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-12}$	$2.3 \cdot 10^{-04}$	0.999	$8.1 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-13}$	$1.1 \cdot 10^{-04}$	0.999	$2.0 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-14}$	$5.9 \cdot 10^{-05}$	0.999	$5.0 \cdot 10^{-09}$	2.000
$1/5 \cdot 2^{-15}$	$2.9 \cdot 10^{-05}$		$1.2 \cdot 10^{-09}$	

h	δ	p	δ	p
$1/5$	$5.2 \cdot 10^{+03}$	4.338	$1.5 \cdot 10^{+04}$	7.958
$1/5 \cdot 2^{-1}$	$2.5 \cdot 10^{+02}$	6.330	$6.3 \cdot 10^{+01}$	2.399
$1/5 \cdot 2^{-2}$	3.2	4.366	$1.2 \cdot 10^{+01}$	7.255
$1/5 \cdot 2^{-3}$	$1.5 \cdot 10^{-01}$	4.940	$7.8 \cdot 10^{-02}$	5.844
$1/5 \cdot 2^{-4}$	$5.0 \cdot 10^{-03}$	3.741	$1.3 \cdot 10^{-03}$	3.728
$1/5 \cdot 2^{-5}$	$3.7 \cdot 10^{-04}$	3.210	$1.0 \cdot 10^{-04}$	3.430
$1/5 \cdot 2^{-6}$	$4.0 \cdot 10^{-05}$	3.130	$9.5 \cdot 10^{-06}$	2.827
$1/5 \cdot 2^{-7}$	$4.6 \cdot 10^{-06}$	3.073	$1.3 \cdot 10^{-06}$	2.954
$1/5 \cdot 2^{-8}$	$5.5 \cdot 10^{-07}$	3.039	$1.7 \cdot 10^{-07}$	2.986
$1/5 \cdot 2^{-9}$	$6.7 \cdot 10^{-08}$	3.020	$2.2 \cdot 10^{-08}$	2.995
$1/5 \cdot 2^{-10}$	$8.3 \cdot 10^{-09}$	3.010	$2.7 \cdot 10^{-09}$	2.998
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-09}$	3.005	$3.4 \cdot 10^{-10}$	2.999
$1/5 \cdot 2^{-12}$	$1.2 \cdot 10^{-10}$	3.002	$4.3 \cdot 10^{-11}$	2.999
$1/5 \cdot 2^{-13}$	$1.6 \cdot 10^{-11}$	3.001	$5.3 \cdot 10^{-12}$	2.999
$1/5 \cdot 2^{-14}$	$2.0 \cdot 10^{-12}$		$6.7 \cdot 10^{-13}$	

h	δ	p	δ	p
$1/5$	$1.2 \cdot 10^{+05}$	6.782	$3.5 \cdot 10^{+05}$	6.786
$1/5 \cdot 2^{-1}$	$1.0 \cdot 10^{+03}$	7.928	$3.1 \cdot 10^{+03}$	7.523
$1/5 \cdot 2^{-2}$	4.5	7.731	$1.7 \cdot 10^{+01}$	9.007
$1/5 \cdot 2^{-3}$	$2.1 \cdot 10^{-02}$	6.198	$3.3 \cdot 10^{-02}$	4.700
$1/5 \cdot 2^{-4}$	$2.8 \cdot 10^{-04}$	2.872	$1.2 \cdot 10^{-03}$	3.095
$1/5 \cdot 2^{-5}$	$3.9 \cdot 10^{-05}$	2.781	$1.5 \cdot 10^{-04}$	3.020
$1/5 \cdot 2^{-6}$	$5.7 \cdot 10^{-06}$	2.977	$1.8 \cdot 10^{-05}$	3.001
$1/5 \cdot 2^{-7}$	$7.2 \cdot 10^{-07}$	2.997	$2.3 \cdot 10^{-06}$	2.999
$1/5 \cdot 2^{-8}$	$9.1 \cdot 10^{-08}$	3.000	$2.9 \cdot 10^{-07}$	2.999
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-08}$	3.000	$3.6 \cdot 10^{-08}$	2.999
$1/5 \cdot 2^{-10}$	$1.4 \cdot 10^{-09}$	3.000	$4.5 \cdot 10^{-09}$	2.999
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-10}$	3.000	$5.6 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-11}$	2.999	$7.1 \cdot 10^{-11}$	2.999
$1/5 \cdot 2^{-13}$	$2.7 \cdot 10^{-12}$		$8.8 \cdot 10^{-12}$	

Table 5.14: Explicit Euler, IDeC method at $t = 0$ for 4.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.1 \cdot 10^{-01}$	-0.0000	1.449	$1.1 \cdot 10^{-01}$	-0.0000	2.863
$1/5 \cdot 2^{-1}$	$4.0 \cdot 10^{-02}$	0.10000	1.317	$1.5 \cdot 10^{-02}$	-0.0000	2.669
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-02}$	0.10000	1.097	$2.4 \cdot 10^{-03}$	0.05000	2.366
$1/5 \cdot 2^{-3}$	$7.6 \cdot 10^{-03}$	0.12500	1.060	$4.8 \cdot 10^{-04}$	0.07500	2.141
$1/5 \cdot 2^{-4}$	$3.6 \cdot 10^{-03}$	0.12500	1.028	$1.0 \cdot 10^{-04}$	0.07500	2.069
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-03}$	0.13125	1.014	$2.5 \cdot 10^{-05}$	0.07500	2.034
$1/5 \cdot 2^{-6}$	$8.8 \cdot 10^{-04}$	0.12812	1.007	$6.3 \cdot 10^{-06}$	0.07500	2.017
$1/5 \cdot 2^{-7}$	$4.4 \cdot 10^{-04}$	0.12968	1.003	$1.5 \cdot 10^{-06}$	0.07500	2.008
$1/5 \cdot 2^{-8}$	$2.1 \cdot 10^{-04}$	0.12968	1.001	$3.8 \cdot 10^{-07}$	0.07421	2.004
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-04}$	0.12968	1.000	$9.6 \cdot 10^{-08}$	0.07460	2.002
$1/5 \cdot 2^{-10}$	$5.4 \cdot 10^{-05}$	0.12968	1.000	$2.4 \cdot 10^{-08}$	0.07480	2.001
$1/5 \cdot 2^{-11}$	$2.7 \cdot 10^{-05}$	0.12968	1.000	$6.0 \cdot 10^{-09}$	0.07470	2.000
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-05}$	0.12973	1.000	$1.5 \cdot 10^{-09}$	0.07475	2.000
$1/5 \cdot 2^{-13}$	$6.8 \cdot 10^{-06}$	0.12973	1.000	$3.7 \cdot 10^{-10}$	0.07478	2.000
$1/5 \cdot 2^{-14}$	$3.4 \cdot 10^{-06}$	0.12973	1.000	$9.4 \cdot 10^{-11}$	0.07478	2.000
$1/5 \cdot 2^{-15}$	$1.7 \cdot 10^{-06}$	0.12973		$2.3 \cdot 10^{-11}$	0.07478	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.1 \cdot 10^{-01}$	0.80000	3.389	$2.7 \cdot 10^{-01}$	0.80000	3.577
$1/5 \cdot 2^{-1}$	$1.0 \cdot 10^{-02}$	-0.0000	3.434	$2.3 \cdot 10^{-02}$	0.40000	3.759
$1/5 \cdot 2^{-2}$	$9.8 \cdot 10^{-04}$	0.00000	3.854	$1.7 \cdot 10^{-03}$	0.20000	3.946
$1/5 \cdot 2^{-3}$	$6.8 \cdot 10^{-05}$	0.00000	3.744	$1.1 \cdot 10^{-04}$	0.10000	4.023
$1/5 \cdot 2^{-4}$	$5.0 \cdot 10^{-06}$	0.02500	3.209	$6.8 \cdot 10^{-06}$	0.05000	4.038
$1/5 \cdot 2^{-5}$	$5.5 \cdot 10^{-07}$	0.03125	3.077	$4.1 \cdot 10^{-07}$	0.02500	4.030
$1/5 \cdot 2^{-6}$	$6.5 \cdot 10^{-08}$	0.02812	3.040	$2.5 \cdot 10^{-08}$	0.01250	4.018
$1/5 \cdot 2^{-7}$	$7.9 \cdot 10^{-09}$	0.02968	3.020	$1.5 \cdot 10^{-09}$	0.00625	4.010
$1/5 \cdot 2^{-8}$	$9.7 \cdot 10^{-10}$	0.03046	3.010	$9.8 \cdot 10^{-11}$	0.00312	4.005
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-10}$	0.03046	3.005	$6.1 \cdot 10^{-12}$	0.00156	4.002
$1/5 \cdot 2^{-10}$	$1.5 \cdot 10^{-11}$	0.03046	3.002	$3.8 \cdot 10^{-13}$	0.00078	4.001
$1/5 \cdot 2^{-11}$	$1.8 \cdot 10^{-12}$	0.03056	3.001	$2.3 \cdot 10^{-14}$	0.00039	4.000
$1/5 \cdot 2^{-12}$	$2.3 \cdot 10^{-13}$	0.03061	3.000	$1.4 \cdot 10^{-15}$	0.00019	4.000
$1/5 \cdot 2^{-13}$	$2.9 \cdot 10^{-14}$	0.03061	3.000	$9.2 \cdot 10^{-17}$	0.00009	4.000
$1/5 \cdot 2^{-14}$	$3.6 \cdot 10^{-15}$	0.03062		$5.8 \cdot 10^{-18}$	0.00004	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$5.3 \cdot 10^{-01}$	0.80000	3.682	$9.2 \cdot 10^{-01}$	0.80000	4.070
$1/5 \cdot 2^{-1}$	$4.1 \cdot 10^{-02}$	0.40000	3.784	$5.5 \cdot 10^{-02}$	0.40000	3.847
$1/5 \cdot 2^{-2}$	$3.0 \cdot 10^{-03}$	0.20000	3.935	$3.8 \cdot 10^{-03}$	0.20000	3.936
$1/5 \cdot 2^{-3}$	$1.9 \cdot 10^{-04}$	0.10000	3.987	$2.4 \cdot 10^{-04}$	0.10000	3.983
$1/5 \cdot 2^{-4}$	$1.2 \cdot 10^{-05}$	0.05000	4.000	$1.5 \cdot 10^{-05}$	0.05000	3.998
$1/5 \cdot 2^{-5}$	$7.8 \cdot 10^{-07}$	0.02500	4.002	$9.8 \cdot 10^{-07}$	0.02500	4.001
$1/5 \cdot 2^{-6}$	$4.8 \cdot 10^{-08}$	0.01250	4.001	$6.1 \cdot 10^{-08}$	0.01250	4.001
$1/5 \cdot 2^{-7}$	$3.0 \cdot 10^{-09}$	0.00625	4.001	$3.8 \cdot 10^{-09}$	0.00625	4.000
$1/5 \cdot 2^{-8}$	$1.9 \cdot 10^{-10}$	0.00312	4.000	$2.4 \cdot 10^{-10}$	0.00312	4.000
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-11}$	0.00156	4.000	$1.5 \cdot 10^{-11}$	0.00156	4.000
$1/5 \cdot 2^{-10}$	$7.4 \cdot 10^{-13}$	0.00078	4.000	$9.3 \cdot 10^{-13}$	0.00078	4.000
$1/5 \cdot 2^{-11}$	$4.6 \cdot 10^{-14}$	0.00039	4.000	$5.8 \cdot 10^{-14}$	0.00039	4.000
$1/5 \cdot 2^{-12}$	$2.9 \cdot 10^{-15}$	0.00019	3.999	$3.6 \cdot 10^{-15}$	0.00019	3.999
$1/5 \cdot 2^{-13}$	$1.8 \cdot 10^{-16}$	0.00009		$2.2 \cdot 10^{-16}$	0.00009	

Table 5.15: Explicit Euler, IDeC method on $[0, 1]$ for 5.2a.

h	δ	p	δ	p
$1/5$	$1.1 \cdot 10^{-01}$	3.537	$1.1 \cdot 10^{-01}$	2.863
$1/5 \cdot 2^{-1}$	$9.5 \cdot 10^{-03}$	3.884	$1.5 \cdot 10^{-02}$	3.703
$1/5 \cdot 2^{-2}$	$6.4 \cdot 10^{-04}$	3.983	$1.2 \cdot 10^{-03}$	3.932
$1/5 \cdot 2^{-3}$	$4.0 \cdot 10^{-05}$	4.005	$7.9 \cdot 10^{-05}$	3.992
$1/5 \cdot 2^{-4}$	$2.5 \cdot 10^{-06}$	4.006	$4.9 \cdot 10^{-06}$	4.003
$1/5 \cdot 2^{-5}$	$1.5 \cdot 10^{-07}$	4.004	$3.1 \cdot 10^{-07}$	4.004
$1/5 \cdot 2^{-6}$	$9.8 \cdot 10^{-09}$	4.002	$1.9 \cdot 10^{-08}$	4.002
$1/5 \cdot 2^{-7}$	$6.1 \cdot 10^{-10}$	4.001	$1.2 \cdot 10^{-09}$	4.001
$1/5 \cdot 2^{-8}$	$3.8 \cdot 10^{-11}$	4.000	$7.5 \cdot 10^{-11}$	4.000
$1/5 \cdot 2^{-9}$	$2.4 \cdot 10^{-12}$	4.000	$4.7 \cdot 10^{-12}$	4.000
$1/5 \cdot 2^{-10}$	$1.5 \cdot 10^{-13}$	4.000	$2.9 \cdot 10^{-13}$	4.000
$1/5 \cdot 2^{-11}$	$9.3 \cdot 10^{-15}$	4.000	$1.8 \cdot 10^{-14}$	4.000
$1/5 \cdot 2^{-12}$	$5.8 \cdot 10^{-16}$	3.999	$1.1 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-13}$	$3.6 \cdot 10^{-17}$	4.000	$7.1 \cdot 10^{-17}$	4.000
$1/5 \cdot 2^{-14}$	$2.2 \cdot 10^{-18}$	4.000	$4.4 \cdot 10^{-18}$	4.000
$1/5 \cdot 2^{-15}$	$1.4 \cdot 10^{-19}$		$2.8 \cdot 10^{-19}$	

h	δ	p	δ	p
$1/5$	$3.1 \cdot 10^{-02}$	1.564	$7.1 \cdot 10^{-02}$	4.469
$1/5 \cdot 2^{-1}$	$1.0 \cdot 10^{-02}$	3.434	$3.2 \cdot 10^{-03}$	8.563
$1/5 \cdot 2^{-2}$	$9.8 \cdot 10^{-04}$	3.854	$8.5 \cdot 10^{-06}$	0.641
$1/5 \cdot 2^{-3}$	$6.8 \cdot 10^{-05}$	3.970	$5.4 \cdot 10^{-06}$	3.574
$1/5 \cdot 2^{-4}$	$4.3 \cdot 10^{-06}$	3.998	$4.5 \cdot 10^{-07}$	3.911
$1/5 \cdot 2^{-5}$	$2.7 \cdot 10^{-07}$	4.003	$3.0 \cdot 10^{-08}$	3.985
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-08}$	4.002	$1.9 \cdot 10^{-09}$	4.001
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-09}$	4.001	$1.2 \cdot 10^{-10}$	4.002
$1/5 \cdot 2^{-8}$	$6.6 \cdot 10^{-11}$	4.000	$7.4 \cdot 10^{-12}$	4.002
$1/5 \cdot 2^{-9}$	$4.1 \cdot 10^{-12}$	4.000	$4.6 \cdot 10^{-13}$	4.001
$1/5 \cdot 2^{-10}$	$2.5 \cdot 10^{-13}$	4.000	$2.9 \cdot 10^{-14}$	4.000
$1/5 \cdot 2^{-11}$	$1.6 \cdot 10^{-14}$	4.000	$1.8 \cdot 10^{-15}$	4.000
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-15}$	3.999	$1.1 \cdot 10^{-16}$	3.999
$1/5 \cdot 2^{-13}$	$6.3 \cdot 10^{-17}$	4.000	$7.1 \cdot 10^{-18}$	4.000
$1/5 \cdot 2^{-14}$	$3.9 \cdot 10^{-18}$		$4.4 \cdot 10^{-19}$	

h	δ	p	δ	p
$1/5$	$1.1 \cdot 10^{-01}$	2.812	$2.3 \cdot 10^{-01}$	3.283
$1/5 \cdot 2^{-1}$	$1.5 \cdot 10^{-02}$	3.785	$2.4 \cdot 10^{-02}$	3.706
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-03}$	3.986	$1.8 \cdot 10^{-03}$	3.920
$1/5 \cdot 2^{-3}$	$7.2 \cdot 10^{-05}$	4.009	$1.2 \cdot 10^{-04}$	3.982
$1/5 \cdot 2^{-4}$	$4.5 \cdot 10^{-06}$	4.006	$7.7 \cdot 10^{-06}$	3.998
$1/5 \cdot 2^{-5}$	$2.8 \cdot 10^{-07}$	4.003	$4.8 \cdot 10^{-07}$	4.001
$1/5 \cdot 2^{-6}$	$1.7 \cdot 10^{-08}$	4.001	$3.0 \cdot 10^{-08}$	4.001
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-09}$	4.000	$1.8 \cdot 10^{-09}$	4.000
$1/5 \cdot 2^{-8}$	$6.8 \cdot 10^{-11}$	4.000	$1.1 \cdot 10^{-10}$	4.000
$1/5 \cdot 2^{-9}$	$4.2 \cdot 10^{-12}$	4.000	$7.3 \cdot 10^{-12}$	4.000
$1/5 \cdot 2^{-10}$	$2.6 \cdot 10^{-13}$	4.000	$4.6 \cdot 10^{-13}$	4.000
$1/5 \cdot 2^{-11}$	$1.6 \cdot 10^{-14}$	4.000	$2.8 \cdot 10^{-14}$	4.000
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-15}$	3.999	$1.8 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-13}$	$6.5 \cdot 10^{-17}$		$1.1 \cdot 10^{-16}$	

Table 5.16: Explicit Euler, IDeC method at $t = 0$ for 5.2a.

$$- 121196250t^5 + 730803150t^6 - 2104987500t^7$$
$$+ 3127667400t^8 - 2310808500t^9 + 671284900t^{10}).$$

For problem 4.1d we observed an order sequence $O(h)$, $O(h^2)$, $O(h^2)$,.... This correlates with the fact that 1 is an eigenvalue of the coefficient matrix M in this case. This fact makes it likely that no sufficiently long error expansion exists for the numerical solution, because the variational equations cannot be expected to have a smooth solution. Even though the problem itself has a smooth solution $z \in C^\infty[0,1]$, for equations of the form (5.1) with an eigenvalue 1 we cannot guarantee to have a unique, differentiable solution. This fact is observed for the variational equations,

$$e_1(t) = \left(\frac{7e}{2} + 2t\ln(t) + O(t), 2t\ln(t) + O(t) \right),$$

$$e_2(t) = \left(-\frac{7}{6} + \frac{5e}{3} + 6t\ln(t), +O(t), -1 + 6t\ln(t) + O(t) \right),$$

$$e_3(t) = \left(-\frac{1}{6}t^{-1} - 3 - 6t\ln(t) + O(t), -\frac{1}{6}t^{-1} - 3 - 6t\ln(t) + O(t) \right).$$

While e_1 and e_2 are continuous functions which are not differentiable, however, no continuous solution of the variational equation for e_3 exists. This is the apparent reason why IDeC fails to improve the convergence order of the iterates after the first step.

We would like to stress that the reason why no satisfactory error expansion exists is different from the reason for the same phenomenon which was observed for the box scheme for problems (4.1). While in the latter case, unsmooth terms appeared in the right-hand member of the variational equations due to the structure of the method, this is not the case here. Rather, the inherent unsmoothness of the solutions of terminal value problems (5.1) when $M(0)$ has positive eigenvalues causes the breakdown of the error expansion. This happens even though the structure of the variational equations is the same as for the implicit Euler method for (4.1), which in contrast permit a satisfactory error expansion.

For example 4.1e, a similar behavior is observed. In this case, Table 5.9 shows an order sequence $O(h)$, $O(h^2)$, $O(h^3)$, $O(h^4)$, $O(h^4)$,... for IDeC with polynomial degree $m = 5$. In the light of the discussion for example 4.1d, this corresponds to the fact that M has got the eigenvalue 3 for this test problem. Indeed, the solutions of the variational equations read[15]

$$e_1(t) = \left(\frac{1}{6}e - 6t^2 + O(t^3\ln(t)), -t - 18t^2 + O(t^3\ln(t)) \right),$$

$$e_2(t) = \left(\frac{1}{6}e + \frac{1}{18} + 4t + O(t^2\ln(t)), 12t + O(t^2\ln(t)) \right),$$

---

[15] $\mathrm{Ei}(1,-1)$ is the *exponential integral* which is obtained as the value for $x = -1$ of the analytic extension of $\mathrm{Ei}(1,x) = \int_1^\infty \frac{e^{-xt}}{t}\,dt$ $(\Re(x) > 0)$ to the left half-plane. Note that $\Im(\mathrm{Ei}(1,-1)) = -i\pi$. γ denotes Euler's constant, see §4.2.3.

$$
e_3(t) = \Bigg(\big(8(\mathrm{Ei}(1,-1)+\mathrm{i}\pi)+\frac{80}{3}+8\gamma+8\ln(t)+\frac{17}{3}\mathrm{e}\big)t + O(t^2\ln(t)),
$$

$$
\big(24(\mathrm{Ei}(1,-1)+\mathrm{i}\pi)+80+17\mathrm{e}+24\gamma+24\ln(t)\big)t + O(t^2\ln(t))\Bigg),
$$

$$
e_4(t) = \Bigg(-\frac{539}{180}-\frac{13}{540}\mathrm{e}+O(t\ln(t)), -9+O(t\ln(t))\Bigg),
$$

$$
e_5(t) = \Bigg(-\frac{19}{30}t^{-1}-\frac{125}{12}+O(t\ln(t)), -\frac{19}{10}t^{-1}-\frac{125}{4}+O(t\ln(t))\Bigg).
$$

Finally, for example 4.1f the smallest positive eigenvalue of M is 10. This suggests that a sufficiently long asymptotic error expansion exists which explains the fact that IDeC reaches the full convergence order 5 in the fourth iteration step. Indeed, the solutions of the variational equation are

$$
e_1(t) = \Bigg(-\mathrm{e}+\frac{7}{9}t+\frac{15}{16}t^2+O(t^3), -\frac{2}{9}t-\frac{9}{8}t^2+O(t^3)\Bigg),
$$

$$
e_2(t) = \Bigg(-\frac{43}{120}\mathrm{e}+\frac{2}{15}+\frac{19}{48}t+\frac{13}{28}t^2+O(t^3), \frac{11}{24}t+\frac{23}{14}t^2+O(t^3)\Bigg),
$$

$$
e_3(t) = \Bigg(-\frac{5}{84}t-\frac{425}{2016}t^2+O(t^3), -\frac{25}{42}t-\frac{2125}{1008}t^2+O(t^3)\Bigg),
$$

$$
e_4(t) = \Bigg(-\frac{1}{100}+\frac{19}{1440}\mathrm{e}+\frac{367}{6048}t+\frac{97}{336}t^2+O(t^3), \frac{2255}{3024}t+\frac{499}{168}t^2+O(t^3)\Bigg),
$$

$$
e_5(t) = \Bigg(-\frac{529}{5040}t-\frac{973}{1920}t^2+O(t^3), -\frac{529}{504}t-\frac{973}{192}t^2+O(t^3)\Bigg),
$$

$$
e_6(t) = \Bigg(-\frac{23}{60480}\mathrm{e}+\frac{1}{2520}+\frac{5171}{28800}t+\frac{491}{432}t^2+O(t^3),
$$

$$
\frac{5159}{2880}t+\frac{4091}{360}t^2+O(t^3)\Bigg).
$$

5.2   The Box Scheme

In this section, Iterated Defect Correction based on the box scheme is discussed for (5.1). However, we found the method to be unreliable even in the case of (4.1) due to an unsmoothness in the variational equations defining the asymptotic expansion of the global error. Due to the symmetry of the method, the general form of the variational equations for the box scheme for (5.1) is the same as the corresponding variational equation for an initial value problem integrated from left to right. Hence, we do not expect a satisfactory error expansion for the box scheme for terminal value problems (5.1). Moreover, we also face the inherent unsmoothness of solutions of singular problems posed at $t=1$ if the coefficient matrix M has eigenvalues with positive real parts which was already discussed for the explicit Euler method. The numerical results given in §5.2.1 show that indeed the method does not perform satisfactorily. We do not attempt a thorough analysis as in the preceding sections, since the method is obviously of no practical relevance due to the (unsystematic) order reductions.

Moreover, the reasons leading to the disadvantageous behavior are most likely the same as those that were discussed in a lot of detail for the other methods.

Before giving the numerical results for IDeC, we discuss the convergence order of the box scheme. From the symmetry of the method we conclude that a representation of the solution similar to that in [14] holds. Accordingly, the estimates for the stability of the scheme can be worked out analogously, if we note that the direction of the integration is reversed as in the proofs in §5.1.1. We do not work out the steps of the proof explicitly, as the result can also be concluded from results in [11]. Moreover, in §5.2.1 we give comprehensive numerical evidence that a convergence order $O(h^2)$ holds.

5.2.1   Iterated Defect Correction

We now give the results for IDeC based on the box scheme for (5.1), where Zadunaisky polynomials of degree $m = 10$ are used.

For the problems 4.1b and 4.1c, we observe a behavior which reminds us of the results for the explicit Euler method. For example 4.1b, we observe the classical order sequence for the maximal error. In the last step, an even higher convergence order is observed. At $t = 0$, the numerical solution assumes the exact value for all IDeC iterates. For 4.1c the maximal error shows the same behavior, at $t = 0$ we observe a convergence order of 10 for every numerical approximation. Refer to Tables 5.17–5.20.

For problems 4.1d, 4.1e and 4.1f, we observe order reductions which seem to correspond to the size of the positive eigenvalue of the coefficient matrix M. For example 4.1d, no convergence order higher than 3 can be obtained. For 4.1e, the second iterate has convergence order 5 which does not increase any further after this. In both cases, the maximal error is assumed at $t = 0$ for the iterates where order reductions are observed. In the case of problem 4.1f the classical order sequence is obtained, even a convergence order of 12 is achieved in the last step. For all 3 problems, the behavior at $t = 0$ is the same as for the maximal error (which, in fact, is assumed at this point for some of the approximations).

Problems 4.2a and 5.2a show the same behavior. An order sequence $O(h^2)$, $O(h^4)$, $O(h^4)$, ... is observed. Again, for the iterates that feature order reductions, the maximal error is assumed at $t = 0$. The results at $t = 0$ are the same as for the maximal error on $[0, 1]$.

For this method, we refrain from computing the error expansions explicitly. The previous investigations showed with sufficient reliability that the order reductions observed for IDeC are to be attributed to the length of the asymptotic expansion of the global error of the method under consideration. Therefore, we do not repeat this exercise, as no novel insights are likely to be gained from the calculations. Moreover, due to the order reductions, IDeC based on the box scheme is of no practical value for (5.1).

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.1 \cdot 10^{-02}$	0.60000	2.010	$1.1 \cdot 10^{-04}$	0.60000	4.021
$1/5 \cdot 2^{-2}$	$2.8 \cdot 10^{-03}$	0.60000	1.999	$7.2 \cdot 10^{-06}$	0.60000	4.002
$1/5 \cdot 2^{-3}$	$7.2 \cdot 10^{-04}$	0.57500	2.000	$4.5 \cdot 10^{-07}$	0.57500	4.001
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-04}$	0.57500	2.000	$2.8 \cdot 10^{-08}$	0.57500	4.000
$1/5 \cdot 2^{-5}$	$4.5 \cdot 10^{-05}$	0.57500	2.000	$1.7 \cdot 10^{-09}$	0.57500	4.000
$1/5 \cdot 2^{-6}$	$1.1 \cdot 10^{-05}$	0.57812	2.000	$1.1 \cdot 10^{-10}$	0.57812	4.000
$1/5 \cdot 2^{-7}$	$2.8 \cdot 10^{-06}$	0.57812	1.999	$6.8 \cdot 10^{-12}$	0.57812	4.000
$1/5 \cdot 2^{-8}$	$7.0 \cdot 10^{-07}$	0.57734	2.000	$4.3 \cdot 10^{-13}$	0.57734	4.000
$1/5 \cdot 2^{-9}$	$1.7 \cdot 10^{-07}$	0.57734	2.000	$2.6 \cdot 10^{-14}$	0.57734	4.000
$1/5 \cdot 2^{-10}$	$4.4 \cdot 10^{-08}$	0.57734	2.000	$1.6 \cdot 10^{-15}$	0.57734	4.000
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-08}$	0.57734	2.000	$1.0 \cdot 10^{-16}$	0.57734	4.000
$1/5 \cdot 2^{-12}$	$2.7 \cdot 10^{-09}$	0.57734	2.000	$6.5 \cdot 10^{-18}$	0.57734	4.000
$1/5 \cdot 2^{-13}$	$6.8 \cdot 10^{-10}$	0.57734	1.999	$4.1 \cdot 10^{-19}$	0.57734	3.999
$1/5 \cdot 2^{-14}$	$1.7 \cdot 10^{-10}$	0.57735		$2.5 \cdot 10^{-20}$	0.57735	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.1 \cdot 10^{-06}$	0.60000	6.032	$1.1 \cdot 10^{-08}$	0.60000	8.043
$1/5 \cdot 2^{-2}$	$1.8 \cdot 10^{-08}$	0.60000	6.004	$4.5 \cdot 10^{-11}$	0.60000	8.007
$1/5 \cdot 2^{-3}$	$2.8 \cdot 10^{-10}$	0.57500	6.002	$1.7 \cdot 10^{-13}$	0.57500	8.002
$1/5 \cdot 2^{-4}$	$4.4 \cdot 10^{-12}$	0.57500	6.000	$6.8 \cdot 10^{-16}$	0.57500	8.000
$1/5 \cdot 2^{-5}$	$6.8 \cdot 10^{-14}$	0.57500	6.000	$2.6 \cdot 10^{-18}$	0.57500	8.000
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{-15}$	0.57812	6.000	$1.0 \cdot 10^{-20}$	0.57812	8.000
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-17}$	0.57812	6.000	$4.1 \cdot 10^{-23}$	0.57812	7.999
$1/5 \cdot 2^{-8}$	$2.6 \cdot 10^{-19}$	0.57734	6.000	$1.6 \cdot 10^{-25}$	0.57734	7.973
$1/5 \cdot 2^{-9}$	$4.1 \cdot 10^{-21}$	0.57734	6.000	$6.3 \cdot 10^{-28}$	0.59765	4.261
$1/5 \cdot 2^{-10}$	$6.4 \cdot 10^{-23}$	0.57734	5.999	$3.3 \cdot 10^{-29}$	0.37773	-2.255
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-24}$	0.57734	6.007	$1.5 \cdot 10^{-28}$	0.03808	-0.834
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-26}$	0.57270	5.526	$2.8 \cdot 10^{-28}$	0.04101	-0.970
$1/5 \cdot 2^{-13}$	$3.3 \cdot 10^{-28}$	0.00017		$5.5 \cdot 10^{-28}$	0.00017	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.2 \cdot 10^{-10}$	0.60000	10.054	$1.2 \cdot 10^{-12}$	0.60000	12.065
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-13}$	0.60000	10.010	$2.8 \cdot 10^{-16}$	0.60000	12.012
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-16}$	0.57500	10.003	$6.9 \cdot 10^{-20}$	0.57500	12.004
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-19}$	0.57500	10.000	$1.6 \cdot 10^{-23}$	0.57500	12.000
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-22}$	0.57500	10.000	$4.1 \cdot 10^{-27}$	0.57500	8.849
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{-25}$	0.57812	10.015	$8.8 \cdot 10^{-30}$	0.01250	-0.953
$1/5 \cdot 2^{-7}$	$9.9 \cdot 10^{-29}$	0.56093	2.267	$1.7 \cdot 10^{-29}$	0.21718	-0.117
$1/5 \cdot 2^{-8}$	$2.0 \cdot 10^{-29}$	0.14453	-1.691	$1.8 \cdot 10^{-29}$	0.14453	-1.781
$1/5 \cdot 2^{-9}$	$6.6 \cdot 10^{-29}$	0.02265	0.739	$6.4 \cdot 10^{-29}$	0.02265	0.772
$1/5 \cdot 2^{-10}$	$3.9 \cdot 10^{-29}$	0.37871	-1.935	$3.7 \cdot 10^{-29}$	0.15585	-2.058
$1/5 \cdot 2^{-11}$	$1.5 \cdot 10^{-28}$	0.03808	-1.039	$1.5 \cdot 10^{-28}$	0.03808	-0.938
$1/5 \cdot 2^{-12}$	$3.1 \cdot 10^{-28}$	0.04101		$3.0 \cdot 10^{-28}$	0.04101	

Table 5.17: Box scheme, IDeC method on $[0, 1]$ for 4.1b.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$3.3 \cdot 10^{-34}$	-6.192	$1.9 \cdot 10^{-31}$	-3.285
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-32}$	0.0000	$1.9 \cdot 10^{-30}$	-0.572
$1/5 \cdot 2^{-3}$	$2.4 \cdot 10^{-32}$	-1.584	$2.8 \cdot 10^{-30}$	2.536
$1/5 \cdot 2^{-4}$	$7.3 \cdot 10^{-32}$	-1.736	$4.9 \cdot 10^{-31}$	-1.137
$1/5 \cdot 2^{-5}$	$2.4 \cdot 10^{-31}$	-0.925	$1.0 \cdot 10^{-30}$	1.067
$1/5 \cdot 2^{-6}$	$4.6 \cdot 10^{-31}$	-0.752	$5.1 \cdot 10^{-31}$	-3.301
$1/5 \cdot 2^{-7}$	$7.8 \cdot 10^{-31}$	-0.672	$5.1 \cdot 10^{-30}$	-1.448
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-30}$	-0.736	$1.3 \cdot 10^{-29}$	0.476
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-30}$	-1.385	$1.0 \cdot 10^{-29}$	-3.347
$1/5 \cdot 2^{-10}$	$5.4 \cdot 10^{-30}$	-0.798	$1.0 \cdot 10^{-28}$	-0.409
$1/5 \cdot 2^{-11}$	$9.5 \cdot 10^{-30}$	-1.198	$1.3 \cdot 10^{-28}$	-0.699
$1/5 \cdot 2^{-12}$	$2.1 \cdot 10^{-29}$	-0.926	$2.1 \cdot 10^{-28}$	-0.986
$1/5 \cdot 2^{-13}$	$4.1 \cdot 10^{-29}$	-1.015	$4.3 \cdot 10^{-28}$	-1.051
$1/5 \cdot 2^{-14}$	$8.3 \cdot 10^{-29}$		$9.0 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$8.1 \cdot 10^{-31}$	-1.825	$1.3 \cdot 10^{-30}$	-1.099
$1/5 \cdot 2^{-2}$	$2.8 \cdot 10^{-30}$	1.315	$2.9 \cdot 10^{-30}$	1.480
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-30}$	-1.103	$1.0 \cdot 10^{-30}$	-1.341
$1/5 \cdot 2^{-4}$	$2.4 \cdot 10^{-30}$	-1.103	$2.6 \cdot 10^{-30}$	-1.191
$1/5 \cdot 2^{-5}$	$5.3 \cdot 10^{-30}$	-0.331	$6.1 \cdot 10^{-30}$	0.492
$1/5 \cdot 2^{-6}$	$6.7 \cdot 10^{-30}$	0.561	$4.3 \cdot 10^{-30}$	-0.756
$1/5 \cdot 2^{-7}$	$4.5 \cdot 10^{-30}$	0.071	$7.3 \cdot 10^{-30}$	0.120
$1/5 \cdot 2^{-8}$	$4.3 \cdot 10^{-30}$	-3.029	$6.7 \cdot 10^{-30}$	-3.168
$1/5 \cdot 2^{-9}$	$3.5 \cdot 10^{-29}$	0.902	$6.0 \cdot 10^{-29}$	4.672
$1/5 \cdot 2^{-10}$	$1.8 \cdot 10^{-29}$	-2.151	$2.3 \cdot 10^{-30}$	-5.934
$1/5 \cdot 2^{-11}$	$8.4 \cdot 10^{-29}$	-0.784	$1.4 \cdot 10^{-28}$	-0.812
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-28}$	-1.215	$2.5 \cdot 10^{-28}$	-1.107
$1/5 \cdot 2^{-13}$	$3.3 \cdot 10^{-28}$		$5.5 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.2 \cdot 10^{-30}$	-1.584	$1.4 \cdot 10^{-30}$	-1.128
$1/5 \cdot 2^{-2}$	$3.6 \cdot 10^{-30}$	3.877	$3.1 \cdot 10^{-30}$	3.203
$1/5 \cdot 2^{-3}$	$2.4 \cdot 10^{-31}$	-3.776	$3.4 \cdot 10^{-31}$	-2.960
$1/5 \cdot 2^{-4}$	$3.3 \cdot 10^{-30}$	-1.294	$2.6 \cdot 10^{-30}$	-1.071
$1/5 \cdot 2^{-5}$	$8.2 \cdot 10^{-30}$	-0.166	$5.6 \cdot 10^{-30}$	-0.582
$1/5 \cdot 2^{-6}$	$9.2 \cdot 10^{-30}$	0.042	$8.4 \cdot 10^{-30}$	-0.572
$1/5 \cdot 2^{-7}$	$9.0 \cdot 10^{-30}$	0.127	$1.2 \cdot 10^{-29}$	0.718
$1/5 \cdot 2^{-8}$	$8.2 \cdot 10^{-30}$	-2.904	$7.6 \cdot 10^{-30}$	-2.958
$1/5 \cdot 2^{-9}$	$6.1 \cdot 10^{-29}$	3.025	$5.9 \cdot 10^{-29}$	3.034
$1/5 \cdot 2^{-10}$	$7.5 \cdot 10^{-30}$	-4.189	$7.2 \cdot 10^{-30}$	-4.316
$1/5 \cdot 2^{-11}$	$1.3 \cdot 10^{-28}$	-1.052	$1.4 \cdot 10^{-28}$	-0.917
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-28}$		$2.7 \cdot 10^{-28}$	

Table 5.18: Box scheme, IDeC method at $t = 0$ for 4.1b.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$3.8 \cdot 10^{-01}$	0.90000	2.141	$4.6 \cdot 10^{-02}$	0.90000	4.366
$1/5 \cdot 2^{-2}$	$8.6 \cdot 10^{-02}$	0.90000	2.032	$2.2 \cdot 10^{-03}$	0.90000	4.032
$1/5 \cdot 2^{-3}$	$2.1 \cdot 10^{-02}$	0.90000	2.007	$1.3 \cdot 10^{-04}$	0.92500	4.020
$1/5 \cdot 2^{-4}$	$5.2 \cdot 10^{-03}$	0.90000	1.999	$8.5 \cdot 10^{-06}$	0.92500	4.002
$1/5 \cdot 2^{-5}$	$1.3 \cdot 10^{-03}$	0.89375	2.000	$5.3 \cdot 10^{-07}$	0.91875	4.001
$1/5 \cdot 2^{-6}$	$3.3 \cdot 10^{-04}$	0.89375	2.000	$3.3 \cdot 10^{-08}$	0.92187	4.000
$1/5 \cdot 2^{-7}$	$8.2 \cdot 10^{-05}$	0.89375	1.999	$2.0 \cdot 10^{-09}$	0.92031	4.000
$1/5 \cdot 2^{-8}$	$2.0 \cdot 10^{-05}$	0.89453	2.000	$1.2 \cdot 10^{-10}$	0.92031	4.000
$1/5 \cdot 2^{-9}$	$5.1 \cdot 10^{-06}$	0.89453	2.000	$8.1 \cdot 10^{-12}$	0.92031	4.000
$1/5 \cdot 2^{-10}$	$1.2 \cdot 10^{-06}$	0.89433	1.999	$5.0 \cdot 10^{-13}$	0.92031	4.000
$1/5 \cdot 2^{-11}$	$3.2 \cdot 10^{-07}$	0.89443	2.000	$3.1 \cdot 10^{-14}$	0.92041	4.000
$1/5 \cdot 2^{-12}$	$8.0 \cdot 10^{-08}$	0.89443	2.000	$1.9 \cdot 10^{-15}$	0.92041	4.000
$1/5 \cdot 2^{-13}$	$2.0 \cdot 10^{-08}$	0.89443	1.999	$1.2 \cdot 10^{-16}$	0.92038	3.999
$1/5 \cdot 2^{-14}$	$5.0 \cdot 10^{-09}$	0.89442		$7.7 \cdot 10^{-18}$	0.92039	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$7.0 \cdot 10^{-03}$	0.90000	6.532	$1.1 \cdot 10^{-03}$	0.90000	8.654
$1/5 \cdot 2^{-2}$	$7.6 \cdot 10^{-05}$	0.95000	6.082	$2.9 \cdot 10^{-06}$	0.95000	8.209
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-06}$	0.92500	6.032	$9.8 \cdot 10^{-09}$	0.95000	8.017
$1/5 \cdot 2^{-4}$	$1.7 \cdot 10^{-08}$	0.93750	6.002	$3.8 \cdot 10^{-11}$	0.93750	8.012
$1/5 \cdot 2^{-5}$	$2.6 \cdot 10^{-10}$	0.93125	6.002	$1.4 \cdot 10^{-13}$	0.93750	8.003
$1/5 \cdot 2^{-6}$	$4.1 \cdot 10^{-12}$	0.93125	6.000	$5.7 \cdot 10^{-16}$	0.93750	8.000
$1/5 \cdot 2^{-7}$	$6.5 \cdot 10^{-14}$	0.93125	6.000	$2.2 \cdot 10^{-18}$	0.93906	8.000
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-15}$	0.93203	6.000	$8.7 \cdot 10^{-21}$	0.93906	8.000
$1/5 \cdot 2^{-9}$	$1.5 \cdot 10^{-17}$	0.93164	6.000	$3.4 \cdot 10^{-23}$	0.93867	8.000
$1/5 \cdot 2^{-10}$	$2.4 \cdot 10^{-19}$	0.93183	6.000	$1.3 \cdot 10^{-25}$	0.93886	8.047
$1/5 \cdot 2^{-11}$	$3.8 \cdot 10^{-21}$	0.93183	6.000	$5.0 \cdot 10^{-28}$	0.94765	0.815
$1/5 \cdot 2^{-12}$	$6.0 \cdot 10^{-23}$	0.93183	6.000	$2.8 \cdot 10^{-28}$	0.04101	-0.928
$1/5 \cdot 2^{-13}$	$9.4 \cdot 10^{-25}$	0.93186		$5.4 \cdot 10^{-28}$	0.00012	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.0 \cdot 10^{-04}$	0.90000	10.809	$3.8 \cdot 10^{-05}$	0.90000	12.990
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-07}$	0.95000	10.275	$4.7 \cdot 10^{-09}$	0.95000	12.343
$1/5 \cdot 2^{-3}$	$9.3 \cdot 10^{-11}$	0.95000	10.065	$9.0 \cdot 10^{-13}$	0.95000	5.060
$1/5 \cdot 2^{-4}$	$8.7 \cdot 10^{-14}$	0.93750	0.846	$2.7 \cdot 10^{-14}$	-0.0000	-0.836
$1/5 \cdot 2^{-5}$	$4.8 \cdot 10^{-14}$	-0.0000	19.163	$4.8 \cdot 10^{-14}$	-0.0000	31.882
$1/5 \cdot 2^{-6}$	$8.2 \cdot 10^{-20}$	0.94375	10.001	$1.2 \cdot 10^{-23}$	0.94687	12.004
$1/5 \cdot 2^{-7}$	$8.0 \cdot 10^{-23}$	0.94375	10.000	$2.9 \cdot 10^{-27}$	0.94687	6.129
$1/5 \cdot 2^{-8}$	$7.8 \cdot 10^{-26}$	0.94375	10.063	$4.2 \cdot 10^{-29}$	0.89765	-0.547
$1/5 \cdot 2^{-9}$	$7.3 \cdot 10^{-29}$	0.94101	0.237	$6.2 \cdot 10^{-29}$	0.02265	-0.005
$1/5 \cdot 2^{-10}$	$6.2 \cdot 10^{-29}$	0.90742	-1.386	$6.2 \cdot 10^{-29}$	0.90742	-1.353
$1/5 \cdot 2^{-11}$	$1.6 \cdot 10^{-28}$	0.03808	-0.810	$1.6 \cdot 10^{-28}$	0.03808	-0.923
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-28}$	0.04101		$3.0 \cdot 10^{-28}$	0.04101	

Table 5.19: Box scheme, IDeC method on $[0, 1]$ for 4.1c.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.3 \cdot 10^{-06}$	10.492	$6.8 \cdot 10^{-06}$	10.605
$1/5 \cdot 2^{-2}$	$9.4 \cdot 10^{-10}$	10.114	$4.3 \cdot 10^{-09}$	10.139
$1/5 \cdot 2^{-3}$	$8.5 \cdot 10^{-13}$	10.028	$3.8 \cdot 10^{-12}$	10.034
$1/5 \cdot 2^{-4}$	$8.1 \cdot 10^{-16}$	10.006	$3.6 \cdot 10^{-15}$	10.008
$1/5 \cdot 2^{-5}$	$7.9 \cdot 10^{-19}$	10.001	$3.5 \cdot 10^{-18}$	10.002
$1/5 \cdot 2^{-6}$	$7.7 \cdot 10^{-22}$	10.000	$3.4 \cdot 10^{-21}$	10.000
$1/5 \cdot 2^{-7}$	$7.5 \cdot 10^{-25}$	10.000	$3.4 \cdot 10^{-24}$	10.000
$1/5 \cdot 2^{-8}$	$7.3 \cdot 10^{-28}$	9.585	$3.3 \cdot 10^{-27}$	9.825
$1/5 \cdot 2^{-9}$	$9.6 \cdot 10^{-31}$	-0.442	$3.6 \cdot 10^{-30}$	-4.717
$1/5 \cdot 2^{-10}$	$1.3 \cdot 10^{-30}$	-1.771	$9.6 \cdot 10^{-29}$	-0.454
$1/5 \cdot 2^{-11}$	$4.4 \cdot 10^{-30}$	-0.762	$1.3 \cdot 10^{-28}$	-0.750
$1/5 \cdot 2^{-12}$	$7.5 \cdot 10^{-30}$	-1.151	$2.2 \cdot 10^{-28}$	-1.008
$1/5 \cdot 2^{-13}$	$1.6 \cdot 10^{-29}$	-0.864	$4.4 \cdot 10^{-28}$	-1.044
$1/5 \cdot 2^{-14}$	$3.0 \cdot 10^{-29}$		$9.2 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.3 \cdot 10^{-05}$	10.788	$1.3 \cdot 10^{-05}$	11.122
$1/5 \cdot 2^{-2}$	$7.6 \cdot 10^{-09}$	10.182	$6.1 \cdot 10^{-09}$	10.265
$1/5 \cdot 2^{-3}$	$6.5 \cdot 10^{-12}$	10.044	$4.9 \cdot 10^{-12}$	10.065
$1/5 \cdot 2^{-4}$	$6.2 \cdot 10^{-15}$	10.011	$4.6 \cdot 10^{-15}$	10.016
$1/5 \cdot 2^{-5}$	$6.0 \cdot 10^{-18}$	10.002	$4.4 \cdot 10^{-18}$	10.004
$1/5 \cdot 2^{-6}$	$5.8 \cdot 10^{-21}$	10.000	$4.3 \cdot 10^{-21}$	10.001
$1/5 \cdot 2^{-7}$	$5.7 \cdot 10^{-24}$	10.000	$4.2 \cdot 10^{-24}$	10.000
$1/5 \cdot 2^{-8}$	$5.6 \cdot 10^{-27}$	6.991	$4.1 \cdot 10^{-27}$	6.266
$1/5 \cdot 2^{-9}$	$4.4 \cdot 10^{-29}$	1.437	$5.4 \cdot 10^{-29}$	5.579
$1/5 \cdot 2^{-10}$	$1.6 \cdot 10^{-29}$	-2.492	$1.1 \cdot 10^{-30}$	-7.077
$1/5 \cdot 2^{-11}$	$9.1 \cdot 10^{-29}$	-0.611	$1.5 \cdot 10^{-28}$	-0.788
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-28}$	-1.110	$2.6 \cdot 10^{-28}$	-1.045
$1/5 \cdot 2^{-13}$	$3.0 \cdot 10^{-28}$		$5.4 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$7.7 \cdot 10^{-06}$	11.838	$2.7 \cdot 10^{-06}$	13.722
$1/5 \cdot 2^{-2}$	$2.1 \cdot 10^{-09}$	10.490	$2.0 \cdot 10^{-10}$	12.416
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-12}$	10.126	$3.7 \cdot 10^{-14}$	12.103
$1/5 \cdot 2^{-4}$	$1.3 \cdot 10^{-15}$	10.031	$8.4 \cdot 10^{-18}$	12.025
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-18}$	10.008	$2.0 \cdot 10^{-21}$	12.006
$1/5 \cdot 2^{-6}$	$1.2 \cdot 10^{-21}$	10.002	$4.9 \cdot 10^{-25}$	12.001
$1/5 \cdot 2^{-7}$	$1.1 \cdot 10^{-24}$	10.000	$1.1 \cdot 10^{-28}$	3.772
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-27}$	4.382	$8.7 \cdot 10^{-30}$	-2.702
$1/5 \cdot 2^{-9}$	$5.5 \cdot 10^{-29}$	3.078	$5.6 \cdot 10^{-29}$	2.551
$1/5 \cdot 2^{-10}$	$6.5 \cdot 10^{-30}$	-4.535	$9.7 \cdot 10^{-30}$	-3.924
$1/5 \cdot 2^{-11}$	$1.5 \cdot 10^{-28}$	-0.791	$1.4 \cdot 10^{-28}$	-0.920
$1/5 \cdot 2^{-12}$	$2.6 \cdot 10^{-28}$		$2.7 \cdot 10^{-28}$	

Table 5.20: Box scheme, IDeC method at $t = 0$ for 4.1c.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5\cdot 2^{-1}$	$3.0\cdot 10^{-02}$	0.20000	2.002	$3.5\cdot 10^{-04}$	-0.0000	3.053
$1/5\cdot 2^{-2}$	$7.6\cdot 10^{-03}$	0.20000	2.000	$4.2\cdot 10^{-05}$	0.00000	3.027
$1/5\cdot 2^{-3}$	$1.9\cdot 10^{-03}$	0.20000	2.000	$5.2\cdot 10^{-06}$	0.00000	3.013
$1/5\cdot 2^{-4}$	$4.7\cdot 10^{-04}$	0.20000	2.000	$6.4\cdot 10^{-07}$	-0.0000	3.006
$1/5\cdot 2^{-5}$	$1.1\cdot 10^{-04}$	0.20000	1.999	$8.0\cdot 10^{-08}$	-0.0000	3.003
$1/5\cdot 2^{-6}$	$2.9\cdot 10^{-05}$	0.19687	2.000	$1.0\cdot 10^{-08}$	0.00000	3.001
$1/5\cdot 2^{-7}$	$7.4\cdot 10^{-06}$	0.19687	2.000	$1.2\cdot 10^{-09}$	0.00000	3.000
$1/5\cdot 2^{-8}$	$1.8\cdot 10^{-06}$	0.19687	1.999	$1.5\cdot 10^{-10}$	-0.0000	3.000
$1/5\cdot 2^{-9}$	$4.6\cdot 10^{-07}$	0.19726	2.000	$1.9\cdot 10^{-11}$	-0.0000	3.000
$1/5\cdot 2^{-10}$	$1.1\cdot 10^{-07}$	0.19726	2.000	$2.4\cdot 10^{-12}$	0.00000	3.000
$1/5\cdot 2^{-11}$	$2.9\cdot 10^{-08}$	0.19716	1.999	$3.0\cdot 10^{-13}$	0.00000	3.000
$1/5\cdot 2^{-12}$	$7.2\cdot 10^{-09}$	0.19721	2.000	$3.8\cdot 10^{-14}$	-0.0000	3.000
$1/5\cdot 2^{-13}$	$1.8\cdot 10^{-09}$	0.19721	1.999	$4.7\cdot 10^{-15}$	-0.0000	2.999
$1/5\cdot 2^{-14}$	$4.5\cdot 10^{-10}$	0.19720		$5.9\cdot 10^{-16}$	0.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5\cdot 2^{-1}$	$5.3\cdot 10^{-05}$	-0.0000	3.001	$1.2\cdot 10^{-05}$	-0.0000	3.001
$1/5\cdot 2^{-2}$	$6.6\cdot 10^{-06}$	0.00000	3.000	$1.5\cdot 10^{-06}$	0.00000	3.000
$1/5\cdot 2^{-3}$	$8.2\cdot 10^{-07}$	0.00000	3.000	$1.9\cdot 10^{-07}$	0.00000	3.000
$1/5\cdot 2^{-4}$	$1.0\cdot 10^{-07}$	-0.0000	3.000	$2.4\cdot 10^{-08}$	-0.0000	3.000
$1/5\cdot 2^{-5}$	$1.2\cdot 10^{-08}$	-0.0000	3.000	$3.1\cdot 10^{-09}$	-0.0000	3.000
$1/5\cdot 2^{-6}$	$1.6\cdot 10^{-09}$	0.00000	3.000	$3.8\cdot 10^{-10}$	0.00000	3.000
$1/5\cdot 2^{-7}$	$2.0\cdot 10^{-10}$	0.00000	3.000	$4.8\cdot 10^{-11}$	0.00000	3.000
$1/5\cdot 2^{-8}$	$2.5\cdot 10^{-11}$	-0.0000	3.000	$6.0\cdot 10^{-12}$	-0.0000	3.000
$1/5\cdot 2^{-9}$	$3.1\cdot 10^{-12}$	-0.0000	3.000	$7.5\cdot 10^{-13}$	-0.0000	3.000
$1/5\cdot 2^{-10}$	$3.9\cdot 10^{-13}$	0.00000	3.000	$9.4\cdot 10^{-14}$	0.00000	3.000
$1/5\cdot 2^{-11}$	$4.9\cdot 10^{-14}$	0.00000	3.000	$1.1\cdot 10^{-14}$	0.00000	3.000
$1/5\cdot 2^{-12}$	$6.1\cdot 10^{-15}$	-0.0000	3.000	$1.4\cdot 10^{-15}$	-0.0000	3.000
$1/5\cdot 2^{-13}$	$7.7\cdot 10^{-16}$	-0.0000		$1.8\cdot 10^{-16}$	-0.0000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5\cdot 2^{-1}$	$3.1\cdot 10^{-06}$	-0.0000	3.001	$7.8\cdot 10^{-07}$	-0.0000	3.003
$1/5\cdot 2^{-2}$	$3.9\cdot 10^{-07}$	0.00000	3.000	$9.7\cdot 10^{-08}$	0.00000	3.000
$1/5\cdot 2^{-3}$	$4.9\cdot 10^{-08}$	0.00000	3.000	$1.2\cdot 10^{-08}$	0.00000	3.000
$1/5\cdot 2^{-4}$	$6.1\cdot 10^{-09}$	-0.0000	3.000	$1.5\cdot 10^{-09}$	-0.0000	3.000
$1/5\cdot 2^{-5}$	$7.6\cdot 10^{-10}$	-0.0000	3.000	$1.9\cdot 10^{-10}$	-0.0000	3.000
$1/5\cdot 2^{-6}$	$9.6\cdot 10^{-11}$	0.00000	3.000	$2.3\cdot 10^{-11}$	0.00000	3.000
$1/5\cdot 2^{-7}$	$1.2\cdot 10^{-11}$	0.00000	3.000	$2.9\cdot 10^{-12}$	0.00000	3.000
$1/5\cdot 2^{-8}$	$1.5\cdot 10^{-12}$	-0.0000	3.000	$3.7\cdot 10^{-13}$	-0.0000	3.000
$1/5\cdot 2^{-9}$	$1.8\cdot 10^{-13}$	-0.0000	3.000	$4.6\cdot 10^{-14}$	-0.0000	3.000
$1/5\cdot 2^{-10}$	$2.3\cdot 10^{-14}$	0.00000	3.000	$5.8\cdot 10^{-15}$	0.00000	3.000
$1/5\cdot 2^{-11}$	$2.9\cdot 10^{-15}$	0.00000	3.000	$7.2\cdot 10^{-16}$	0.00000	3.000
$1/5\cdot 2^{-12}$	$3.6\cdot 10^{-16}$	-0.0000		$9.1\cdot 10^{-17}$	-0.0000	

Table 5.21: Box scheme, IDeC method on $[0, 1]$ for 4.1d.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$2.7 \cdot 10^{-02}$	2.020	$3.5 \cdot 10^{-04}$	3.053
$1/5 \cdot 2^{-2}$	$6.7 \cdot 10^{-03}$	2.010	$4.2 \cdot 10^{-05}$	3.027
$1/5 \cdot 2^{-3}$	$1.6 \cdot 10^{-03}$	2.005	$5.2 \cdot 10^{-06}$	3.013
$1/5 \cdot 2^{-4}$	$4.2 \cdot 10^{-04}$	2.002	$6.4 \cdot 10^{-07}$	3.006
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-04}$	2.001	$8.0 \cdot 10^{-08}$	3.003
$1/5 \cdot 2^{-6}$	$2.6 \cdot 10^{-05}$	2.000	$1.0 \cdot 10^{-08}$	3.001
$1/5 \cdot 2^{-7}$	$6.5 \cdot 10^{-06}$	2.000	$1.2 \cdot 10^{-09}$	3.000
$1/5 \cdot 2^{-8}$	$1.6 \cdot 10^{-06}$	2.000	$1.5 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-9}$	$4.0 \cdot 10^{-07}$	2.000	$1.9 \cdot 10^{-11}$	3.000
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-07}$	2.000	$2.4 \cdot 10^{-12}$	3.000
$1/5 \cdot 2^{-11}$	$2.5 \cdot 10^{-08}$	2.000	$3.0 \cdot 10^{-13}$	3.000
$1/5 \cdot 2^{-12}$	$6.3 \cdot 10^{-09}$	2.000	$3.8 \cdot 10^{-14}$	3.000
$1/5 \cdot 2^{-13}$	$1.5 \cdot 10^{-09}$	1.999	$4.7 \cdot 10^{-15}$	2.999
$1/5 \cdot 2^{-14}$	$3.9 \cdot 10^{-10}$		$5.9 \cdot 10^{-16}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$5.3 \cdot 10^{-05}$	3.001	$1.2 \cdot 10^{-05}$	3.001
$1/5 \cdot 2^{-2}$	$6.6 \cdot 10^{-06}$	3.000	$1.5 \cdot 10^{-06}$	3.000
$1/5 \cdot 2^{-3}$	$8.2 \cdot 10^{-07}$	3.000	$1.9 \cdot 10^{-07}$	3.000
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-07}$	3.000	$2.4 \cdot 10^{-08}$	3.000
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-08}$	3.000	$3.1 \cdot 10^{-09}$	3.000
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-09}$	3.000	$3.8 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-10}$	3.000	$4.8 \cdot 10^{-11}$	3.000
$1/5 \cdot 2^{-8}$	$2.5 \cdot 10^{-11}$	3.000	$6.0 \cdot 10^{-12}$	3.000
$1/5 \cdot 2^{-9}$	$3.1 \cdot 10^{-12}$	3.000	$7.5 \cdot 10^{-13}$	3.000
$1/5 \cdot 2^{-10}$	$3.9 \cdot 10^{-13}$	3.000	$9.4 \cdot 10^{-14}$	3.000
$1/5 \cdot 2^{-11}$	$4.9 \cdot 10^{-14}$	3.000	$1.1 \cdot 10^{-14}$	3.000
$1/5 \cdot 2^{-12}$	$6.1 \cdot 10^{-15}$	3.000	$1.4 \cdot 10^{-15}$	3.000
$1/5 \cdot 2^{-13}$	$7.7 \cdot 10^{-16}$		$1.8 \cdot 10^{-16}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$3.1 \cdot 10^{-06}$	3.001	$7.8 \cdot 10^{-07}$	3.003
$1/5 \cdot 2^{-2}$	$3.9 \cdot 10^{-07}$	3.000	$9.7 \cdot 10^{-08}$	3.000
$1/5 \cdot 2^{-3}$	$4.9 \cdot 10^{-08}$	3.000	$1.2 \cdot 10^{-08}$	3.000
$1/5 \cdot 2^{-4}$	$6.1 \cdot 10^{-09}$	3.000	$1.5 \cdot 10^{-09}$	3.000
$1/5 \cdot 2^{-5}$	$7.6 \cdot 10^{-10}$	3.000	$1.9 \cdot 10^{-10}$	3.000
$1/5 \cdot 2^{-6}$	$9.6 \cdot 10^{-11}$	3.000	$2.3 \cdot 10^{-11}$	3.000
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-11}$	3.000	$2.9 \cdot 10^{-12}$	3.000
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-12}$	3.000	$3.7 \cdot 10^{-13}$	3.000
$1/5 \cdot 2^{-9}$	$1.8 \cdot 10^{-13}$	3.000	$4.6 \cdot 10^{-14}$	3.000
$1/5 \cdot 2^{-10}$	$2.3 \cdot 10^{-14}$	3.000	$5.8 \cdot 10^{-15}$	3.000
$1/5 \cdot 2^{-11}$	$2.9 \cdot 10^{-15}$	3.000	$7.2 \cdot 10^{-16}$	3.000
$1/5 \cdot 2^{-12}$	$3.6 \cdot 10^{-16}$		$9.1 \cdot 10^{-17}$	

Table 5.22: Box scheme, IDeC method at $t = 0$ for 4.1d.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.3 \cdot 10^{-02}$	0.60000	2.012	$4.8 \cdot 10^{-04}$	0.70000	4.010
$1/5 \cdot 2^{-2}$	$1.0 \cdot 10^{-02}$	0.60000	1.998	$3.0 \cdot 10^{-05}$	0.65000	4.007
$1/5 \cdot 2^{-3}$	$2.7 \cdot 10^{-03}$	0.62500	2.000	$1.8 \cdot 10^{-06}$	0.65000	4.001
$1/5 \cdot 2^{-4}$	$6.7 \cdot 10^{-04}$	0.62500	2.000	$1.1 \cdot 10^{-07}$	0.66250	4.000
$1/5 \cdot 2^{-5}$	$1.6 \cdot 10^{-04}$	0.62500	2.000	$7.3 \cdot 10^{-09}$	0.65625	4.000
$1/5 \cdot 2^{-6}$	$4.2 \cdot 10^{-05}$	0.62500	2.000	$4.5 \cdot 10^{-10}$	0.65625	4.000
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-05}$	0.62500	2.000	$2.8 \cdot 10^{-11}$	0.65625	4.000
$1/5 \cdot 2^{-8}$	$2.6 \cdot 10^{-06}$	0.62500	2.000	$1.7 \cdot 10^{-12}$	0.65703	4.000
$1/5 \cdot 2^{-9}$	$6.6 \cdot 10^{-07}$	0.62500	2.000	$1.1 \cdot 10^{-13}$	0.65703	4.000
$1/5 \cdot 2^{-10}$	$1.6 \cdot 10^{-07}$	0.62500	2.000	$6.9 \cdot 10^{-15}$	0.65683	4.000
$1/5 \cdot 2^{-11}$	$4.1 \cdot 10^{-08}$	0.62509	2.000	$4.3 \cdot 10^{-16}$	0.65693	4.000
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-08}$	0.62509	2.000	$2.7 \cdot 10^{-17}$	0.65688	4.000
$1/5 \cdot 2^{-13}$	$2.5 \cdot 10^{-09}$	0.62507	1.999	$1.7 \cdot 10^{-18}$	0.65690	3.999
$1/5 \cdot 2^{-14}$	$6.4 \cdot 10^{-10}$	0.62507		$1.0 \cdot 10^{-19}$	0.65690	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$9.3 \cdot 10^{-06}$	-0.0000	5.123	$9.8 \cdot 10^{-07}$	-0.0000	4.981
$1/5 \cdot 2^{-2}$	$2.6 \cdot 10^{-07}$	0.00000	5.061	$3.1 \cdot 10^{-08}$	0.00000	4.995
$1/5 \cdot 2^{-3}$	$8.0 \cdot 10^{-09}$	0.00000	5.031	$9.7 \cdot 10^{-10}$	0.00000	4.998
$1/5 \cdot 2^{-4}$	$2.4 \cdot 10^{-10}$	-0.0000	5.015	$3.0 \cdot 10^{-11}$	-0.0000	4.999
$1/5 \cdot 2^{-5}$	$7.6 \cdot 10^{-12}$	-0.0000	5.007	$9.5 \cdot 10^{-13}$	-0.0000	4.999
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-13}$	0.00000	5.003	$2.9 \cdot 10^{-14}$	0.00000	4.999
$1/5 \cdot 2^{-7}$	$7.3 \cdot 10^{-15}$	0.00000	5.001	$9.3 \cdot 10^{-16}$	0.00000	4.999
$1/5 \cdot 2^{-8}$	$2.3 \cdot 10^{-16}$	-0.0000	5.000	$2.9 \cdot 10^{-17}$	-0.0000	4.999
$1/5 \cdot 2^{-9}$	$7.2 \cdot 10^{-18}$	-0.0000	5.000	$9.1 \cdot 10^{-19}$	-0.0000	4.999
$1/5 \cdot 2^{-10}$	$2.2 \cdot 10^{-19}$	0.00000	5.000	$2.8 \cdot 10^{-20}$	0.00000	4.999
$1/5 \cdot 2^{-11}$	$7.0 \cdot 10^{-21}$	0.00000	5.000	$8.8 \cdot 10^{-22}$	0.00000	4.999
$1/5 \cdot 2^{-12}$	$2.1 \cdot 10^{-22}$	-0.0000	5.000	$2.7 \cdot 10^{-23}$	-0.0000	5.000
$1/5 \cdot 2^{-13}$	$6.8 \cdot 10^{-24}$	-0.0000		$8.6 \cdot 10^{-25}$	-0.0000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.1 \cdot 10^{-07}$	-0.0000	4.998	$5.3 \cdot 10^{-08}$	-0.0000	5.023
$1/5 \cdot 2^{-2}$	$6.8 \cdot 10^{-09}$	0.00000	4.997	$1.6 \cdot 10^{-09}$	0.00000	4.998
$1/5 \cdot 2^{-3}$	$2.1 \cdot 10^{-10}$	0.00000	4.999	$5.1 \cdot 10^{-11}$	0.00000	4.999
$1/5 \cdot 2^{-4}$	$6.7 \cdot 10^{-12}$	-0.0000	4.999	$1.6 \cdot 10^{-12}$	-0.0000	4.999
$1/5 \cdot 2^{-5}$	$2.0 \cdot 10^{-13}$	-0.0000	4.999	$5.0 \cdot 10^{-14}$	-0.0000	4.999
$1/5 \cdot 2^{-6}$	$6.5 \cdot 10^{-15}$	0.00000	4.999	$1.5 \cdot 10^{-15}$	0.00000	4.999
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-16}$	0.00000	4.999	$4.9 \cdot 10^{-17}$	0.00000	4.999
$1/5 \cdot 2^{-8}$	$6.4 \cdot 10^{-18}$	-0.0000	4.999	$1.5 \cdot 10^{-18}$	-0.0000	4.999
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-19}$	-0.0000	4.999	$4.8 \cdot 10^{-20}$	-0.0000	4.999
$1/5 \cdot 2^{-10}$	$6.2 \cdot 10^{-21}$	0.00000	4.999	$1.5 \cdot 10^{-21}$	0.00000	4.999
$1/5 \cdot 2^{-11}$	$1.9 \cdot 10^{-22}$	0.00000	4.999	$4.6 \cdot 10^{-23}$	0.00000	4.999
$1/5 \cdot 2^{-12}$	$6.1 \cdot 10^{-24}$	-0.0000		$1.4 \cdot 10^{-24}$	-0.0000	

Table 5.23: Box scheme, IDeC method on $[0, 1]$ for 4.1e.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$4.8 \cdot 10^{-03}$	1.958	$2.1 \cdot 10^{-04}$	4.132
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-03}$	1.990	$1.2 \cdot 10^{-05}$	4.066
$1/5 \cdot 2^{-3}$	$3.1 \cdot 10^{-04}$	1.997	$7.1 \cdot 10^{-07}$	4.033
$1/5 \cdot 2^{-4}$	$7.8 \cdot 10^{-05}$	1.999	$4.3 \cdot 10^{-08}$	4.016
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-05}$	1.999	$2.7 \cdot 10^{-09}$	4.008
$1/5 \cdot 2^{-6}$	$4.8 \cdot 10^{-06}$	1.999	$1.6 \cdot 10^{-10}$	4.004
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-06}$	1.999	$1.0 \cdot 10^{-11}$	4.002
$1/5 \cdot 2^{-8}$	$3.0 \cdot 10^{-07}$	1.999	$6.5 \cdot 10^{-13}$	4.001
$1/5 \cdot 2^{-9}$	$7.6 \cdot 10^{-08}$	1.999	$4.0 \cdot 10^{-14}$	4.000
$1/5 \cdot 2^{-10}$	$1.9 \cdot 10^{-08}$	1.999	$2.5 \cdot 10^{-15}$	4.000
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-09}$	1.999	$1.5 \cdot 10^{-16}$	4.000
$1/5 \cdot 2^{-12}$	$1.1 \cdot 10^{-09}$	2.000	$9.9 \cdot 10^{-18}$	4.000
$1/5 \cdot 2^{-13}$	$2.9 \cdot 10^{-10}$	1.999	$6.2 \cdot 10^{-19}$	3.999
$1/5 \cdot 2^{-14}$	$7.4 \cdot 10^{-11}$		$3.8 \cdot 10^{-20}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$9.3 \cdot 10^{-06}$	5.123	$9.8 \cdot 10^{-07}$	4.981
$1/5 \cdot 2^{-2}$	$2.6 \cdot 10^{-07}$	5.061	$3.1 \cdot 10^{-08}$	4.995
$1/5 \cdot 2^{-3}$	$8.0 \cdot 10^{-09}$	5.031	$9.7 \cdot 10^{-10}$	4.998
$1/5 \cdot 2^{-4}$	$2.4 \cdot 10^{-10}$	5.015	$3.0 \cdot 10^{-11}$	4.999
$1/5 \cdot 2^{-5}$	$7.6 \cdot 10^{-12}$	5.007	$9.5 \cdot 10^{-13}$	4.999
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-13}$	5.003	$2.9 \cdot 10^{-14}$	4.999
$1/5 \cdot 2^{-7}$	$7.3 \cdot 10^{-15}$	5.001	$9.3 \cdot 10^{-16}$	4.999
$1/5 \cdot 2^{-8}$	$2.3 \cdot 10^{-16}$	5.000	$2.9 \cdot 10^{-17}$	4.999
$1/5 \cdot 2^{-9}$	$7.2 \cdot 10^{-18}$	5.000	$9.1 \cdot 10^{-19}$	4.999
$1/5 \cdot 2^{-10}$	$2.2 \cdot 10^{-19}$	5.000	$2.8 \cdot 10^{-20}$	4.999
$1/5 \cdot 2^{-11}$	$7.0 \cdot 10^{-21}$	5.000	$8.8 \cdot 10^{-22}$	4.999
$1/5 \cdot 2^{-12}$	$2.1 \cdot 10^{-22}$	5.000	$2.7 \cdot 10^{-23}$	5.000
$1/5 \cdot 2^{-13}$	$6.8 \cdot 10^{-24}$		$8.6 \cdot 10^{-25}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$2.1 \cdot 10^{-07}$	4.998	$5.3 \cdot 10^{-08}$	5.023
$1/5 \cdot 2^{-2}$	$6.8 \cdot 10^{-09}$	4.997	$1.6 \cdot 10^{-09}$	4.998
$1/5 \cdot 2^{-3}$	$2.1 \cdot 10^{-10}$	4.999	$5.1 \cdot 10^{-11}$	4.999
$1/5 \cdot 2^{-4}$	$6.7 \cdot 10^{-12}$	4.999	$1.6 \cdot 10^{-12}$	4.999
$1/5 \cdot 2^{-5}$	$2.0 \cdot 10^{-13}$	4.999	$5.0 \cdot 10^{-14}$	4.999
$1/5 \cdot 2^{-6}$	$6.5 \cdot 10^{-15}$	4.999	$1.5 \cdot 10^{-15}$	4.999
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-16}$	4.999	$4.9 \cdot 10^{-17}$	4.999
$1/5 \cdot 2^{-8}$	$6.4 \cdot 10^{-18}$	4.999	$1.5 \cdot 10^{-18}$	4.999
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-19}$	4.999	$4.8 \cdot 10^{-20}$	4.999
$1/5 \cdot 2^{-10}$	$6.2 \cdot 10^{-21}$	4.999	$1.5 \cdot 10^{-21}$	4.999
$1/5 \cdot 2^{-11}$	$1.9 \cdot 10^{-22}$	4.999	$4.6 \cdot 10^{-23}$	4.999
$1/5 \cdot 2^{-12}$	$6.1 \cdot 10^{-24}$		$1.4 \cdot 10^{-24}$	

Table 5.24: Box scheme, IDeC method at $t = 0$ for 4.1e.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.3 \cdot 10^{-01}$	0.90000	2.134	$5.1 \cdot 10^{-02}$	0.90000	4.358
$1/5 \cdot 2^{-2}$	$9.9 \cdot 10^{-02}$	0.90000	2.030	$2.4 \cdot 10^{-03}$	0.90000	4.040
$1/5 \cdot 2^{-3}$	$2.4 \cdot 10^{-02}$	0.90000	1.999	$1.5 \cdot 10^{-04}$	0.92500	4.020
$1/5 \cdot 2^{-4}$	$6.0 \cdot 10^{-03}$	0.88750	2.001	$9.3 \cdot 10^{-06}$	0.92500	4.000
$1/5 \cdot 2^{-5}$	$1.5 \cdot 10^{-03}$	0.88750	2.000	$5.8 \cdot 10^{-07}$	0.91875	4.001
$1/5 \cdot 2^{-6}$	$3.7 \cdot 10^{-04}$	0.88750	1.999	$3.6 \cdot 10^{-08}$	0.91875	4.000
$1/5 \cdot 2^{-7}$	$9.4 \cdot 10^{-05}$	0.88906	2.000	$2.2 \cdot 10^{-09}$	0.91875	4.000
$1/5 \cdot 2^{-8}$	$2.3 \cdot 10^{-05}$	0.88906	2.000	$1.4 \cdot 10^{-10}$	0.91875	4.000
$1/5 \cdot 2^{-9}$	$5.9 \cdot 10^{-06}$	0.88906	2.000	$8.8 \cdot 10^{-12}$	0.91914	4.000
$1/5 \cdot 2^{-10}$	$1.4 \cdot 10^{-06}$	0.88886	1.999	$5.5 \cdot 10^{-13}$	0.91894	4.000
$1/5 \cdot 2^{-11}$	$3.7 \cdot 10^{-07}$	0.88896	2.000	$3.4 \cdot 10^{-14}$	0.91904	4.000
$1/5 \cdot 2^{-12}$	$9.2 \cdot 10^{-08}$	0.88896	2.000	$2.1 \cdot 10^{-15}$	0.91899	4.000
$1/5 \cdot 2^{-13}$	$2.3 \cdot 10^{-08}$	0.88896	1.999	$1.3 \cdot 10^{-16}$	0.91901	3.999
$1/5 \cdot 2^{-14}$	$5.7 \cdot 10^{-09}$	0.88896		$8.4 \cdot 10^{-18}$	0.91901	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$7.6 \cdot 10^{-03}$	0.90000	6.537	$1.2 \cdot 10^{-03}$	0.90000	8.657
$1/5 \cdot 2^{-2}$	$8.1 \cdot 10^{-05}$	0.95000	6.075	$3.1 \cdot 10^{-06}$	0.95000	8.208
$1/5 \cdot 2^{-3}$	$1.2 \cdot 10^{-06}$	0.92500	6.034	$1.0 \cdot 10^{-08}$	0.95000	8.015
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-08}$	0.92500	6.001	$4.0 \cdot 10^{-11}$	0.93750	8.012
$1/5 \cdot 2^{-5}$	$2.8 \cdot 10^{-10}$	0.93125	6.002	$1.5 \cdot 10^{-13}$	0.93750	8.003
$1/5 \cdot 2^{-6}$	$4.5 \cdot 10^{-12}$	0.93125	6.000	$6.1 \cdot 10^{-16}$	0.93750	8.000
$1/5 \cdot 2^{-7}$	$7.0 \cdot 10^{-14}$	0.93125	6.000	$2.4 \cdot 10^{-18}$	0.93906	8.000
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-15}$	0.93125	6.000	$9.4 \cdot 10^{-21}$	0.93828	8.000
$1/5 \cdot 2^{-9}$	$1.7 \cdot 10^{-17}$	0.93125	6.000	$3.6 \cdot 10^{-23}$	0.93828	8.000
$1/5 \cdot 2^{-10}$	$2.6 \cdot 10^{-19}$	0.93125	6.000	$1.4 \cdot 10^{-25}$	0.93847	7.910
$1/5 \cdot 2^{-11}$	$4.2 \cdot 10^{-21}$	0.93115	6.000	$5.9 \cdot 10^{-28}$	0.94765	0.433
$1/5 \cdot 2^{-12}$	$6.5 \cdot 10^{-23}$	0.93120	6.000	$4.4 \cdot 10^{-28}$	0.89838	-0.097
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-24}$	0.93110		$4.7 \cdot 10^{-28}$	0.00012	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.2 \cdot 10^{-04}$	0.90000	10.810	$4.1 \cdot 10^{-05}$	0.90000	12.990
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-07}$	0.95000	10.273	$5.0 \cdot 10^{-09}$	0.95000	12.341
$1/5 \cdot 2^{-3}$	$1.0 \cdot 10^{-10}$	0.95000	10.063	$9.7 \cdot 10^{-13}$	0.95000	5.157
$1/5 \cdot 2^{-4}$	$9.3 \cdot 10^{-14}$	0.93750	0.947	$2.7 \cdot 10^{-14}$	-0.0000	-0.836
$1/5 \cdot 2^{-5}$	$4.8 \cdot 10^{-14}$	-0.0000	19.063	$4.8 \cdot 10^{-14}$	-0.0000	31.778
$1/5 \cdot 2^{-6}$	$8.8 \cdot 10^{-20}$	0.94375	10.001	$1.3 \cdot 10^{-23}$	0.94687	11.977
$1/5 \cdot 2^{-7}$	$8.6 \cdot 10^{-23}$	0.94375	10.000	$3.2 \cdot 10^{-27}$	0.94843	5.495
$1/5 \cdot 2^{-8}$	$8.4 \cdot 10^{-26}$	0.94375	9.996	$7.2 \cdot 10^{-29}$	0.89765	0.562
$1/5 \cdot 2^{-9}$	$8.2 \cdot 10^{-29}$	0.96054	-0.346	$4.9 \cdot 10^{-29}$	0.02656	-0.912
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-28}$	0.84550	-0.183	$9.2 \cdot 10^{-29}$	0.84433	-0.360
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-28}$	0.03808	-1.810	$1.1 \cdot 10^{-28}$	0.03808	-1.844
$1/5 \cdot 2^{-12}$	$4.1 \cdot 10^{-28}$	0.89838		$4.2 \cdot 10^{-28}$	0.89838	

Table 5.25: Box scheme, IDeC method on $[0, 1]$ for 4.1f.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-03}$	1.984	$7.9 \cdot 10^{-05}$	3.868
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-03}$	1.995	$5.4 \cdot 10^{-06}$	3.995
$1/5 \cdot 2^{-3}$	$3.1 \cdot 10^{-04}$	1.998	$3.4 \cdot 10^{-07}$	3.999
$1/5 \cdot 2^{-4}$	$7.8 \cdot 10^{-05}$	1.999	$2.1 \cdot 10^{-08}$	3.999
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-05}$	1.999	$1.3 \cdot 10^{-09}$	3.999
$1/5 \cdot 2^{-6}$	$4.8 \cdot 10^{-06}$	1.999	$8.3 \cdot 10^{-11}$	3.999
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-06}$	1.999	$5.2 \cdot 10^{-12}$	3.999
$1/5 \cdot 2^{-8}$	$3.0 \cdot 10^{-07}$	1.999	$3.2 \cdot 10^{-13}$	3.999
$1/5 \cdot 2^{-9}$	$7.6 \cdot 10^{-08}$	1.999	$2.0 \cdot 10^{-14}$	3.999
$1/5 \cdot 2^{-10}$	$1.9 \cdot 10^{-08}$	1.999	$1.2 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-09}$	1.999	$7.9 \cdot 10^{-17}$	3.999
$1/5 \cdot 2^{-12}$	$1.1 \cdot 10^{-09}$	2.000	$4.9 \cdot 10^{-18}$	4.000
$1/5 \cdot 2^{-13}$	$2.9 \cdot 10^{-10}$	1.999	$3.1 \cdot 10^{-19}$	3.999
$1/5 \cdot 2^{-14}$	$7.4 \cdot 10^{-11}$		$1.9 \cdot 10^{-20}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.4 \cdot 10^{-05}$	9.728	$1.4 \cdot 10^{-05}$	11.148
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-08}$	6.805	$6.1 \cdot 10^{-09}$	10.285
$1/5 \cdot 2^{-3}$	$1.5 \cdot 10^{-10}$	6.059	$4.9 \cdot 10^{-12}$	10.120
$1/5 \cdot 2^{-4}$	$2.2 \cdot 10^{-12}$	6.003	$4.4 \cdot 10^{-15}$	10.252
$1/5 \cdot 2^{-5}$	$3.5 \cdot 10^{-14}$	6.000	$3.6 \cdot 10^{-18}$	11.790
$1/5 \cdot 2^{-6}$	$5.5 \cdot 10^{-16}$	5.999	$1.0 \cdot 10^{-21}$	6.863
$1/5 \cdot 2^{-7}$	$8.6 \cdot 10^{-18}$	5.999	$8.8 \cdot 10^{-24}$	7.552
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-19}$	5.999	$4.6 \cdot 10^{-26}$	7.906
$1/5 \cdot 2^{-9}$	$2.1 \cdot 10^{-21}$	5.999	$1.9 \cdot 10^{-28}$	6.726
$1/5 \cdot 2^{-10}$	$3.2 \cdot 10^{-23}$	5.999	$1.8 \cdot 10^{-30}$	-5.739
$1/5 \cdot 2^{-11}$	$5.1 \cdot 10^{-25}$	5.999	$9.8 \cdot 10^{-29}$	-0.864
$1/5 \cdot 2^{-12}$	$8.0 \cdot 10^{-27}$	4.750	$1.7 \cdot 10^{-28}$	-1.392
$1/5 \cdot 2^{-13}$	$2.9 \cdot 10^{-28}$		$4.7 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$7.9 \cdot 10^{-06}$	11.866	$2.8 \cdot 10^{-06}$	13.733
$1/5 \cdot 2^{-2}$	$2.1 \cdot 10^{-09}$	10.501	$2.0 \cdot 10^{-10}$	12.415
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-12}$	10.129	$3.8 \cdot 10^{-14}$	12.095
$1/5 \cdot 2^{-4}$	$1.3 \cdot 10^{-15}$	10.032	$8.7 \cdot 10^{-18}$	12.007
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-18}$	10.008	$2.1 \cdot 10^{-21}$	11.969
$1/5 \cdot 2^{-6}$	$1.2 \cdot 10^{-21}$	10.002	$5.3 \cdot 10^{-25}$	11.930
$1/5 \cdot 2^{-7}$	$1.1 \cdot 10^{-24}$	10.000	$1.3 \cdot 10^{-28}$	5.487
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-27}$	4.666	$3.0 \cdot 10^{-30}$	-3.894
$1/5 \cdot 2^{-9}$	$4.5 \cdot 10^{-29}$	3.285	$4.5 \cdot 10^{-29}$	7.136
$1/5 \cdot 2^{-10}$	$4.6 \cdot 10^{-30}$	-4.519	$3.2 \cdot 10^{-31}$	-8.367
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-28}$	-0.828	$1.0 \cdot 10^{-28}$	-0.857
$1/5 \cdot 2^{-12}$	$1.9 \cdot 10^{-28}$		$1.9 \cdot 10^{-28}$	

Table 5.26: Box scheme, IDeC method at $t = 0$ for 4.1f.

5.3　Implicit Methods

Implicit methods like the implicit Euler method or the trapezoidal rule have to be treated separately for problem (5.1). The reason is that the computation of the numerical solution requires an evaluation of the right-hand side of the differential equation at $t = 0$. Due to the singularity, this evaluation is not possible. For initial value problems, we were able to modify the scheme by replacing this evaluation by the value of $z'(0)$ which can be computed using Taylor expansion. For the terminal value problem (5.1), this is not possible in general. Thus, different modifications of the scheme were tried out in order to enable the computation of the numerical solution. The variant that we are mainly going to discuss is to replace the evaluation of the right-hand side of the differential equation at $t = 0$ with the evaluation at a point $t = \varepsilon > 0$ which is of the order of magnitude of the roundoff error EPS. Surprisingly, this method, which looks rather simple and unstable at first sight, yields the best results. Alternatives like using an explicit method for the last step or freezing the Jacobian in the last step were found to yield no better performance. We are going to give a brief discussion of these modifications at the end of this section.

First, we consider IDeC based on the implicit Euler method applied to (5.1). Again, a polynomial degree $m = 5$ is used to define the Zadunaisky polynomials. Here, we replace the evaluation of the right-hand side by the evaluation at $t \approx \text{EPS}$. Strikingly, the results are very similar to the results obtained by the explicit Euler rule for the same problems. The main difference between the two methods is observed in the last step of the computation. While the error on the interval $[0, 1]$ shows a similar behavior as the maximal error for the explicit Euler method (except for problems 4.1b and 4.1c, where the error damping was extensively analyzed in §5.1.2), the special treatment of the endpoint in the implicit Euler method seems to improve the asymptotic qualities noticeably. Throughout, either the exact value at $t = 0$ is assumed or the classical order sequence with maximal attainable order 5 is observed.

Modifications of the last step of the implicit Euler scheme are discussed at the end of this section.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	1.5	0.50000	1.917	$3.6 \cdot 10^{-01}$	-0.0000	4.241
$1/5 \cdot 2^{-2}$	$4.0 \cdot 10^{-01}$	0.50000	1.985	$1.9 \cdot 10^{-02}$	0.35000	3.926
$1/5 \cdot 2^{-3}$	$1.0 \cdot 10^{-01}$	0.50000	1.983	$1.2 \cdot 10^{-03}$	0.37500	3.981
$1/5 \cdot 2^{-4}$	$2.6 \cdot 10^{-02}$	0.51250	1.998	$7.9 \cdot 10^{-05}$	0.37500	3.991
$1/5 \cdot 2^{-5}$	$6.5 \cdot 10^{-03}$	0.51250	1.999	$4.9 \cdot 10^{-06}$	0.36875	3.999
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-03}$	0.51250	1.999	$3.1 \cdot 10^{-07}$	0.36875	3.999
$1/5 \cdot 2^{-7}$	$4.0 \cdot 10^{-04}$	0.51250	1.999	$1.9 \cdot 10^{-08}$	0.36875	3.999
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-04}$	0.51250	1.999	$1.2 \cdot 10^{-09}$	0.36953	3.999
$1/5 \cdot 2^{-9}$	$2.5 \cdot 10^{-05}$	0.51289	1.999	$7.6 \cdot 10^{-11}$	0.36953	3.999
$1/5 \cdot 2^{-10}$	$6.3 \cdot 10^{-06}$	0.51289	1.999	$4.7 \cdot 10^{-12}$	0.36953	3.999
$1/5 \cdot 2^{-11}$	$1.5 \cdot 10^{-06}$	0.51289	1.999	$2.9 \cdot 10^{-13}$	0.36943	3.999
$1/5 \cdot 2^{-12}$	$3.9 \cdot 10^{-07}$	0.51289	2.000	$1.8 \cdot 10^{-14}$	0.36948	4.000
$1/5 \cdot 2^{-13}$	$9.9 \cdot 10^{-08}$	0.51289	1.999	$1.1 \cdot 10^{-15}$	0.36948	3.936
$1/5 \cdot 2^{-14}$	$2.4 \cdot 10^{-08}$	0.51289		$7.5 \cdot 10^{-17}$	0.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.5 \cdot 10^{-01}$	-0.0000	5.768	$1.2 \cdot 10^{-01}$	0.90000	7.696
$1/5 \cdot 2^{-2}$	$2.7 \cdot 10^{-03}$	0.00000	5.023	$5.9 \cdot 10^{-04}$	0.00000	5.743
$1/5 \cdot 2^{-3}$	$8.4 \cdot 10^{-05}$	0.00000	4.574	$1.1 \cdot 10^{-05}$	0.00000	4.127
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-06}$	-0.0000	4.203	$6.3 \cdot 10^{-07}$	-0.0000	4.013
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-07}$	-0.0000	4.055	$3.9 \cdot 10^{-08}$	-0.0000	3.999
$1/5 \cdot 2^{-6}$	$1.1 \cdot 10^{-08}$	0.00000	4.013	$2.4 \cdot 10^{-09}$	0.00000	3.998
$1/5 \cdot 2^{-7}$	$7.2 \cdot 10^{-10}$	0.00000	4.002	$1.5 \cdot 10^{-10}$	0.00000	3.999
$1/5 \cdot 2^{-8}$	$4.4 \cdot 10^{-11}$	-0.0000	4.000	$9.6 \cdot 10^{-12}$	-0.0000	3.999
$1/5 \cdot 2^{-9}$	$2.8 \cdot 10^{-12}$	-0.0000	4.000	$6.0 \cdot 10^{-13}$	-0.0000	3.999
$1/5 \cdot 2^{-10}$	$1.7 \cdot 10^{-13}$	0.00000	3.999	$3.7 \cdot 10^{-14}$	0.00000	3.999
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-14}$	0.00000	3.999	$2.3 \cdot 10^{-15}$	0.00000	3.999
$1/5 \cdot 2^{-12}$	$6.8 \cdot 10^{-16}$	-0.0000	4.000	$1.4 \cdot 10^{-16}$	-0.0000	4.000
$1/5 \cdot 2^{-13}$	$4.2 \cdot 10^{-17}$	-0.0000		$9.1 \cdot 10^{-18}$	-0.0000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.2 \cdot 10^{-01}$	-0.0000	9.228	$1.2 \cdot 10^{-01}$	-0.0000	9.321
$1/5 \cdot 2^{-2}$	$2.0 \cdot 10^{-04}$	0.00000	6.305	$1.8 \cdot 10^{-04}$	0.00000	8.104
$1/5 \cdot 2^{-3}$	$2.6 \cdot 10^{-06}$	0.00000	4.028	$6.8 \cdot 10^{-07}$	0.00000	4.009
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-07}$	-0.0000	4.009	$4.2 \cdot 10^{-08}$	-0.0000	4.013
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-08}$	-0.0000	3.999	$2.6 \cdot 10^{-09}$	-0.0000	4.000
$1/5 \cdot 2^{-6}$	$6.2 \cdot 10^{-10}$	0.00000	3.998	$1.6 \cdot 10^{-10}$	0.00000	3.999
$1/5 \cdot 2^{-7}$	$3.9 \cdot 10^{-11}$	0.00000	3.999	$1.0 \cdot 10^{-11}$	0.00000	3.999
$1/5 \cdot 2^{-8}$	$2.4 \cdot 10^{-12}$	-0.0000	3.999	$6.4 \cdot 10^{-13}$	-0.0000	3.999
$1/5 \cdot 2^{-9}$	$1.5 \cdot 10^{-13}$	-0.0000	3.999	$4.0 \cdot 10^{-14}$	-0.0000	3.999
$1/5 \cdot 2^{-10}$	$9.6 \cdot 10^{-15}$	0.00000	3.999	$2.5 \cdot 10^{-15}$	0.00000	3.999
$1/5 \cdot 2^{-11}$	$6.0 \cdot 10^{-16}$	0.00000	3.999	$1.5 \cdot 10^{-16}$	0.00000	3.999
$1/5 \cdot 2^{-12}$	$3.7 \cdot 10^{-17}$	-0.0000		$9.8 \cdot 10^{-18}$	-0.0000	

Table 5.27: Box scheme, IDeC method on $[0, 1]$ for 4.2a.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$3.2 \cdot 10^{-01}$	2.413	$3.6 \cdot 10^{-01}$	4.844
$1/5 \cdot 2^{-2}$	$6.1 \cdot 10^{-02}$	2.081	$1.2 \cdot 10^{-02}$	4.818
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-02}$	2.011	$4.4 \cdot 10^{-04}$	4.928
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-03}$	2.001	$1.4 \cdot 10^{-05}$	5.862
$1/5 \cdot 2^{-5}$	$8.9 \cdot 10^{-04}$	1.999	$2.5 \cdot 10^{-07}$	3.447
$1/5 \cdot 2^{-6}$	$2.2 \cdot 10^{-04}$	1.999	$2.2 \cdot 10^{-08}$	2.592
$1/5 \cdot 2^{-7}$	$5.5 \cdot 10^{-05}$	1.999	$3.8 \cdot 10^{-09}$	3.303
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-05}$	1.999	$3.8 \cdot 10^{-10}$	3.532
$1/5 \cdot 2^{-9}$	$3.4 \cdot 10^{-06}$	1.999	$3.3 \cdot 10^{-11}$	3.647
$1/5 \cdot 2^{-10}$	$8.7 \cdot 10^{-07}$	1.999	$2.6 \cdot 10^{-12}$	3.716
$1/5 \cdot 2^{-11}$	$2.1 \cdot 10^{-07}$	1.999	$2.0 \cdot 10^{-13}$	3.763
$1/5 \cdot 2^{-12}$	$5.4 \cdot 10^{-08}$	2.000	$1.4 \cdot 10^{-14}$	3.797
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-08}$	1.999	$1.0 \cdot 10^{-15}$	3.821
$1/5 \cdot 2^{-14}$	$3.4 \cdot 10^{-09}$		$7.5 \cdot 10^{-17}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.5 \cdot 10^{-01}$	5.768	$1.0 \cdot 10^{-01}$	7.416
$1/5 \cdot 2^{-2}$	$2.7 \cdot 10^{-03}$	5.023	$5.9 \cdot 10^{-04}$	5.743
$1/5 \cdot 2^{-3}$	$8.4 \cdot 10^{-05}$	4.574	$1.1 \cdot 10^{-05}$	4.127
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-06}$	4.203	$6.3 \cdot 10^{-07}$	4.013
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-07}$	4.055	$3.9 \cdot 10^{-08}$	3.999
$1/5 \cdot 2^{-6}$	$1.1 \cdot 10^{-08}$	4.013	$2.4 \cdot 10^{-09}$	3.998
$1/5 \cdot 2^{-7}$	$7.2 \cdot 10^{-10}$	4.002	$1.5 \cdot 10^{-10}$	3.999
$1/5 \cdot 2^{-8}$	$4.4 \cdot 10^{-11}$	4.000	$9.6 \cdot 10^{-12}$	3.999
$1/5 \cdot 2^{-9}$	$2.8 \cdot 10^{-12}$	4.000	$6.0 \cdot 10^{-13}$	3.999
$1/5 \cdot 2^{-10}$	$1.7 \cdot 10^{-13}$	3.999	$3.7 \cdot 10^{-14}$	3.999
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-14}$	3.999	$2.3 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-12}$	$6.8 \cdot 10^{-16}$	4.000	$1.4 \cdot 10^{-16}$	4.000
$1/5 \cdot 2^{-13}$	$4.2 \cdot 10^{-17}$		$9.1 \cdot 10^{-18}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.2 \cdot 10^{-01}$	9.228	$1.2 \cdot 10^{-01}$	9.321
$1/5 \cdot 2^{-2}$	$2.0 \cdot 10^{-04}$	6.305	$1.8 \cdot 10^{-04}$	8.104
$1/5 \cdot 2^{-3}$	$2.6 \cdot 10^{-06}$	4.028	$6.8 \cdot 10^{-07}$	4.009
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-07}$	4.009	$4.2 \cdot 10^{-08}$	4.013
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-08}$	3.999	$2.6 \cdot 10^{-09}$	4.000
$1/5 \cdot 2^{-6}$	$6.2 \cdot 10^{-10}$	3.998	$1.6 \cdot 10^{-10}$	3.999
$1/5 \cdot 2^{-7}$	$3.9 \cdot 10^{-11}$	3.999	$1.0 \cdot 10^{-11}$	3.999
$1/5 \cdot 2^{-8}$	$2.4 \cdot 10^{-12}$	3.999	$6.4 \cdot 10^{-13}$	3.999
$1/5 \cdot 2^{-9}$	$1.5 \cdot 10^{-13}$	3.999	$4.0 \cdot 10^{-14}$	3.999
$1/5 \cdot 2^{-10}$	$9.6 \cdot 10^{-15}$	3.999	$2.5 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-11}$	$6.0 \cdot 10^{-16}$	3.999	$1.5 \cdot 10^{-16}$	3.999
$1/5 \cdot 2^{-12}$	$3.7 \cdot 10^{-17}$		$9.8 \cdot 10^{-18}$	

Table 5.28: Box scheme, IDeC method at $t = 0$ for 4.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.3 \cdot 10^{-02}$	-0.0000	1.971	$3.9 \cdot 10^{-04}$	-0.0000	4.108
$1/5 \cdot 2^{-2}$	$3.3 \cdot 10^{-03}$	0.00000	1.993	$2.3 \cdot 10^{-05}$	0.00000	4.060
$1/5 \cdot 2^{-3}$	$8.4 \cdot 10^{-04}$	0.00000	1.998	$1.3 \cdot 10^{-06}$	0.02500	4.020
$1/5 \cdot 2^{-4}$	$2.1 \cdot 10^{-04}$	-0.0000	1.999	$8.5 \cdot 10^{-08}$	0.02500	4.002
$1/5 \cdot 2^{-5}$	$5.3 \cdot 10^{-05}$	-0.0000	1.999	$5.3 \cdot 10^{-09}$	0.03125	4.000
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-05}$	0.00000	2.000	$3.3 \cdot 10^{-10}$	0.02812	4.001
$1/5 \cdot 2^{-7}$	$3.3 \cdot 10^{-06}$	0.00000	2.000	$2.0 \cdot 10^{-11}$	0.03125	3.999
$1/5 \cdot 2^{-8}$	$8.2 \cdot 10^{-07}$	-0.0000	2.000	$1.3 \cdot 10^{-12}$	0.03046	4.000
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-07}$	-0.0000	2.000	$8.1 \cdot 10^{-14}$	0.03046	4.000
$1/5 \cdot 2^{-10}$	$5.1 \cdot 10^{-08}$	0.00000	2.000	$5.0 \cdot 10^{-15}$	0.03046	3.771
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-08}$	0.00000	2.000	$3.7 \cdot 10^{-16}$	0.00000	3.701
$1/5 \cdot 2^{-12}$	$3.2 \cdot 10^{-09}$	-0.0000	2.000	$2.8 \cdot 10^{-17}$	-0.0000	3.752
$1/5 \cdot 2^{-13}$	$8.1 \cdot 10^{-10}$	-0.0000	1.999	$2.1 \cdot 10^{-18}$	-0.0000	3.788
$1/5 \cdot 2^{-14}$	$2.0 \cdot 10^{-10}$	0.00000		$1.5 \cdot 10^{-19}$	0.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$5.2 \cdot 10^{-06}$	0.10000	2.739	$1.8 \cdot 10^{-06}$	-0.0000	2.916
$1/5 \cdot 2^{-2}$	$7.8 \cdot 10^{-07}$	0.00000	3.297	$2.5 \cdot 10^{-07}$	0.00000	3.669
$1/5 \cdot 2^{-3}$	$7.9 \cdot 10^{-08}$	0.00000	3.739	$1.9 \cdot 10^{-08}$	0.00000	3.857
$1/5 \cdot 2^{-4}$	$5.9 \cdot 10^{-09}$	-0.0000	3.882	$1.3 \cdot 10^{-09}$	-0.0000	3.932
$1/5 \cdot 2^{-5}$	$4.0 \cdot 10^{-10}$	-0.0000	3.943	$8.9 \cdot 10^{-11}$	-0.0000	3.966
$1/5 \cdot 2^{-6}$	$2.6 \cdot 10^{-11}$	0.00000	3.972	$5.7 \cdot 10^{-12}$	0.00000	3.983
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-12}$	0.00000	3.986	$3.6 \cdot 10^{-13}$	0.00000	3.991
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-13}$	-0.0000	3.993	$2.2 \cdot 10^{-14}$	-0.0000	3.995
$1/5 \cdot 2^{-9}$	$6.6 \cdot 10^{-15}$	-0.0000	3.996	$1.4 \cdot 10^{-15}$	-0.0000	3.997
$1/5 \cdot 2^{-10}$	$4.1 \cdot 10^{-16}$	0.00000	3.998	$8.9 \cdot 10^{-17}$	0.00000	3.998
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-17}$	0.00000	3.999	$5.5 \cdot 10^{-18}$	0.00000	3.999
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-18}$	-0.0000	3.999	$3.4 \cdot 10^{-19}$	-0.0000	3.999
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-19}$	-0.0000		$2.1 \cdot 10^{-20}$	-0.0000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$6.4 \cdot 10^{-07}$	-0.0000	3.229	$1.9 \cdot 10^{-07}$	-0.0000	3.341
$1/5 \cdot 2^{-2}$	$6.9 \cdot 10^{-08}$	0.00000	3.730	$1.8 \cdot 10^{-08}$	0.00000	3.759
$1/5 \cdot 2^{-3}$	$5.1 \cdot 10^{-09}$	0.00000	3.879	$1.3 \cdot 10^{-09}$	0.00000	3.890
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-10}$	-0.0000	3.942	$9.3 \cdot 10^{-11}$	-0.0000	3.947
$1/5 \cdot 2^{-5}$	$2.2 \cdot 10^{-11}$	-0.0000	3.971	$6.0 \cdot 10^{-12}$	-0.0000	3.974
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-12}$	0.00000	3.985	$3.8 \cdot 10^{-13}$	0.00000	3.987
$1/5 \cdot 2^{-7}$	$9.2 \cdot 10^{-14}$	0.00000	3.992	$2.4 \cdot 10^{-14}$	0.00000	3.993
$1/5 \cdot 2^{-8}$	$5.8 \cdot 10^{-15}$	-0.0000	3.996	$1.5 \cdot 10^{-15}$	-0.0000	3.996
$1/5 \cdot 2^{-9}$	$3.6 \cdot 10^{-16}$	-0.0000	3.998	$9.5 \cdot 10^{-17}$	-0.0000	3.998
$1/5 \cdot 2^{-10}$	$2.2 \cdot 10^{-17}$	0.00000	3.999	$5.9 \cdot 10^{-18}$	0.00000	3.999
$1/5 \cdot 2^{-11}$	$1.4 \cdot 10^{-18}$	0.00000	3.999	$3.7 \cdot 10^{-19}$	0.00000	3.999
$1/5 \cdot 2^{-12}$	$8.9 \cdot 10^{-20}$	-0.0000		$2.3 \cdot 10^{-20}$	-0.0000	

Table 5.29: Box scheme, IDeC method on $[0, 1]$ for 5.2a.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.3 \cdot 10^{-02}$	1.971	$3.9 \cdot 10^{-04}$	4.108
$1/5 \cdot 2^{-2}$	$3.3 \cdot 10^{-03}$	1.993	$2.3 \cdot 10^{-05}$	4.257
$1/5 \cdot 2^{-3}$	$8.4 \cdot 10^{-04}$	1.998	$1.2 \cdot 10^{-06}$	4.412
$1/5 \cdot 2^{-4}$	$2.1 \cdot 10^{-04}$	1.999	$5.6 \cdot 10^{-08}$	4.659
$1/5 \cdot 2^{-5}$	$5.3 \cdot 10^{-05}$	1.999	$2.2 \cdot 10^{-09}$	4.722
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-05}$	2.000	$8.5 \cdot 10^{-11}$	3.397
$1/5 \cdot 2^{-7}$	$3.3 \cdot 10^{-06}$	2.000	$8.0 \cdot 10^{-12}$	3.573
$1/5 \cdot 2^{-8}$	$8.2 \cdot 10^{-07}$	2.000	$6.7 \cdot 10^{-13}$	3.669
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-07}$	2.000	$5.3 \cdot 10^{-14}$	3.542
$1/5 \cdot 2^{-10}$	$5.1 \cdot 10^{-08}$	2.000	$4.5 \cdot 10^{-15}$	3.623
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-08}$	2.000	$3.7 \cdot 10^{-16}$	3.701
$1/5 \cdot 2^{-12}$	$3.2 \cdot 10^{-09}$	2.000	$2.8 \cdot 10^{-17}$	3.752
$1/5 \cdot 2^{-13}$	$8.1 \cdot 10^{-10}$	1.999	$2.1 \cdot 10^{-18}$	3.788
$1/5 \cdot 2^{-14}$	$2.0 \cdot 10^{-10}$		$1.5 \cdot 10^{-19}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$4.0 \cdot 10^{-06}$	2.365	$1.8 \cdot 10^{-06}$	2.916
$1/5 \cdot 2^{-2}$	$7.8 \cdot 10^{-07}$	3.297	$2.5 \cdot 10^{-07}$	3.669
$1/5 \cdot 2^{-3}$	$7.9 \cdot 10^{-08}$	3.739	$1.9 \cdot 10^{-08}$	3.857
$1/5 \cdot 2^{-4}$	$5.9 \cdot 10^{-09}$	3.882	$1.3 \cdot 10^{-09}$	3.932
$1/5 \cdot 2^{-5}$	$4.0 \cdot 10^{-10}$	3.943	$8.9 \cdot 10^{-11}$	3.966
$1/5 \cdot 2^{-6}$	$2.6 \cdot 10^{-11}$	3.972	$5.7 \cdot 10^{-12}$	3.983
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-12}$	3.986	$3.6 \cdot 10^{-13}$	3.991
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-13}$	3.993	$2.2 \cdot 10^{-14}$	3.995
$1/5 \cdot 2^{-9}$	$6.6 \cdot 10^{-15}$	3.996	$1.4 \cdot 10^{-15}$	3.997
$1/5 \cdot 2^{-10}$	$4.1 \cdot 10^{-16}$	3.998	$8.9 \cdot 10^{-17}$	3.998
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-17}$	3.999	$5.5 \cdot 10^{-18}$	3.999
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-18}$	3.999	$3.4 \cdot 10^{-19}$	3.999
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-19}$		$2.1 \cdot 10^{-20}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$6.4 \cdot 10^{-07}$	3.229	$1.9 \cdot 10^{-07}$	3.341
$1/5 \cdot 2^{-2}$	$6.9 \cdot 10^{-08}$	3.730	$1.8 \cdot 10^{-08}$	3.759
$1/5 \cdot 2^{-3}$	$5.1 \cdot 10^{-09}$	3.879	$1.3 \cdot 10^{-09}$	3.890
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-10}$	3.942	$9.3 \cdot 10^{-11}$	3.947
$1/5 \cdot 2^{-5}$	$2.2 \cdot 10^{-11}$	3.971	$6.0 \cdot 10^{-12}$	3.974
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-12}$	3.985	$3.8 \cdot 10^{-13}$	3.987
$1/5 \cdot 2^{-7}$	$9.2 \cdot 10^{-14}$	3.992	$2.4 \cdot 10^{-14}$	3.993
$1/5 \cdot 2^{-8}$	$5.8 \cdot 10^{-15}$	3.996	$1.5 \cdot 10^{-15}$	3.996
$1/5 \cdot 2^{-9}$	$3.6 \cdot 10^{-16}$	3.998	$9.5 \cdot 10^{-17}$	3.998
$1/5 \cdot 2^{-10}$	$2.2 \cdot 10^{-17}$	3.999	$5.9 \cdot 10^{-18}$	3.999
$1/5 \cdot 2^{-11}$	$1.4 \cdot 10^{-18}$	3.999	$3.7 \cdot 10^{-19}$	3.999
$1/5 \cdot 2^{-12}$	$8.9 \cdot 10^{-20}$		$2.3 \cdot 10^{-20}$	

Table 5.30: Box scheme, IDeC method at $t = 0$ for 5.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.5 \cdot 10^{-33}$	0.20000	-5.000	$2.4 \cdot 10^{-32}$	0.20000	-1.000
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-32}$	0.30000	1.000	$4.9 \cdot 10^{-32}$	-0.0000	-0.584
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-32}$	0.20000	-2.000	$7.3 \cdot 10^{-32}$	0.50000	-0.736
$1/5 \cdot 2^{-3}$	$9.8 \cdot 10^{-32}$	0.00000	0.752	$1.2 \cdot 10^{-31}$	0.65000	-1.263
$1/5 \cdot 2^{-4}$	$5.8 \cdot 10^{-32}$	0.42500	-4.000	$2.9 \cdot 10^{-31}$	-0.0000	-0.624
$1/5 \cdot 2^{-5}$	$9.3 \cdot 10^{-31}$	-0.0000	0.633	$4.5 \cdot 10^{-31}$	0.06875	-0.878
$1/5 \cdot 2^{-6}$	$6.0 \cdot 10^{-31}$	0.55937	-1.385	$8.3 \cdot 10^{-31}$	0.00000	-1.363
$1/5 \cdot 2^{-7}$	$1.5 \cdot 10^{-30}$	0.00312	1.173	$2.1 \cdot 10^{-30}$	0.40625	0.102
$1/5 \cdot 2^{-8}$	$6.9 \cdot 10^{-31}$	0.28593	-4.407	$2.0 \cdot 10^{-30}$	0.00234	-0.634
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-29}$	0.00117	0.555	$3.1 \cdot 10^{-30}$	0.48320	-3.087
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-29}$	0.50937	-1.566	$2.6 \cdot 10^{-29}$	0.00000	-0.621
$1/5 \cdot 2^{-11}$	$2.9 \cdot 10^{-29}$	0.00000	1.418	$4.0 \cdot 10^{-29}$	0.00000	0.633
$1/5 \cdot 2^{-12}$	$1.1 \cdot 10^{-29}$	0.29965	-4.408	$2.6 \cdot 10^{-29}$	0.51215	-0.944
$1/5 \cdot 2^{-13}$	$2.3 \cdot 10^{-28}$	-0.0000	0.552	$5.0 \cdot 10^{-29}$	0.49895	-3.020
$1/5 \cdot 2^{-14}$	$1.6 \cdot 10^{-28}$	0.49998		$4.1 \cdot 10^{-28}$	0.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.2 \cdot 10^{-31}$	-0.0000	0.514	$1.9 \cdot 10^{-31}$	-0.0000	-0.584
$1/5 \cdot 2^{-1}$	$8.6 \cdot 10^{-32}$	0.10000	-0.362	$2.9 \cdot 10^{-31}$	-0.0000	1.263
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-31}$	0.20000	-2.000	$1.2 \cdot 10^{-31}$	0.50000	-2.350
$1/5 \cdot 2^{-3}$	$4.4 \cdot 10^{-31}$	0.10000	-0.611	$6.2 \cdot 10^{-31}$	0.12500	0.0000
$1/5 \cdot 2^{-4}$	$6.7 \cdot 10^{-31}$	0.18750	-0.368	$6.2 \cdot 10^{-31}$	0.22500	-1.556
$1/5 \cdot 2^{-5}$	$8.7 \cdot 10^{-31}$	0.22500	0.940	$1.8 \cdot 10^{-30}$	0.00625	0.370
$1/5 \cdot 2^{-6}$	$4.5 \cdot 10^{-31}$	0.73437	-3.438	$1.4 \cdot 10^{-30}$	0.00000	-0.048
$1/5 \cdot 2^{-7}$	$4.9 \cdot 10^{-30}$	0.25000	-1.542	$1.4 \cdot 10^{-30}$	0.04687	-3.112
$1/5 \cdot 2^{-8}$	$1.4 \cdot 10^{-29}$	-0.0000	0.115	$1.2 \cdot 10^{-29}$	-0.0000	-0.216
$1/5 \cdot 2^{-9}$	$1.3 \cdot 10^{-29}$	0.09609	1.279	$1.4 \cdot 10^{-29}$	-0.0000	-0.605
$1/5 \cdot 2^{-10}$	$5.4 \cdot 10^{-30}$	0.06914	-3.605	$2.2 \cdot 10^{-29}$	0.00292	-0.063
$1/5 \cdot 2^{-11}$	$6.6 \cdot 10^{-29}$	0.30273	-1.238	$2.3 \cdot 10^{-29}$	0.00292	-1.589
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-28}$	-0.0000	0.052	$7.1 \cdot 10^{-29}$	0.09379	-1.359
$1/5 \cdot 2^{-13}$	$1.5 \cdot 10^{-28}$	0.09233		$1.8 \cdot 10^{-28}$	-0.0000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.2 \cdot 10^{-31}$	-0.0000	-0.847	$1.7 \cdot 10^{-31}$	-0.0000	-1.099
$1/5 \cdot 2^{-1}$	$2.2 \cdot 10^{-31}$	-0.0000	-0.289	$3.6 \cdot 10^{-31}$	-0.0000	1.000
$1/5 \cdot 2^{-2}$	$2.7 \cdot 10^{-31}$	0.20000	-1.184	$1.8 \cdot 10^{-31}$	0.50000	-0.793
$1/5 \cdot 2^{-3}$	$6.1 \cdot 10^{-31}$	0.17500	-0.604	$3.2 \cdot 10^{-31}$	0.00000	-1.000
$1/5 \cdot 2^{-4}$	$9.3 \cdot 10^{-31}$	0.23750	-0.127	$6.4 \cdot 10^{-31}$	-0.0000	-1.928
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-30}$	0.33750	0.867	$2.4 \cdot 10^{-30}$	-0.0000	1.563
$1/5 \cdot 2^{-6}$	$5.6 \cdot 10^{-31}$	0.73125	-2.849	$8.2 \cdot 10^{-31}$	0.43750	-2.389
$1/5 \cdot 2^{-7}$	$4.0 \cdot 10^{-30}$	0.23437	-1.633	$4.3 \cdot 10^{-30}$	0.30937	-0.583
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-29}$	-0.0000	0.358	$6.4 \cdot 10^{-30}$	0.08359	-0.742
$1/5 \cdot 2^{-9}$	$9.7 \cdot 10^{-30}$	0.23554	0.308	$1.0 \cdot 10^{-29}$	-0.0000	0.523
$1/5 \cdot 2^{-10}$	$7.9 \cdot 10^{-30}$	0.00253	-3.148	$7.5 \cdot 10^{-30}$	0.57519	-2.433
$1/5 \cdot 2^{-11}$	$7.0 \cdot 10^{-29}$	0.30214	-0.763	$4.0 \cdot 10^{-29}$	0.00693	-0.424
$1/5 \cdot 2^{-12}$	$1.1 \cdot 10^{-28}$	0.08413		$5.4 \cdot 10^{-29}$	0.46435	

Table 5.31: Implicit Euler, IDeC method on $[0, 1]$ for 4.1a.

h	δ	p	δ	p
$1/5$	$6.1 \cdot 10^{-33}$	-3.000	$2.4 \cdot 10^{-32}$	-1.000
$1/5 \cdot 2^{-1}$	$4.9 \cdot 10^{-32}$	0.0000	$4.9 \cdot 10^{-32}$	1.000
$1/5 \cdot 2^{-2}$	$4.9 \cdot 10^{-32}$	-1.321	$2.4 \cdot 10^{-32}$	-1.584
$1/5 \cdot 2^{-3}$	$1.2 \cdot 10^{-31}$	0.321	$7.3 \cdot 10^{-32}$	-2.321
$1/5 \cdot 2^{-4}$	$9.8 \cdot 10^{-32}$	-3.000	$3.6 \cdot 10^{-31}$	1.321
$1/5 \cdot 2^{-5}$	$7.8 \cdot 10^{-31}$	0.0000	$1.4 \cdot 10^{-31}$	-1.807
$1/5 \cdot 2^{-6}$	$7.8 \cdot 10^{-31}$	-1.442	$5.1 \cdot 10^{-31}$	-1.389
$1/5 \cdot 2^{-7}$	$2.1 \cdot 10^{-30}$	0.584	$1.3 \cdot 10^{-30}$	0.064
$1/5 \cdot 2^{-8}$	$1.4 \cdot 10^{-30}$	-3.142	$1.2 \cdot 10^{-30}$	-0.779
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-29}$	0.0000	$2.2 \cdot 10^{-30}$	-3.258
$1/5 \cdot 2^{-10}$	$1.2 \cdot 10^{-29}$	-1.620	$2.1 \cdot 10^{-29}$	-0.581
$1/5 \cdot 2^{-11}$	$3.8 \cdot 10^{-29}$	0.655	$3.1 \cdot 10^{-29}$	0.616
$1/5 \cdot 2^{-12}$	$2.4 \cdot 10^{-29}$	-3.035	$2.0 \cdot 10^{-29}$	-0.777
$1/5 \cdot 2^{-13}$	$2.0 \cdot 10^{-28}$	0.0000	$3.5 \cdot 10^{-29}$	-3.199
$1/5 \cdot 2^{-14}$	$2.0 \cdot 10^{-28}$		$3.2 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5$	$1.2 \cdot 10^{-31}$	0.736	$1.9 \cdot 10^{-31}$	-0.584
$1/5 \cdot 2^{-1}$	$7.3 \cdot 10^{-32}$	0.584	$2.9 \cdot 10^{-31}$	3.884
$1/5 \cdot 2^{-2}$	$4.9 \cdot 10^{-32}$	-2.459	$2.0 \cdot 10^{-32}$	-4.469
$1/5 \cdot 2^{-3}$	$2.7 \cdot 10^{-31}$	-1.347	$4.4 \cdot 10^{-31}$	0.082
$1/5 \cdot 2^{-4}$	$6.9 \cdot 10^{-31}$	0.559	$4.1 \cdot 10^{-31}$	-2.021
$1/5 \cdot 2^{-5}$	$4.6 \cdot 10^{-31}$	0.440	$1.7 \cdot 10^{-30}$	0.616
$1/5 \cdot 2^{-6}$	$3.4 \cdot 10^{-31}$	-3.269	$1.1 \cdot 10^{-30}$	0.994
$1/5 \cdot 2^{-7}$	$3.3 \cdot 10^{-30}$	-2.238	$5.5 \cdot 10^{-31}$	-4.366
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-29}$	0.649	$1.1 \cdot 10^{-29}$	-0.138
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-29}$	0.072	$1.2 \cdot 10^{-29}$	-0.435
$1/5 \cdot 2^{-10}$	$9.5 \cdot 10^{-30}$	-2.092	$1.7 \cdot 10^{-29}$	0.214
$1/5 \cdot 2^{-11}$	$4.0 \cdot 10^{-29}$	-2.132	$1.4 \cdot 10^{-29}$	-1.465
$1/5 \cdot 2^{-12}$	$1.7 \cdot 10^{-28}$	0.804	$4.0 \cdot 10^{-29}$	-1.851
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-28}$		$1.4 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5$	$1.2 \cdot 10^{-31}$	-0.847	$1.7 \cdot 10^{-31}$	-1.099
$1/5 \cdot 2^{-1}$	$2.2 \cdot 10^{-31}$	0.0000	$3.6 \cdot 10^{-31}$	2.906
$1/5 \cdot 2^{-2}$	$2.2 \cdot 10^{-31}$	0.0000	$4.9 \cdot 10^{-32}$	-2.584
$1/5 \cdot 2^{-3}$	$2.2 \cdot 10^{-31}$	-1.688	$2.9 \cdot 10^{-31}$	-1.273
$1/5 \cdot 2^{-4}$	$7.1 \cdot 10^{-31}$	0.688	$7.1 \cdot 10^{-31}$	-1.681
$1/5 \cdot 2^{-5}$	$4.4 \cdot 10^{-31}$	-0.584	$2.2 \cdot 10^{-30}$	2.819
$1/5 \cdot 2^{-6}$	$6.6 \cdot 10^{-31}$	-1.752	$3.2 \cdot 10^{-31}$	-3.138
$1/5 \cdot 2^{-7}$	$2.2 \cdot 10^{-30}$	-2.626	$2.8 \cdot 10^{-30}$	-0.481
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-29}$	2.637	$3.9 \cdot 10^{-30}$	-1.111
$1/5 \cdot 2^{-9}$	$2.2 \cdot 10^{-30}$	-2.544	$8.6 \cdot 10^{-30}$	0.731
$1/5 \cdot 2^{-10}$	$1.2 \cdot 10^{-29}$	-1.641	$5.1 \cdot 10^{-30}$	-2.591
$1/5 \cdot 2^{-11}$	$4.0 \cdot 10^{-29}$	-1.773	$3.1 \cdot 10^{-29}$	0.592
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-28}$		$2.0 \cdot 10^{-29}$	

Table 5.32: Implicit Euler, IDeC method at $t = 0$ for 4.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$2.0 \cdot 10^{-01}$	0.60000	0.844	$5.0 \cdot 10^{-02}$	0.60000	1.552
$1/5 \cdot 2^{-1}$	$1.1 \cdot 10^{-01}$	0.70000	0.898	$1.7 \cdot 10^{-02}$	0.70000	1.788
$1/5 \cdot 2^{-2}$	$6.2 \cdot 10^{-02}$	0.65000	0.954	$5.0 \cdot 10^{-03}$	0.70000	1.887
$1/5 \cdot 2^{-3}$	$3.2 \cdot 10^{-02}$	0.67500	0.975	$1.3 \cdot 10^{-03}$	0.70000	1.941
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-02}$	0.66250	0.988	$3.5 \cdot 10^{-04}$	0.70000	1.969
$1/5 \cdot 2^{-5}$	$8.2 \cdot 10^{-03}$	0.66875	0.993	$9.0 \cdot 10^{-05}$	0.70625	1.984
$1/5 \cdot 2^{-6}$	$4.1 \cdot 10^{-03}$	0.66562	0.997	$2.2 \cdot 10^{-05}$	0.70625	1.992
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-03}$	0.66718	0.998	$5.7 \cdot 10^{-06}$	0.70468	1.996
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-03}$	0.66640	0.999	$1.4 \cdot 10^{-06}$	0.70546	1.998
$1/5 \cdot 2^{-9}$	$5.2 \cdot 10^{-04}$	0.66679	0.999	$3.6 \cdot 10^{-07}$	0.70546	1.999
$1/5 \cdot 2^{-10}$	$2.6 \cdot 10^{-04}$	0.66660	0.999	$9.0 \cdot 10^{-08}$	0.70546	1.999
$1/5 \cdot 2^{-11}$	$1.3 \cdot 10^{-04}$	0.66669	0.999	$2.2 \cdot 10^{-08}$	0.70546	1.999
$1/5 \cdot 2^{-12}$	$6.5 \cdot 10^{-05}$	0.66665	0.999	$5.6 \cdot 10^{-09}$	0.70551	1.999
$1/5 \cdot 2^{-13}$	$3.2 \cdot 10^{-05}$	0.66667	0.999	$1.4 \cdot 10^{-09}$	0.70549	1.999
$1/5 \cdot 2^{-14}$	$1.6 \cdot 10^{-05}$	0.66666		$3.5 \cdot 10^{-10}$	0.70550	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.4 \cdot 10^{-02}$	0.60000	2.278	$4.7 \cdot 10^{-03}$	0.60000	3.027
$1/5 \cdot 2^{-1}$	$3.0 \cdot 10^{-03}$	0.70000	2.654	$5.8 \cdot 10^{-04}$	0.70000	3.520
$1/5 \cdot 2^{-2}$	$4.9 \cdot 10^{-04}$	0.70000	2.813	$5.1 \cdot 10^{-05}$	0.70000	3.739
$1/5 \cdot 2^{-3}$	$6.9 \cdot 10^{-05}$	0.72500	2.902	$3.8 \cdot 10^{-06}$	0.72500	3.866
$1/5 \cdot 2^{-4}$	$9.3 \cdot 10^{-06}$	0.71250	2.950	$2.6 \cdot 10^{-07}$	0.72500	3.931
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-06}$	0.71875	2.975	$1.7 \cdot 10^{-08}$	0.71875	3.965
$1/5 \cdot 2^{-6}$	$1.5 \cdot 10^{-07}$	0.71562	2.987	$1.0 \cdot 10^{-09}$	0.71875	3.982
$1/5 \cdot 2^{-7}$	$1.9 \cdot 10^{-08}$	0.71718	2.993	$6.9 \cdot 10^{-11}$	0.72031	3.991
$1/5 \cdot 2^{-8}$	$2.4 \cdot 10^{-09}$	0.71718	2.996	$4.3 \cdot 10^{-12}$	0.72031	3.995
$1/5 \cdot 2^{-9}$	$3.0 \cdot 10^{-10}$	0.71718	2.998	$2.7 \cdot 10^{-13}$	0.72031	3.997
$1/5 \cdot 2^{-10}$	$3.8 \cdot 10^{-11}$	0.71718	2.999	$1.7 \cdot 10^{-14}$	0.72031	3.998
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-12}$	0.71708	2.999	$1.0 \cdot 10^{-15}$	0.72031	3.999
$1/5 \cdot 2^{-12}$	$5.9 \cdot 10^{-13}$	0.71713	2.999	$6.7 \cdot 10^{-17}$	0.72031	3.999
$1/5 \cdot 2^{-13}$	$7.4 \cdot 10^{-14}$	0.71713		$4.1 \cdot 10^{-18}$	0.72033	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.5 \cdot 10^{-03}$	0.60000	3.787	$5.2 \cdot 10^{-04}$	0.60000	4.553
$1/5 \cdot 2^{-1}$	$1.1 \cdot 10^{-04}$	0.70000	4.388	$2.2 \cdot 10^{-05}$	0.70000	5.256
$1/5 \cdot 2^{-2}$	$5.4 \cdot 10^{-06}$	0.70000	4.668	$5.8 \cdot 10^{-07}$	0.70000	5.598
$1/5 \cdot 2^{-3}$	$2.1 \cdot 10^{-07}$	0.72500	4.829	$1.1 \cdot 10^{-08}$	0.72500	5.793
$1/5 \cdot 2^{-4}$	$7.5 \cdot 10^{-09}$	0.72500	4.913	$2.1 \cdot 10^{-10}$	0.72500	5.894
$1/5 \cdot 2^{-5}$	$2.4 \cdot 10^{-10}$	0.71875	4.956	$3.6 \cdot 10^{-12}$	0.71875	5.946
$1/5 \cdot 2^{-6}$	$8.0 \cdot 10^{-12}$	0.72187	4.978	$5.8 \cdot 10^{-14}$	0.72187	5.973
$1/5 \cdot 2^{-7}$	$2.5 \cdot 10^{-13}$	0.72031	4.988	$9.3 \cdot 10^{-16}$	0.72187	5.986
$1/5 \cdot 2^{-8}$	$8.0 \cdot 10^{-15}$	0.72109	4.994	$1.4 \cdot 10^{-17}$	0.72109	5.993
$1/5 \cdot 2^{-9}$	$2.5 \cdot 10^{-16}$	0.72109	4.997	$2.3 \cdot 10^{-19}$	0.72148	5.996
$1/5 \cdot 2^{-10}$	$7.8 \cdot 10^{-18}$	0.72109	4.998	$3.6 \cdot 10^{-21}$	0.72148	5.998
$1/5 \cdot 2^{-11}$	$2.4 \cdot 10^{-19}$	0.72119	4.999	$5.6 \cdot 10^{-23}$	0.72138	5.999
$1/5 \cdot 2^{-12}$	$7.7 \cdot 10^{-21}$	0.72119		$8.8 \cdot 10^{-25}$	0.72148	

Table 5.33: Implicit Euler, IDeC method on $[0, 1]$ for 4.1b.

h	δ	p	δ	p
$1/5$	$2.4 \cdot 10^{-32}$	5.874	$7.3 \cdot 10^{-32}$	-0.736
$1/5 \cdot 2^{-1}$	$4.2 \cdot 10^{-34}$	-5.874	$1.2 \cdot 10^{-31}$	0.0000
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-32}$	-2.000	$1.2 \cdot 10^{-31}$	-0.485
$1/5 \cdot 2^{-3}$	$9.8 \cdot 10^{-32}$	0.0000	$1.7 \cdot 10^{-31}$	-1.584
$1/5 \cdot 2^{-4}$	$9.8 \cdot 10^{-32}$	-0.321	$5.1 \cdot 10^{-31}$	-1.630
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-31}$	-1.925	$1.6 \cdot 10^{-30}$	0.321
$1/5 \cdot 2^{-6}$	$4.6 \cdot 10^{-31}$	0.440	$1.2 \cdot 10^{-30}$	-1.000
$1/5 \cdot 2^{-7}$	$3.4 \cdot 10^{-31}$	-2.050	$2.5 \cdot 10^{-30}$	-0.621
$1/5 \cdot 2^{-8}$	$1.4 \cdot 10^{-30}$	-0.910	$3.9 \cdot 10^{-30}$	-1.189
$1/5 \cdot 2^{-9}$	$2.6 \cdot 10^{-30}$	-1.051	$8.9 \cdot 10^{-30}$	-1.142
$1/5 \cdot 2^{-10}$	$5.5 \cdot 10^{-30}$	-0.951	$1.9 \cdot 10^{-29}$	-0.935
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-29}$	-0.845	$3.8 \cdot 10^{-29}$	-1.052
$1/5 \cdot 2^{-12}$	$1.9 \cdot 10^{-29}$	-1.077	$7.8 \cdot 10^{-29}$	-0.961
$1/5 \cdot 2^{-13}$	$4.0 \cdot 10^{-29}$	-1.001	$1.5 \cdot 10^{-28}$	-0.969
$1/5 \cdot 2^{-14}$	$8.1 \cdot 10^{-29}$		$3.0 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5$	$2.2 \cdot 10^{-31}$	0.847	$1.9 \cdot 10^{-31}$	-0.321
$1/5 \cdot 2^{-1}$	$1.2 \cdot 10^{-31}$	-1.263	$2.4 \cdot 10^{-31}$	-0.925
$1/5 \cdot 2^{-2}$	$2.9 \cdot 10^{-31}$	0.777	$4.6 \cdot 10^{-31}$	1.662
$1/5 \cdot 2^{-3}$	$1.7 \cdot 10^{-31}$	0.222	$1.4 \cdot 10^{-31}$	-2.222
$1/5 \cdot 2^{-4}$	$1.4 \cdot 10^{-31}$	-1.874	$6.9 \cdot 10^{-31}$	1.807
$1/5 \cdot 2^{-5}$	$5.4 \cdot 10^{-31}$	1.000	$1.9 \cdot 10^{-31}$	-1.906
$1/5 \cdot 2^{-6}$	$2.7 \cdot 10^{-31}$	-2.669	$7.3 \cdot 10^{-31}$	-2.058
$1/5 \cdot 2^{-7}$	$1.7 \cdot 10^{-30}$	-0.870	$3.0 \cdot 10^{-30}$	-0.142
$1/5 \cdot 2^{-8}$	$3.1 \cdot 10^{-30}$	4.192	$3.4 \cdot 10^{-30}$	2.108
$1/5 \cdot 2^{-9}$	$1.7 \cdot 10^{-31}$	-2.099	$7.8 \cdot 10^{-31}$	-0.672
$1/5 \cdot 2^{-10}$	$7.3 \cdot 10^{-31}$	-2.700	$1.2 \cdot 10^{-30}$	-0.851
$1/5 \cdot 2^{-11}$	$4.8 \cdot 10^{-30}$	0.349	$2.2 \cdot 10^{-30}$	-0.334
$1/5 \cdot 2^{-12}$	$3.7 \cdot 10^{-30}$	-1.164	$2.8 \cdot 10^{-30}$	-2.377
$1/5 \cdot 2^{-13}$	$8.4 \cdot 10^{-30}$		$1.4 \cdot 10^{-29}$	

h	δ	p	δ	p
$1/5$	$1.4 \cdot 10^{-31}$	0.0000	$2.4 \cdot 10^{-32}$	-1.584
$1/5 \cdot 2^{-1}$	$1.4 \cdot 10^{-31}$	-0.736	$7.3 \cdot 10^{-32}$	1.584
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-31}$	-1.000	$2.4 \cdot 10^{-32}$	-4.643
$1/5 \cdot 2^{-3}$	$4.9 \cdot 10^{-31}$	-0.070	$6.1 \cdot 10^{-31}$	0.943
$1/5 \cdot 2^{-4}$	$5.1 \cdot 10^{-31}$	1.392	$3.2 \cdot 10^{-31}$	0.115
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-31}$	2.000	$2.9 \cdot 10^{-31}$	0.415
$1/5 \cdot 2^{-6}$	$4.9 \cdot 10^{-32}$	-5.247	$2.2 \cdot 10^{-31}$	-2.384
$1/5 \cdot 2^{-7}$	$1.8 \cdot 10^{-30}$	3.440	$1.1 \cdot 10^{-30}$	1.854
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-31}$	-4.684	$3.2 \cdot 10^{-31}$	-4.316
$1/5 \cdot 2^{-9}$	$4.4 \cdot 10^{-30}$	-1.366	$6.3 \cdot 10^{-30}$	0.380
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-29}$	-0.268	$4.9 \cdot 10^{-30}$	-0.875
$1/5 \cdot 2^{-11}$	$1.3 \cdot 10^{-29}$	3.082	$8.9 \cdot 10^{-30}$	-1.228
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-30}$		$2.1 \cdot 10^{-29}$	

Table 5.34: Implicit Euler, IDeC method at $t = 0$ for 4.1b.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	1.7	0.80000	0.512	$8.5 \cdot 10^{-01}$	0.80000	0.761
$1/5 \cdot 2^{-1}$	1.2	0.90000	0.785	$5.0 \cdot 10^{-01}$	0.90000	1.446
$1/5 \cdot 2^{-2}$	$7.2 \cdot 10^{-01}$	0.90000	0.875	$1.8 \cdot 10^{-01}$	0.90000	1.649
$1/5 \cdot 2^{-3}$	$3.9 \cdot 10^{-01}$	0.90000	0.931	$5.8 \cdot 10^{-02}$	0.92500	1.794
$1/5 \cdot 2^{-4}$	$2.0 \cdot 10^{-01}$	0.90000	0.964	$1.6 \cdot 10^{-02}$	0.91250	1.890
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-01}$	0.90000	0.981	$4.5 \cdot 10^{-03}$	0.91875	1.945
$1/5 \cdot 2^{-6}$	$5.3 \cdot 10^{-02}$	0.90000	0.990	$1.1 \cdot 10^{-03}$	0.91875	1.971
$1/5 \cdot 2^{-7}$	$2.7 \cdot 10^{-02}$	0.90000	0.995	$3.0 \cdot 10^{-04}$	0.92031	1.985
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-02}$	0.90000	0.997	$7.6 \cdot 10^{-05}$	0.92031	1.992
$1/5 \cdot 2^{-9}$	$6.7 \cdot 10^{-03}$	0.90000	0.998	$1.9 \cdot 10^{-05}$	0.92070	1.996
$1/5 \cdot 2^{-10}$	$3.4 \cdot 10^{-03}$	0.90000	0.999	$4.8 \cdot 10^{-06}$	0.92070	1.998
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-03}$	0.90000	0.999	$1.2 \cdot 10^{-06}$	0.92080	1.999
$1/5 \cdot 2^{-12}$	$8.5 \cdot 10^{-04}$	0.90000	0.999	$3.0 \cdot 10^{-07}$	0.92075	1.999
$1/5 \cdot 2^{-13}$	$4.2 \cdot 10^{-04}$	0.90000	0.999	$7.5 \cdot 10^{-08}$	0.92077	1.999
$1/5 \cdot 2^{-14}$	$2.1 \cdot 10^{-04}$	0.90000		$1.8 \cdot 10^{-08}$	0.92077	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$5.3 \cdot 10^{-01}$	0.80000	1.087	$4.0 \cdot 10^{-01}$	0.80000	1.453
$1/5 \cdot 2^{-1}$	$2.5 \cdot 10^{-01}$	0.90000	2.070	$1.4 \cdot 10^{-01}$	0.90000	2.713
$1/5 \cdot 2^{-2}$	$6.0 \cdot 10^{-02}$	0.90000	2.346	$2.2 \cdot 10^{-02}$	0.90000	3.050
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-02}$	0.92500	2.650	$2.6 \cdot 10^{-03}$	0.92500	3.493
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-03}$	0.92500	2.812	$2.3 \cdot 10^{-04}$	0.92500	3.725
$1/5 \cdot 2^{-5}$	$2.6 \cdot 10^{-04}$	0.92500	2.901	$1.8 \cdot 10^{-05}$	0.93125	3.858
$1/5 \cdot 2^{-6}$	$3.5 \cdot 10^{-05}$	0.92812	2.949	$1.2 \cdot 10^{-06}$	0.93125	3.927
$1/5 \cdot 2^{-7}$	$4.6 \cdot 10^{-06}$	0.92812	2.974	$8.1 \cdot 10^{-08}$	0.93125	3.963
$1/5 \cdot 2^{-8}$	$5.9 \cdot 10^{-07}$	0.92812	2.987	$5.2 \cdot 10^{-09}$	0.93203	3.981
$1/5 \cdot 2^{-9}$	$7.4 \cdot 10^{-08}$	0.92812	2.993	$3.3 \cdot 10^{-10}$	0.93203	3.990
$1/5 \cdot 2^{-10}$	$9.3 \cdot 10^{-09}$	0.92832	2.996	$2.0 \cdot 10^{-11}$	0.93203	3.995
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-09}$	0.92832	2.998	$1.3 \cdot 10^{-12}$	0.93193	3.997
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-10}$	0.92832	2.999	$8.2 \cdot 10^{-14}$	0.93198	3.998
$1/5 \cdot 2^{-13}$	$1.8 \cdot 10^{-11}$	0.92832		$5.1 \cdot 10^{-15}$	0.93198	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$3.3 \cdot 10^{-01}$	0.80000	1.802	$3.0 \cdot 10^{-01}$	0.80000	2.082
$1/5 \cdot 2^{-1}$	$9.6 \cdot 10^{-02}$	0.90000	3.292	$7.1 \cdot 10^{-02}$	0.90000	3.760
$1/5 \cdot 2^{-2}$	$9.8 \cdot 10^{-03}$	0.95000	3.873	$5.2 \cdot 10^{-03}$	0.95000	4.774
$1/5 \cdot 2^{-3}$	$6.7 \cdot 10^{-04}$	0.92500	4.315	$1.9 \cdot 10^{-04}$	0.95000	4.953
$1/5 \cdot 2^{-4}$	$3.3 \cdot 10^{-05}$	0.92500	4.650	$6.2 \cdot 10^{-06}$	0.92500	5.429
$1/5 \cdot 2^{-5}$	$1.3 \cdot 10^{-06}$	0.93125	4.821	$1.4 \cdot 10^{-07}$	0.93125	5.511
$1/5 \cdot 2^{-6}$	$4.7 \cdot 10^{-08}$	0.93437	4.909	$3.1 \cdot 10^{-09}$	0.91875	5.440
$1/5 \cdot 2^{-7}$	$1.5 \cdot 10^{-09}$	0.93593	4.952	$7.3 \cdot 10^{-11}$	0.91250	5.303
$1/5 \cdot 2^{-8}$	$5.0 \cdot 10^{-11}$	0.93281	4.976	$1.8 \cdot 10^{-12}$	0.90156	5.173
$1/5 \cdot 2^{-9}$	$1.6 \cdot 10^{-12}$	0.93320	4.988	$5.1 \cdot 10^{-14}$	0.88828	5.089
$1/5 \cdot 2^{-10}$	$5.0 \cdot 10^{-14}$	0.93339	4.994	$1.5 \cdot 10^{-15}$	0.88066	5.044
$1/5 \cdot 2^{-11}$	$1.5 \cdot 10^{-15}$	0.93349	4.997	$4.5 \cdot 10^{-17}$	0.87587	5.022
$1/5 \cdot 2^{-12}$	$5.0 \cdot 10^{-17}$	0.93378		$1.4 \cdot 10^{-18}$	0.87324	

Table 5.35: Implicit Euler, IDeC method on $[0, 1]$ for 4.1c.

h	δ	p	δ	p
$1/5$	$3.0 \cdot 10^{-35}$	4.943	$2.4 \cdot 10^{-32}$	0.0000
$1/5 \cdot 2^{-1}$	$1.0 \cdot 10^{-36}$	-14.588	$2.4 \cdot 10^{-32}$	-3.000
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-32}$	-1.000	$1.9 \cdot 10^{-31}$	2.000
$1/5 \cdot 2^{-3}$	$4.9 \cdot 10^{-32}$	0.0000	$4.9 \cdot 10^{-32}$	0.0000
$1/5 \cdot 2^{-4}$	$4.9 \cdot 10^{-32}$	-1.584	$4.9 \cdot 10^{-32}$	-1.321
$1/5 \cdot 2^{-5}$	$1.4 \cdot 10^{-31}$	-0.222	$1.2 \cdot 10^{-31}$	-3.560
$1/5 \cdot 2^{-6}$	$1.7 \cdot 10^{-31}$	-0.652	$1.4 \cdot 10^{-30}$	-1.024
$1/5 \cdot 2^{-7}$	$2.7 \cdot 10^{-31}$	-0.710	$2.9 \cdot 10^{-30}$	0.277
$1/5 \cdot 2^{-8}$	$4.4 \cdot 10^{-31}$	-1.688	$2.4 \cdot 10^{-30}$	-2.010
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-30}$	0.050	$9.8 \cdot 10^{-30}$	-0.979
$1/5 \cdot 2^{-10}$	$1.3 \cdot 10^{-30}$	-1.111	$1.9 \cdot 10^{-29}$	-0.921
$1/5 \cdot 2^{-11}$	$2.9 \cdot 10^{-30}$	-1.429	$3.6 \cdot 10^{-29}$	-1.091
$1/5 \cdot 2^{-12}$	$8.0 \cdot 10^{-30}$	-0.870	$7.8 \cdot 10^{-29}$	-0.971
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-29}$	-0.968	$1.5 \cdot 10^{-28}$	-1.011
$1/5 \cdot 2^{-14}$	$2.8 \cdot 10^{-29}$		$3.0 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5$	$1.4 \cdot 10^{-31}$	1.000	$9.8 \cdot 10^{-34}$	-6.964
$1/5 \cdot 2^{-1}$	$7.3 \cdot 10^{-32}$	1.584	$1.2 \cdot 10^{-31}$	0.736
$1/5 \cdot 2^{-2}$	$2.4 \cdot 10^{-32}$	-3.459	$7.3 \cdot 10^{-32}$	-2.321
$1/5 \cdot 2^{-3}$	$2.7 \cdot 10^{-31}$	-2.032	$3.6 \cdot 10^{-31}$	-1.707
$1/5 \cdot 2^{-4}$	$1.1 \cdot 10^{-30}$	1.791	$1.2 \cdot 10^{-30}$	-0.707
$1/5 \cdot 2^{-5}$	$3.2 \cdot 10^{-31}$	0.378	$1.9 \cdot 10^{-30}$	0.463
$1/5 \cdot 2^{-6}$	$2.4 \cdot 10^{-31}$	-2.350	$1.4 \cdot 10^{-30}$	0.770
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-30}$	-0.497	$8.3 \cdot 10^{-31}$	0.280
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-30}$	0.147	$6.9 \cdot 10^{-31}$	0.807
$1/5 \cdot 2^{-9}$	$1.6 \cdot 10^{-30}$	-0.745	$3.9 \cdot 10^{-31}$	-2.507
$1/5 \cdot 2^{-10}$	$2.6 \cdot 10^{-30}$	-0.498	$2.2 \cdot 10^{-30}$	1.259
$1/5 \cdot 2^{-11}$	$3.7 \cdot 10^{-30}$	-1.340	$9.3 \cdot 10^{-31}$	-3.287
$1/5 \cdot 2^{-12}$	$9.6 \cdot 10^{-30}$	-0.503	$9.1 \cdot 10^{-30}$	0.204
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-29}$		$7.9 \cdot 10^{-30}$	

h	δ	p	δ	p
$1/5$	$7.3 \cdot 10^{-32}$	-1.584	$1.1 \cdot 10^{-33}$	-5.966
$1/5 \cdot 2^{-1}$	$2.2 \cdot 10^{-31}$	1.584	$7.3 \cdot 10^{-32}$	0.0000
$1/5 \cdot 2^{-2}$	$7.3 \cdot 10^{-32}$	-2.874	$7.3 \cdot 10^{-32}$	-3.000
$1/5 \cdot 2^{-3}$	$5.4 \cdot 10^{-31}$	1.137	$5.9 \cdot 10^{-31}$	1.415
$1/5 \cdot 2^{-4}$	$2.4 \cdot 10^{-31}$	-2.722	$2.2 \cdot 10^{-31}$	-2.637
$1/5 \cdot 2^{-5}$	$1.6 \cdot 10^{-30}$	-0.750	$1.3 \cdot 10^{-30}$	-0.850
$1/5 \cdot 2^{-6}$	$2.7 \cdot 10^{-30}$	0.644	$2.4 \cdot 10^{-30}$	2.850
$1/5 \cdot 2^{-7}$	$1.7 \cdot 10^{-30}$	0.535	$3.4 \cdot 10^{-31}$	-1.652
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-30}$	4.029	$1.0 \cdot 10^{-30}$	4.459
$1/5 \cdot 2^{-9}$	$7.3 \cdot 10^{-32}$	-2.115	$4.9 \cdot 10^{-32}$	-6.700
$1/5 \cdot 2^{-10}$	$3.2 \cdot 10^{-31}$	2.700	$5.1 \cdot 10^{-30}$	-0.616
$1/5 \cdot 2^{-11}$	$4.9 \cdot 10^{-32}$	-7.554	$7.8 \cdot 10^{-30}$	-0.872
$1/5 \cdot 2^{-12}$	$9.2 \cdot 10^{-30}$		$1.4 \cdot 10^{-29}$	

Table 5.36: Implicit Euler, IDeC method at $t = 0$ for 4.1c.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	1.7	-0.0000	0.929	$4.5 \cdot 10^{-01}$	-0.0000	1.894
$1/5 \cdot 2^{-1}$	$9.0 \cdot 10^{-01}$	-0.0000	0.965	$1.2 \cdot 10^{-01}$	-0.0000	1.946
$1/5 \cdot 2^{-2}$	$4.6 \cdot 10^{-01}$	0.00000	0.983	$3.1 \cdot 10^{-02}$	0.00000	1.973
$1/5 \cdot 2^{-3}$	$2.3 \cdot 10^{-01}$	0.00000	0.991	$8.0 \cdot 10^{-03}$	0.00000	1.986
$1/5 \cdot 2^{-4}$	$1.1 \cdot 10^{-01}$	-0.0000	0.995	$2.0 \cdot 10^{-03}$	-0.0000	1.993
$1/5 \cdot 2^{-5}$	$5.9 \cdot 10^{-02}$	-0.0000	0.997	$5.0 \cdot 10^{-04}$	0.50625	1.992
$1/5 \cdot 2^{-6}$	$2.9 \cdot 10^{-02}$	0.00000	0.998	$1.2 \cdot 10^{-04}$	0.50937	1.996
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-02}$	0.00000	0.999	$3.2 \cdot 10^{-05}$	0.50781	1.998
$1/5 \cdot 2^{-8}$	$7.4 \cdot 10^{-03}$	-0.0000	0.999	$8.0 \cdot 10^{-06}$	0.50781	1.999
$1/5 \cdot 2^{-9}$	$3.7 \cdot 10^{-03}$	-0.0000	0.999	$2.0 \cdot 10^{-06}$	0.50820	1.999
$1/5 \cdot 2^{-10}$	$1.8 \cdot 10^{-03}$	0.00000	0.999	$5.0 \cdot 10^{-07}$	0.50820	1.999
$1/5 \cdot 2^{-11}$	$9.2 \cdot 10^{-04}$	0.00000	1.000	$1.2 \cdot 10^{-07}$	0.50810	1.999
$1/5 \cdot 2^{-12}$	$4.6 \cdot 10^{-04}$	-0.0000	0.999	$3.1 \cdot 10^{-08}$	0.50810	1.999
$1/5 \cdot 2^{-13}$	$2.3 \cdot 10^{-04}$	-0.0000	0.999	$7.8 \cdot 10^{-09}$	0.50808	1.999
$1/5 \cdot 2^{-14}$	$1.1 \cdot 10^{-04}$	0.00000		$1.9 \cdot 10^{-09}$	0.50808	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.0 \cdot 10^{-01}$	0.40000	2.658	$2.4 \cdot 10^{-02}$	0.40000	3.562
$1/5 \cdot 2^{-1}$	$1.6 \cdot 10^{-02}$	0.50000	2.836	$2.0 \cdot 10^{-03}$	0.50000	3.263
$1/5 \cdot 2^{-2}$	$2.2 \cdot 10^{-03}$	0.45000	2.919	$2.1 \cdot 10^{-04}$	0.20000	1.739
$1/5 \cdot 2^{-3}$	$2.9 \cdot 10^{-04}$	0.45000	2.959	$6.4 \cdot 10^{-05}$	0.10000	1.958
$1/5 \cdot 2^{-4}$	$3.8 \cdot 10^{-05}$	0.46250	2.980	$1.6 \cdot 10^{-05}$	0.05000	1.990
$1/5 \cdot 2^{-5}$	$4.8 \cdot 10^{-06}$	0.45625	2.614	$4.1 \cdot 10^{-06}$	0.02500	1.996
$1/5 \cdot 2^{-6}$	$7.9 \cdot 10^{-07}$	0.00312	2.116	$1.0 \cdot 10^{-06}$	0.01250	1.998
$1/5 \cdot 2^{-7}$	$1.8 \cdot 10^{-07}$	0.00156	2.058	$2.6 \cdot 10^{-07}$	0.00625	1.999
$1/5 \cdot 2^{-8}$	$4.3 \cdot 10^{-08}$	0.00078	2.028	$6.5 \cdot 10^{-08}$	0.00312	1.999
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-08}$	0.00039	2.014	$1.6 \cdot 10^{-08}$	0.00156	1.999
$1/5 \cdot 2^{-10}$	$2.6 \cdot 10^{-09}$	0.00019	2.007	$4.0 \cdot 10^{-09}$	0.00078	1.999
$1/5 \cdot 2^{-11}$	$6.6 \cdot 10^{-10}$	0.00009	2.003	$1.0 \cdot 10^{-09}$	0.00039	2.000
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-10}$	0.00004	2.001	$2.5 \cdot 10^{-10}$	0.00019	1.999
$1/5 \cdot 2^{-13}$	$4.1 \cdot 10^{-11}$	0.00002		$6.4 \cdot 10^{-11}$	0.00009	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$7.1 \cdot 10^{-03}$	-0.0000	2.407	$7.9 \cdot 10^{-03}$	0.80000	2.287
$1/5 \cdot 2^{-1}$	$1.3 \cdot 10^{-03}$	0.40000	1.935	$1.6 \cdot 10^{-03}$	0.40000	2.165
$1/5 \cdot 2^{-2}$	$3.5 \cdot 10^{-04}$	0.20000	2.006	$3.6 \cdot 10^{-04}$	0.20000	2.071
$1/5 \cdot 2^{-3}$	$8.7 \cdot 10^{-05}$	0.10000	2.007	$8.5 \cdot 10^{-05}$	0.10000	2.032
$1/5 \cdot 2^{-4}$	$2.1 \cdot 10^{-05}$	0.05000	2.004	$2.1 \cdot 10^{-05}$	0.05000	2.015
$1/5 \cdot 2^{-5}$	$5.4 \cdot 10^{-06}$	0.02500	2.002	$5.1 \cdot 10^{-06}$	0.02500	2.007
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-06}$	0.01250	2.001	$1.2 \cdot 10^{-06}$	0.01250	2.003
$1/5 \cdot 2^{-7}$	$3.3 \cdot 10^{-07}$	0.00625	2.000	$3.2 \cdot 10^{-07}$	0.00625	2.001
$1/5 \cdot 2^{-8}$	$8.4 \cdot 10^{-08}$	0.00312	2.000	$8.0 \cdot 10^{-08}$	0.00312	2.000
$1/5 \cdot 2^{-9}$	$2.1 \cdot 10^{-08}$	0.00156	2.000	$2.0 \cdot 10^{-08}$	0.00156	2.000
$1/5 \cdot 2^{-10}$	$5.2 \cdot 10^{-09}$	0.00078	2.000	$5.0 \cdot 10^{-09}$	0.00078	2.000
$1/5 \cdot 2^{-11}$	$1.3 \cdot 10^{-09}$	0.00039	2.000	$1.2 \cdot 10^{-09}$	0.00039	2.000
$1/5 \cdot 2^{-12}$	$3.3 \cdot 10^{-10}$	0.00019		$3.1 \cdot 10^{-10}$	0.00019	

Table 5.37: Implicit Euler, IDeC method on $[0, 1]$ for 4.1d.

h	δ	p	δ	p
$1/5$	1.7	0.929	$4.5 \cdot 10^{-01}$	1.894
$1/5 \cdot 2^{-1}$	$9.0 \cdot 10^{-01}$	0.965	$1.2 \cdot 10^{-01}$	1.946
$1/5 \cdot 2^{-2}$	$4.6 \cdot 10^{-01}$	0.983	$3.1 \cdot 10^{-02}$	1.973
$1/5 \cdot 2^{-3}$	$2.3 \cdot 10^{-01}$	0.991	$8.0 \cdot 10^{-03}$	1.986
$1/5 \cdot 2^{-4}$	$1.1 \cdot 10^{-01}$	0.995	$2.0 \cdot 10^{-03}$	1.993
$1/5 \cdot 2^{-5}$	$5.9 \cdot 10^{-02}$	0.997	$5.0 \cdot 10^{-04}$	1.996
$1/5 \cdot 2^{-6}$	$2.9 \cdot 10^{-02}$	0.998	$1.2 \cdot 10^{-04}$	1.998
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-02}$	0.999	$3.1 \cdot 10^{-05}$	1.999
$1/5 \cdot 2^{-8}$	$7.4 \cdot 10^{-03}$	0.999	$7.9 \cdot 10^{-06}$	1.999
$1/5 \cdot 2^{-9}$	$3.7 \cdot 10^{-03}$	0.999	$1.9 \cdot 10^{-06}$	1.999
$1/5 \cdot 2^{-10}$	$1.8 \cdot 10^{-03}$	0.999	$4.9 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-11}$	$9.2 \cdot 10^{-04}$	1.000	$1.2 \cdot 10^{-07}$	2.000
$1/5 \cdot 2^{-12}$	$4.6 \cdot 10^{-04}$	0.999	$3.1 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-13}$	$2.3 \cdot 10^{-04}$	0.999	$7.8 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-14}$	$1.1 \cdot 10^{-04}$		$1.9 \cdot 10^{-09}$	

h	δ	p	δ	p
$1/5$	$9.3 \cdot 10^{-02}$	2.835	$2.3 \cdot 10^{-02}$	4.027
$1/5 \cdot 2^{-1}$	$1.3 \cdot 10^{-02}$	2.923	$1.4 \cdot 10^{-03}$	4.040
$1/5 \cdot 2^{-2}$	$1.7 \cdot 10^{-03}$	2.963	$8.9 \cdot 10^{-05}$	4.027
$1/5 \cdot 2^{-3}$	$2.2 \cdot 10^{-04}$	2.982	$5.4 \cdot 10^{-06}$	4.015
$1/5 \cdot 2^{-4}$	$2.7 \cdot 10^{-05}$	2.991	$3.3 \cdot 10^{-07}$	4.008
$1/5 \cdot 2^{-5}$	$3.5 \cdot 10^{-06}$	2.995	$2.1 \cdot 10^{-08}$	4.004
$1/5 \cdot 2^{-6}$	$4.4 \cdot 10^{-07}$	2.997	$1.3 \cdot 10^{-09}$	4.002
$1/5 \cdot 2^{-7}$	$5.5 \cdot 10^{-08}$	2.998	$8.1 \cdot 10^{-11}$	4.001
$1/5 \cdot 2^{-8}$	$6.8 \cdot 10^{-09}$	2.999	$5.1 \cdot 10^{-12}$	4.000
$1/5 \cdot 2^{-9}$	$8.6 \cdot 10^{-10}$	2.999	$3.1 \cdot 10^{-13}$	4.000
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-10}$	2.999	$1.9 \cdot 10^{-14}$	4.000
$1/5 \cdot 2^{-11}$	$1.3 \cdot 10^{-11}$	3.000	$1.2 \cdot 10^{-15}$	4.000
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-12}$	2.999	$7.7 \cdot 10^{-17}$	3.999
$1/5 \cdot 2^{-13}$	$2.1 \cdot 10^{-13}$		$4.8 \cdot 10^{-18}$	

h	δ	p	δ	p
$1/5$	$7.1 \cdot 10^{-03}$	4.900	$7.1 \cdot 10^{-03}$	4.900
$1/5 \cdot 2^{-1}$	$2.3 \cdot 10^{-04}$	4.971	$2.3 \cdot 10^{-04}$	4.971
$1/5 \cdot 2^{-2}$	$7.6 \cdot 10^{-06}$	4.990	$7.6 \cdot 10^{-06}$	4.990
$1/5 \cdot 2^{-3}$	$2.3 \cdot 10^{-07}$	4.996	$2.3 \cdot 10^{-07}$	4.996
$1/5 \cdot 2^{-4}$	$7.5 \cdot 10^{-09}$	4.998	$7.5 \cdot 10^{-09}$	4.998
$1/5 \cdot 2^{-5}$	$2.3 \cdot 10^{-10}$	4.999	$2.3 \cdot 10^{-10}$	4.999
$1/5 \cdot 2^{-6}$	$7.3 \cdot 10^{-12}$	4.999	$7.3 \cdot 10^{-12}$	4.999
$1/5 \cdot 2^{-7}$	$2.2 \cdot 10^{-13}$	4.999	$2.2 \cdot 10^{-13}$	4.999
$1/5 \cdot 2^{-8}$	$7.1 \cdot 10^{-15}$	4.999	$7.1 \cdot 10^{-15}$	4.999
$1/5 \cdot 2^{-9}$	$2.2 \cdot 10^{-16}$	4.999	$2.2 \cdot 10^{-16}$	4.999
$1/5 \cdot 2^{-10}$	$7.0 \cdot 10^{-18}$	4.999	$7.0 \cdot 10^{-18}$	4.999
$1/5 \cdot 2^{-11}$	$2.1 \cdot 10^{-19}$	5.000	$2.1 \cdot 10^{-19}$	5.000
$1/5 \cdot 2^{-12}$	$6.8 \cdot 10^{-21}$		$6.8 \cdot 10^{-21}$	

Table 5.38: Implicit Euler, IDeC method at $t = 0$ for 4.1d.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$8.5 \cdot 10^{-01}$	0.60000	0.767	$2.5 \cdot 10^{-01}$	0.80000	1.512
$1/5 \cdot 2^{-1}$	$5.0 \cdot 10^{-01}$	0.70000	0.905	$9.0 \cdot 10^{-02}$	0.70000	1.757
$1/5 \cdot 2^{-2}$	$2.6 \cdot 10^{-01}$	0.70000	0.949	$2.6 \cdot 10^{-02}$	0.75000	1.874
$1/5 \cdot 2^{-3}$	$1.3 \cdot 10^{-01}$	0.70000	0.974	$7.2 \cdot 10^{-03}$	0.75000	1.932
$1/5 \cdot 2^{-4}$	$7.0 \cdot 10^{-02}$	0.70000	0.986	$1.9 \cdot 10^{-03}$	0.73750	1.967
$1/5 \cdot 2^{-5}$	$3.5 \cdot 10^{-02}$	0.70000	0.993	$4.8 \cdot 10^{-04}$	0.73750	1.983
$1/5 \cdot 2^{-6}$	$1.7 \cdot 10^{-02}$	0.70312	0.996	$1.2 \cdot 10^{-04}$	0.74062	1.991
$1/5 \cdot 2^{-7}$	$9.0 \cdot 10^{-03}$	0.70312	0.998	$3.1 \cdot 10^{-05}$	0.74062	1.995
$1/5 \cdot 2^{-8}$	$4.5 \cdot 10^{-03}$	0.70312	0.999	$7.7 \cdot 10^{-06}$	0.73984	1.997
$1/5 \cdot 2^{-9}$	$2.2 \cdot 10^{-03}$	0.70273	0.999	$1.9 \cdot 10^{-06}$	0.73984	1.998
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-03}$	0.70292	0.999	$4.8 \cdot 10^{-07}$	0.74003	1.999
$1/5 \cdot 2^{-11}$	$5.6 \cdot 10^{-04}$	0.70283	0.999	$1.2 \cdot 10^{-07}$	0.74003	1.999
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-04}$	0.70288	0.999	$3.0 \cdot 10^{-08}$	0.74003	1.999
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-04}$	0.70285	0.999	$7.6 \cdot 10^{-09}$	0.74006	1.999
$1/5 \cdot 2^{-14}$	$7.0 \cdot 10^{-05}$	0.70286		$1.9 \cdot 10^{-09}$	0.74006	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$9.6 \cdot 10^{-02}$	0.80000	2.301	$3.9 \cdot 10^{-02}$	0.80000	3.097
$1/5 \cdot 2^{-1}$	$1.9 \cdot 10^{-02}$	0.70000	2.602	$4.5 \cdot 10^{-03}$	0.70000	3.471
$1/5 \cdot 2^{-2}$	$3.2 \cdot 10^{-03}$	0.75000	2.798	$4.1 \cdot 10^{-04}$	0.75000	3.732
$1/5 \cdot 2^{-3}$	$4.6 \cdot 10^{-04}$	0.75000	2.895	$3.0 \cdot 10^{-05}$	0.75000	3.861
$1/5 \cdot 2^{-4}$	$6.2 \cdot 10^{-05}$	0.75000	2.947	$2.1 \cdot 10^{-06}$	0.75000	3.929
$1/5 \cdot 2^{-5}$	$8.1 \cdot 10^{-06}$	0.75000	2.973	$1.3 \cdot 10^{-07}$	0.74375	3.964
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{-06}$	0.74687	2.986	$8.9 \cdot 10^{-09}$	0.74375	3.982
$1/5 \cdot 2^{-7}$	$1.3 \cdot 10^{-07}$	0.74687	2.993	$5.6 \cdot 10^{-10}$	0.74531	3.991
$1/5 \cdot 2^{-8}$	$1.6 \cdot 10^{-08}$	0.74765	2.996	$3.5 \cdot 10^{-11}$	0.74531	3.995
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-09}$	0.74726	2.998	$2.2 \cdot 10^{-12}$	0.74531	3.997
$1/5 \cdot 2^{-10}$	$2.5 \cdot 10^{-10}$	0.74746	2.999	$1.3 \cdot 10^{-13}$	0.74511	3.998
$1/5 \cdot 2^{-11}$	$3.2 \cdot 10^{-11}$	0.74736	2.999	$8.7 \cdot 10^{-15}$	0.74511	3.999
$1/5 \cdot 2^{-12}$	$4.0 \cdot 10^{-12}$	0.74736	2.999	$5.4 \cdot 10^{-16}$	0.74516	3.999
$1/5 \cdot 2^{-13}$	$5.0 \cdot 10^{-13}$	0.74738		$3.4 \cdot 10^{-17}$	0.74516	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.6 \cdot 10^{-02}$	0.80000	3.946	$8.0 \cdot 10^{-03}$	0.80000	4.763
$1/5 \cdot 2^{-1}$	$1.0 \cdot 10^{-03}$	0.70000	4.353	$2.9 \cdot 10^{-04}$	0.80000	5.017
$1/5 \cdot 2^{-2}$	$5.3 \cdot 10^{-05}$	0.70000	4.668	$9.0 \cdot 10^{-06}$	0.70000	5.408
$1/5 \cdot 2^{-3}$	$2.0 \cdot 10^{-06}$	0.72500	4.835	$2.1 \cdot 10^{-07}$	0.72500	4.845
$1/5 \cdot 2^{-4}$	$7.3 \cdot 10^{-08}$	0.73750	4.918	$7.4 \cdot 10^{-09}$	0.05000	4.018
$1/5 \cdot 2^{-5}$	$2.4 \cdot 10^{-09}$	0.74375	4.959	$4.5 \cdot 10^{-10}$	0.02500	4.001
$1/5 \cdot 2^{-6}$	$7.7 \cdot 10^{-11}$	0.74687	4.978	$2.8 \cdot 10^{-11}$	0.01250	4.000
$1/5 \cdot 2^{-7}$	$2.4 \cdot 10^{-12}$	0.74062	4.519	$1.7 \cdot 10^{-12}$	0.00625	4.000
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-13}$	0.00312	3.979	$1.1 \cdot 10^{-13}$	0.00312	4.000
$1/5 \cdot 2^{-9}$	$6.8 \cdot 10^{-15}$	0.00156	3.989	$7.0 \cdot 10^{-15}$	0.00156	4.000
$1/5 \cdot 2^{-10}$	$4.2 \cdot 10^{-16}$	0.00078	3.994	$4.3 \cdot 10^{-16}$	0.00078	4.000
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-17}$	0.00039	3.997	$2.7 \cdot 10^{-17}$	0.00039	4.000
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-18}$	0.00019		$1.7 \cdot 10^{-18}$	0.00019	

Table 5.39: Implicit Euler, IDeC method on $[0, 1]$ for 4.1e.

h	δ	p	δ	p
$1/5$	$7.0 \cdot 10^{-02}$	0.806	$4.9 \cdot 10^{-02}$	1.849
$1/5 \cdot 2^{-1}$	$4.0 \cdot 10^{-02}$	0.911	$1.3 \cdot 10^{-02}$	1.923
$1/5 \cdot 2^{-2}$	$2.1 \cdot 10^{-02}$	0.957	$3.6 \cdot 10^{-03}$	1.961
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-02}$	0.979	$9.2 \cdot 10^{-04}$	1.981
$1/5 \cdot 2^{-4}$	$5.5 \cdot 10^{-03}$	0.989	$2.3 \cdot 10^{-04}$	1.990
$1/5 \cdot 2^{-5}$	$2.8 \cdot 10^{-03}$	0.994	$5.9 \cdot 10^{-05}$	1.995
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-03}$	0.997	$1.4 \cdot 10^{-05}$	1.997
$1/5 \cdot 2^{-7}$	$7.0 \cdot 10^{-04}$	0.998	$3.7 \cdot 10^{-06}$	1.998
$1/5 \cdot 2^{-8}$	$3.5 \cdot 10^{-04}$	0.999	$9.3 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-9}$	$1.7 \cdot 10^{-04}$	0.999	$2.3 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-10}$	$8.8 \cdot 10^{-05}$	0.999	$5.8 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-11}$	$4.4 \cdot 10^{-05}$	0.999	$1.4 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-05}$	0.999	$3.6 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-13}$	$1.1 \cdot 10^{-05}$	0.999	$9.0 \cdot 10^{-10}$	1.999
$1/5 \cdot 2^{-14}$	$5.5 \cdot 10^{-06}$		$2.2 \cdot 10^{-10}$	

h	δ	p	δ	p
$1/5$	$1.4 \cdot 10^{-02}$	2.798	$4.7 \cdot 10^{-03}$	4.022
$1/5 \cdot 2^{-1}$	$2.1 \cdot 10^{-03}$	2.906	$2.9 \cdot 10^{-04}$	4.042
$1/5 \cdot 2^{-2}$	$2.8 \cdot 10^{-04}$	2.956	$1.7 \cdot 10^{-05}$	4.029
$1/5 \cdot 2^{-3}$	$3.6 \cdot 10^{-05}$	2.978	$1.0 \cdot 10^{-06}$	4.016
$1/5 \cdot 2^{-4}$	$4.5 \cdot 10^{-06}$	2.989	$6.6 \cdot 10^{-08}$	4.008
$1/5 \cdot 2^{-5}$	$5.7 \cdot 10^{-07}$	2.994	$4.1 \cdot 10^{-09}$	4.004
$1/5 \cdot 2^{-6}$	$7.2 \cdot 10^{-08}$	2.997	$2.5 \cdot 10^{-10}$	4.002
$1/5 \cdot 2^{-7}$	$9.0 \cdot 10^{-09}$	2.998	$1.6 \cdot 10^{-11}$	4.001
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-09}$	2.999	$1.0 \cdot 10^{-12}$	4.000
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-10}$	2.999	$6.2 \cdot 10^{-14}$	4.000
$1/5 \cdot 2^{-10}$	$1.7 \cdot 10^{-11}$	2.999	$3.9 \cdot 10^{-15}$	4.000
$1/5 \cdot 2^{-11}$	$2.2 \cdot 10^{-12}$	3.000	$2.4 \cdot 10^{-16}$	4.000
$1/5 \cdot 2^{-12}$	$2.7 \cdot 10^{-13}$	2.999	$1.5 \cdot 10^{-17}$	3.999
$1/5 \cdot 2^{-13}$	$3.4 \cdot 10^{-14}$		$9.5 \cdot 10^{-19}$	

h	δ	p	δ	p
$1/5$	$1.5 \cdot 10^{-03}$	4.888	$1.5 \cdot 10^{-03}$	4.888
$1/5 \cdot 2^{-1}$	$5.2 \cdot 10^{-05}$	4.967	$5.2 \cdot 10^{-05}$	4.967
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-06}$	4.989	$1.6 \cdot 10^{-06}$	4.989
$1/5 \cdot 2^{-3}$	$5.2 \cdot 10^{-08}$	4.996	$5.2 \cdot 10^{-08}$	4.996
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-09}$	4.998	$1.6 \cdot 10^{-09}$	4.998
$1/5 \cdot 2^{-5}$	$5.1 \cdot 10^{-11}$	4.999	$5.1 \cdot 10^{-11}$	4.999
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-12}$	4.999	$1.6 \cdot 10^{-12}$	4.999
$1/5 \cdot 2^{-7}$	$5.0 \cdot 10^{-14}$	4.999	$5.0 \cdot 10^{-14}$	4.999
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-15}$	4.999	$1.5 \cdot 10^{-15}$	4.999
$1/5 \cdot 2^{-9}$	$4.9 \cdot 10^{-17}$	4.999	$4.9 \cdot 10^{-17}$	4.999
$1/5 \cdot 2^{-10}$	$1.5 \cdot 10^{-18}$	4.999	$1.5 \cdot 10^{-18}$	4.999
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-20}$	5.000	$4.7 \cdot 10^{-20}$	5.000
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-21}$		$1.4 \cdot 10^{-21}$	

Table 5.40: Implicit Euler, IDeC method at $t = 0$ for 4.1e.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
1/5	2.0	0.80000	0.581	$9.3 \cdot 10^{-01}$	0.80000	0.795
$1/5 \cdot 2^{-1}$	1.3	0.90000	0.789	$5.3 \cdot 10^{-01}$	0.90000	1.448
$1/5 \cdot 2^{-2}$	$8.0 \cdot 10^{-01}$	0.90000	0.877	$1.9 \cdot 10^{-01}$	0.90000	1.656
$1/5 \cdot 2^{-3}$	$4.4 \cdot 10^{-01}$	0.90000	0.932	$6.2 \cdot 10^{-02}$	0.92500	1.793
$1/5 \cdot 2^{-4}$	$2.3 \cdot 10^{-01}$	0.88750	0.963	$1.8 \cdot 10^{-02}$	0.91250	1.891
$1/5 \cdot 2^{-5}$	$1.1 \cdot 10^{-01}$	0.89375	0.982	$4.8 \cdot 10^{-03}$	0.91875	1.945
$1/5 \cdot 2^{-6}$	$5.9 \cdot 10^{-02}$	0.89687	0.991	$1.2 \cdot 10^{-03}$	0.91875	1.972
$1/5 \cdot 2^{-7}$	$3.0 \cdot 10^{-02}$	0.89531	0.995	$3.2 \cdot 10^{-04}$	0.92031	1.985
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-02}$	0.89609	0.997	$8.1 \cdot 10^{-05}$	0.91953	1.992
$1/5 \cdot 2^{-9}$	$7.5 \cdot 10^{-03}$	0.89648	0.998	$2.0 \cdot 10^{-05}$	0.91992	1.996
$1/5 \cdot 2^{-10}$	$3.7 \cdot 10^{-03}$	0.89628	0.999	$5.1 \cdot 10^{-06}$	0.91992	1.998
$1/5 \cdot 2^{-11}$	$1.8 \cdot 10^{-03}$	0.89638	0.999	$1.2 \cdot 10^{-06}$	0.92001	1.999
$1/5 \cdot 2^{-12}$	$9.4 \cdot 10^{-04}$	0.89638	0.999	$3.2 \cdot 10^{-07}$	0.92001	1.999
$1/5 \cdot 2^{-13}$	$4.7 \cdot 10^{-04}$	0.89641	0.999	$8.0 \cdot 10^{-08}$	0.92001	1.999
$1/5 \cdot 2^{-14}$	$2.3 \cdot 10^{-04}$	0.89641		$2.0 \cdot 10^{-08}$	0.92001	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
1/5	$5.6 \cdot 10^{-01}$	0.80000	1.099	$4.1 \cdot 10^{-01}$	0.80000	1.450
$1/5 \cdot 2^{-1}$	$2.6 \cdot 10^{-01}$	0.90000	2.068	$1.5 \cdot 10^{-01}$	0.90000	2.705
$1/5 \cdot 2^{-2}$	$6.3 \cdot 10^{-02}$	0.90000	2.349	$2.3 \cdot 10^{-02}$	0.90000	3.050
$1/5 \cdot 2^{-3}$	$1.2 \cdot 10^{-02}$	0.92500	2.650	$2.8 \cdot 10^{-03}$	0.92500	3.492
$1/5 \cdot 2^{-4}$	$1.9 \cdot 10^{-03}$	0.92500	2.812	$2.5 \cdot 10^{-04}$	0.92500	3.725
$1/5 \cdot 2^{-5}$	$2.8 \cdot 10^{-04}$	0.92500	2.901	$1.8 \cdot 10^{-05}$	0.93125	3.858
$1/5 \cdot 2^{-6}$	$3.7 \cdot 10^{-05}$	0.92812	2.949	$1.3 \cdot 10^{-06}$	0.93125	3.927
$1/5 \cdot 2^{-7}$	$4.8 \cdot 10^{-06}$	0.92812	2.974	$8.5 \cdot 10^{-08}$	0.93125	3.963
$1/5 \cdot 2^{-8}$	$6.2 \cdot 10^{-07}$	0.92812	2.987	$5.4 \cdot 10^{-09}$	0.93203	3.981
$1/5 \cdot 2^{-9}$	$7.8 \cdot 10^{-08}$	0.92812	2.993	$3.4 \cdot 10^{-10}$	0.93164	3.990
$1/5 \cdot 2^{-10}$	$9.8 \cdot 10^{-09}$	0.92792	2.996	$2.1 \cdot 10^{-11}$	0.93183	3.995
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-09}$	0.92802	2.998	$1.3 \cdot 10^{-12}$	0.93183	3.997
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-10}$	0.92802	2.999	$8.5 \cdot 10^{-14}$	0.93183	3.998
$1/5 \cdot 2^{-13}$	$1.9 \cdot 10^{-11}$	0.92802		$5.3 \cdot 10^{-15}$	0.93186	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
1/5	$3.4 \cdot 10^{-01}$	0.80000	1.793	$3.0 \cdot 10^{-01}$	0.80000	2.074
$1/5 \cdot 2^{-1}$	$9.9 \cdot 10^{-02}$	0.90000	3.286	$7.3 \cdot 10^{-02}$	0.90000	3.755
$1/5 \cdot 2^{-2}$	$1.0 \cdot 10^{-02}$	0.95000	3.864	$5.4 \cdot 10^{-03}$	0.95000	4.762
$1/5 \cdot 2^{-3}$	$6.9 \cdot 10^{-04}$	0.92500	4.315	$1.9 \cdot 10^{-04}$	0.92500	4.957
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-05}$	0.92500	4.649	$6.4 \cdot 10^{-06}$	0.92500	5.432
$1/5 \cdot 2^{-5}$	$1.3 \cdot 10^{-06}$	0.93125	4.820	$1.4 \cdot 10^{-07}$	0.93125	5.518
$1/5 \cdot 2^{-6}$	$4.9 \cdot 10^{-08}$	0.93437	4.909	$3.2 \cdot 10^{-09}$	0.91875	5.449
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-09}$	0.93593	4.952	$7.4 \cdot 10^{-11}$	0.91250	5.312
$1/5 \cdot 2^{-8}$	$5.3 \cdot 10^{-11}$	0.93281	4.976	$1.8 \cdot 10^{-12}$	0.90156	5.179
$1/5 \cdot 2^{-9}$	$1.6 \cdot 10^{-12}$	0.93320	4.988	$5.1 \cdot 10^{-14}$	0.88828	5.092
$1/5 \cdot 2^{-10}$	$5.3 \cdot 10^{-14}$	0.93339	4.994	$1.5 \cdot 10^{-15}$	0.88066	5.046
$1/5 \cdot 2^{-11}$	$1.6 \cdot 10^{-15}$	0.93349	4.997	$4.5 \cdot 10^{-17}$	0.87587	5.023
$1/5 \cdot 2^{-12}$	$5.2 \cdot 10^{-17}$	0.93354		$1.4 \cdot 10^{-18}$	0.87324	

Table 5.41: Implicit Euler, IDeC method on $[0, 1]$ for 4.1f.

h	δ	p	δ	p
$1/5$	$5.1 \cdot 10^{-01}$	0.953	$9.1 \cdot 10^{-02}$	1.928
$1/5 \cdot 2^{-1}$	$2.6 \cdot 10^{-01}$	0.977	$2.4 \cdot 10^{-02}$	1.964
$1/5 \cdot 2^{-2}$	$1.3 \cdot 10^{-01}$	0.988	$6.1 \cdot 10^{-03}$	1.982
$1/5 \cdot 2^{-3}$	$6.7 \cdot 10^{-02}$	0.994	$1.5 \cdot 10^{-03}$	1.991
$1/5 \cdot 2^{-4}$	$3.3 \cdot 10^{-02}$	0.997	$3.9 \cdot 10^{-04}$	1.995
$1/5 \cdot 2^{-5}$	$1.6 \cdot 10^{-02}$	0.998	$9.8 \cdot 10^{-05}$	1.997
$1/5 \cdot 2^{-6}$	$8.4 \cdot 10^{-03}$	0.999	$2.4 \cdot 10^{-05}$	1.998
$1/5 \cdot 2^{-7}$	$4.2 \cdot 10^{-03}$	0.999	$6.1 \cdot 10^{-06}$	1.999
$1/5 \cdot 2^{-8}$	$2.1 \cdot 10^{-03}$	0.999	$1.5 \cdot 10^{-06}$	1.999
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-03}$	0.999	$3.8 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-10}$	$5.3 \cdot 10^{-04}$	0.999	$9.6 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-04}$	1.000	$2.4 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-04}$	0.999	$6.0 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-13}$	$6.6 \cdot 10^{-05}$	0.999	$1.5 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-14}$	$3.3 \cdot 10^{-05}$		$3.7 \cdot 10^{-10}$	

h	δ	p	δ	p
$1/5$	$1.2 \cdot 10^{-02}$	2.895	$2.0 \cdot 10^{-03}$	4.044
$1/5 \cdot 2^{-1}$	$1.7 \cdot 10^{-03}$	2.950	$1.2 \cdot 10^{-04}$	4.033
$1/5 \cdot 2^{-2}$	$2.2 \cdot 10^{-04}$	2.976	$7.4 \cdot 10^{-06}$	4.019
$1/5 \cdot 2^{-3}$	$2.8 \cdot 10^{-05}$	2.988	$4.6 \cdot 10^{-07}$	4.010
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-06}$	2.994	$2.8 \cdot 10^{-08}$	4.005
$1/5 \cdot 2^{-5}$	$4.5 \cdot 10^{-07}$	2.997	$1.7 \cdot 10^{-09}$	4.002
$1/5 \cdot 2^{-6}$	$5.6 \cdot 10^{-08}$	2.998	$1.1 \cdot 10^{-10}$	4.001
$1/5 \cdot 2^{-7}$	$7.0 \cdot 10^{-09}$	2.999	$6.9 \cdot 10^{-12}$	4.000
$1/5 \cdot 2^{-8}$	$8.8 \cdot 10^{-10}$	2.999	$4.3 \cdot 10^{-13}$	4.000
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-10}$	2.999	$2.7 \cdot 10^{-14}$	4.000
$1/5 \cdot 2^{-10}$	$1.3 \cdot 10^{-11}$	2.999	$1.6 \cdot 10^{-15}$	4.000
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-12}$	3.000	$1.0 \cdot 10^{-16}$	4.000
$1/5 \cdot 2^{-12}$	$2.1 \cdot 10^{-13}$	2.999	$6.6 \cdot 10^{-18}$	3.999
$1/5 \cdot 2^{-13}$	$2.6 \cdot 10^{-14}$		$4.1 \cdot 10^{-19}$	

h	δ	p	δ	p
$1/5$	$4.1 \cdot 10^{-04}$	4.964	$4.1 \cdot 10^{-04}$	4.964
$1/5 \cdot 2^{-1}$	$1.3 \cdot 10^{-05}$	4.988	$1.3 \cdot 10^{-05}$	4.988
$1/5 \cdot 2^{-2}$	$4.1 \cdot 10^{-07}$	4.996	$4.1 \cdot 10^{-07}$	4.996
$1/5 \cdot 2^{-3}$	$1.3 \cdot 10^{-08}$	4.998	$1.3 \cdot 10^{-08}$	4.998
$1/5 \cdot 2^{-4}$	$4.1 \cdot 10^{-10}$	4.999	$4.1 \cdot 10^{-10}$	4.999
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-11}$	4.999	$1.2 \cdot 10^{-11}$	4.999
$1/5 \cdot 2^{-6}$	$4.0 \cdot 10^{-13}$	4.999	$4.0 \cdot 10^{-13}$	4.999
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-14}$	4.999	$1.2 \cdot 10^{-14}$	4.999
$1/5 \cdot 2^{-8}$	$3.9 \cdot 10^{-16}$	4.999	$3.9 \cdot 10^{-16}$	4.999
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-17}$	4.999	$1.2 \cdot 10^{-17}$	4.999
$1/5 \cdot 2^{-10}$	$3.8 \cdot 10^{-19}$	4.999	$3.8 \cdot 10^{-19}$	4.999
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-20}$	5.000	$1.1 \cdot 10^{-20}$	5.000
$1/5 \cdot 2^{-12}$	$3.7 \cdot 10^{-22}$		$3.7 \cdot 10^{-22}$	

Table 5.42: Implicit Euler, IDeC method at $t = 0$ for 4.1f.

The order sequences of IDeC based on the trapezoidal rule with Zadunaisky polynomials of degree 10 applied to (5.1) are the same as for the box scheme if the maximal error is considered. However, we observe an improvement of the results in the last step, where, analogous to the implicit Euler method, the evaluation of the right-hand side at $t = 0$ is replaced by the evaluation at $t \approx \text{EPS}$. As for the implicit Euler method, in most cases either the exact value of the solution at $t = 0$ is assumed or the classical order sequence (with maximal order 10 in this case) is observed. The only exception is example 4.2a, where an order reduction down to 6 is observed even in the last step of the integration.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	5.4	0.60000	0.257	4.1	0.60000	0.637
$1/5 \cdot 2^{-1}$	4.5	0.70000	0.435	2.6	0.60000	0.917
$1/5 \cdot 2^{-2}$	3.3	0.65000	0.711	1.4	0.60000	1.348
$1/5 \cdot 2^{-3}$	2.0	0.65000	0.839	$5.5 \cdot 10^{-01}$	0.60000	1.668
$1/5 \cdot 2^{-4}$	1.1	0.63750	0.912	$1.7 \cdot 10^{-01}$	0.58750	1.827
$1/5 \cdot 2^{-5}$	$6.1 \cdot 10^{-01}$	0.63750	0.955	$4.9 \cdot 10^{-02}$	0.58125	1.912
$1/5 \cdot 2^{-6}$	$3.1 \cdot 10^{-01}$	0.63750	0.977	$1.3 \cdot 10^{-02}$	0.58125	1.956
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-01}$	0.63750	0.988	$3.3 \cdot 10^{-03}$	0.57812	1.978
$1/5 \cdot 2^{-8}$	$8.0 \cdot 10^{-02}$	0.63671	0.994	$8.5 \cdot 10^{-04}$	0.57734	1.989
$1/5 \cdot 2^{-9}$	$4.0 \cdot 10^{-02}$	0.63632	0.997	$2.1 \cdot 10^{-04}$	0.57695	1.994
$1/5 \cdot 2^{-10}$	$2.0 \cdot 10^{-02}$	0.63613	0.998	$5.4 \cdot 10^{-05}$	0.57656	1.997
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-02}$	0.63613	0.999	$1.3 \cdot 10^{-05}$	0.57636	1.998
$1/5 \cdot 2^{-12}$	$5.0 \cdot 10^{-03}$	0.63608	0.999	$3.4 \cdot 10^{-06}$	0.57631	1.999
$1/5 \cdot 2^{-13}$	$2.5 \cdot 10^{-03}$	0.63605	0.999	$8.5 \cdot 10^{-07}$	0.57629	1.999
$1/5 \cdot 2^{-14}$	$1.2 \cdot 10^{-03}$	0.63605		$2.1 \cdot 10^{-07}$	0.57628	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	3.7	0.60000	0.898	3.6	0.60000	1.325
$1/5 \cdot 2^{-1}$	1.9	0.60000	1.459	1.4	0.60000	1.906
$1/5 \cdot 2^{-2}$	$7.2 \cdot 10^{-01}$	0.60000	2.060	$3.8 \cdot 10^{-01}$	0.55000	2.780
$1/5 \cdot 2^{-3}$	$1.7 \cdot 10^{-01}$	0.55000	2.514	$5.6 \cdot 10^{-02}$	0.50000	3.362
$1/5 \cdot 2^{-4}$	$3.0 \cdot 10^{-02}$	0.53750	2.749	$5.4 \cdot 10^{-03}$	0.47500	3.675
$1/5 \cdot 2^{-5}$	$4.5 \cdot 10^{-03}$	0.51875	2.874	$4.2 \cdot 10^{-04}$	0.45625	3.838
$1/5 \cdot 2^{-6}$	$6.1 \cdot 10^{-04}$	0.51562	2.937	$2.9 \cdot 10^{-05}$	0.44687	3.919
$1/5 \cdot 2^{-7}$	$8.0 \cdot 10^{-05}$	0.51250	2.968	$1.9 \cdot 10^{-06}$	0.44375	3.959
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-05}$	0.51015	2.984	$1.2 \cdot 10^{-07}$	0.44140	3.979
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-06}$	0.50937	2.992	$8.0 \cdot 10^{-09}$	0.43984	3.989
$1/5 \cdot 2^{-10}$	$1.6 \cdot 10^{-07}$	0.50898	2.996	$5.0 \cdot 10^{-10}$	0.43945	3.994
$1/5 \cdot 2^{-11}$	$2.0 \cdot 10^{-08}$	0.50869	2.998	$3.1 \cdot 10^{-11}$	0.43916	3.055
$1/5 \cdot 2^{-12}$	$2.5 \cdot 10^{-09}$	0.50859	2.998	$3.8 \cdot 10^{-12}$	0.00019	2.995
$1/5 \cdot 2^{-13}$	$3.2 \cdot 10^{-10}$	0.50854		$4.8 \cdot 10^{-13}$	0.00009	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	3.8	0.60000	1.414	4.0	0.60000	1.555
$1/5 \cdot 2^{-1}$	1.4	0.80000	2.751	1.3	0.80000	3.253
$1/5 \cdot 2^{-2}$	$2.1 \cdot 10^{-01}$	0.50000	3.539	$1.4 \cdot 10^{-01}$	0.65000	4.488
$1/5 \cdot 2^{-3}$	$1.8 \cdot 10^{-02}$	0.45000	4.209	$6.4 \cdot 10^{-03}$	0.60000	5.162
$1/5 \cdot 2^{-4}$	$9.8 \cdot 10^{-04}$	0.41250	4.608	$1.7 \cdot 10^{-04}$	0.53750	3.933
$1/5 \cdot 2^{-5}$	$4.0 \cdot 10^{-05}$	0.38750	4.804	$1.1 \cdot 10^{-05}$	0.02500	3.052
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-06}$	0.38125	3.078	$1.4 \cdot 10^{-06}$	0.01250	3.006
$1/5 \cdot 2^{-7}$	$1.7 \cdot 10^{-07}$	0.00625	2.984	$1.7 \cdot 10^{-07}$	0.00625	3.002
$1/5 \cdot 2^{-8}$	$2.1 \cdot 10^{-08}$	0.00312	2.994	$2.1 \cdot 10^{-08}$	0.00312	3.001
$1/5 \cdot 2^{-9}$	$2.7 \cdot 10^{-09}$	0.00156	2.998	$2.7 \cdot 10^{-09}$	0.00156	3.000
$1/5 \cdot 2^{-10}$	$3.4 \cdot 10^{-10}$	0.00078	2.999	$3.4 \cdot 10^{-10}$	0.00078	3.000
$1/5 \cdot 2^{-11}$	$4.2 \cdot 10^{-11}$	0.00039	3.000	$4.2 \cdot 10^{-11}$	0.00039	3.000
$1/5 \cdot 2^{-12}$	$5.3 \cdot 10^{-12}$	0.00019		$5.3 \cdot 10^{-12}$	0.00019	

Table 5.43: Implicit Euler, IDeC method on $[0, 1]$ for 4.2a.

h	δ	p	δ	p
$1/5$	$8.0 \cdot 10^{-01}$	0.749	$9.5 \cdot 10^{-01}$	3.011
$1/5 \cdot 2^{-1}$	$4.8 \cdot 10^{-01}$	0.898	$1.1 \cdot 10^{-01}$	1.726
$1/5 \cdot 2^{-2}$	$2.5 \cdot 10^{-01}$	1.000	$3.5 \cdot 10^{-02}$	1.680
$1/5 \cdot 2^{-3}$	$1.2 \cdot 10^{-01}$	1.027	$1.1 \cdot 10^{-02}$	1.669
$1/5 \cdot 2^{-4}$	$6.3 \cdot 10^{-02}$	1.022	$3.5 \cdot 10^{-03}$	1.742
$1/5 \cdot 2^{-5}$	$3.1 \cdot 10^{-02}$	1.013	$1.0 \cdot 10^{-03}$	1.842
$1/5 \cdot 2^{-6}$	$1.5 \cdot 10^{-02}$	1.007	$2.9 \cdot 10^{-04}$	1.913
$1/5 \cdot 2^{-7}$	$7.6 \cdot 10^{-03}$	1.003	$7.7 \cdot 10^{-05}$	1.955
$1/5 \cdot 2^{-8}$	$3.8 \cdot 10^{-03}$	1.001	$2.0 \cdot 10^{-05}$	1.977
$1/5 \cdot 2^{-9}$	$1.9 \cdot 10^{-03}$	1.000	$5.1 \cdot 10^{-06}$	1.988
$1/5 \cdot 2^{-10}$	$9.5 \cdot 10^{-04}$	1.000	$1.2 \cdot 10^{-06}$	1.994
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-04}$	1.000	$3.2 \cdot 10^{-07}$	1.997
$1/5 \cdot 2^{-12}$	$2.3 \cdot 10^{-04}$	1.000	$8.0 \cdot 10^{-08}$	1.998
$1/5 \cdot 2^{-13}$	$1.1 \cdot 10^{-04}$	1.000	$2.0 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-14}$	$5.9 \cdot 10^{-05}$		$5.0 \cdot 10^{-09}$	

h	δ	p	δ	p
$1/5$	$4.1 \cdot 10^{-01}$	2.115	$9.1 \cdot 10^{-01}$	4.922
$1/5 \cdot 2^{-1}$	$9.5 \cdot 10^{-02}$	1.641	$3.0 \cdot 10^{-02}$	1.341
$1/5 \cdot 2^{-2}$	$3.0 \cdot 10^{-02}$	2.013	$1.1 \cdot 10^{-02}$	1.771
$1/5 \cdot 2^{-3}$	$7.5 \cdot 10^{-03}$	2.234	$3.4 \cdot 10^{-03}$	2.908
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-03}$	2.552	$4.6 \cdot 10^{-04}$	3.466
$1/5 \cdot 2^{-5}$	$2.7 \cdot 10^{-04}$	2.763	$4.1 \cdot 10^{-05}$	3.738
$1/5 \cdot 2^{-6}$	$4.0 \cdot 10^{-05}$	2.879	$3.1 \cdot 10^{-06}$	3.870
$1/5 \cdot 2^{-7}$	$5.4 \cdot 10^{-06}$	2.939	$2.1 \cdot 10^{-07}$	3.935
$1/5 \cdot 2^{-8}$	$7.1 \cdot 10^{-07}$	2.969	$1.4 \cdot 10^{-08}$	3.968
$1/5 \cdot 2^{-9}$	$9.1 \cdot 10^{-08}$	2.984	$8.9 \cdot 10^{-10}$	3.984
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-08}$	2.992	$5.6 \cdot 10^{-11}$	3.992
$1/5 \cdot 2^{-11}$	$1.4 \cdot 10^{-09}$	2.996	$3.5 \cdot 10^{-12}$	3.996
$1/5 \cdot 2^{-12}$	$1.8 \cdot 10^{-10}$	2.997	$2.2 \cdot 10^{-13}$	3.997
$1/5 \cdot 2^{-13}$	$2.2 \cdot 10^{-11}$		$1.3 \cdot 10^{-14}$	

h	δ	p	δ	p
$1/5$	$1.3 \cdot 10^{-01}$	1.515	$9.4 \cdot 10^{-01}$	5.731
$1/5 \cdot 2^{-1}$	$4.7 \cdot 10^{-02}$	1.861	$1.7 \cdot 10^{-02}$	2.121
$1/5 \cdot 2^{-2}$	$1.3 \cdot 10^{-02}$	2.982	$4.0 \cdot 10^{-03}$	3.606
$1/5 \cdot 2^{-3}$	$1.6 \cdot 10^{-03}$	3.970	$3.3 \cdot 10^{-04}$	5.346
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-04}$	4.519	$8.2 \cdot 10^{-06}$	6.743
$1/5 \cdot 2^{-5}$	$4.6 \cdot 10^{-06}$	4.772	$7.6 \cdot 10^{-08}$	5.538
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-07}$	4.886	$1.6 \cdot 10^{-09}$	3.762
$1/5 \cdot 2^{-7}$	$5.7 \cdot 10^{-09}$	4.940	$1.2 \cdot 10^{-10}$	4.625
$1/5 \cdot 2^{-8}$	$1.8 \cdot 10^{-10}$	4.966	$4.9 \cdot 10^{-12}$	4.842
$1/5 \cdot 2^{-9}$	$5.9 \cdot 10^{-12}$	4.978	$1.7 \cdot 10^{-13}$	4.926
$1/5 \cdot 2^{-10}$	$1.8 \cdot 10^{-13}$	4.984	$5.6 \cdot 10^{-15}$	4.964
$1/5 \cdot 2^{-11}$	$5.9 \cdot 10^{-15}$	4.988	$1.8 \cdot 10^{-16}$	4.982
$1/5 \cdot 2^{-12}$	$1.8 \cdot 10^{-16}$		$5.7 \cdot 10^{-18}$	

Table 5.44: Implicit Euler, IDeC method at $t = 0$ for 4.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$3.5 \cdot 10^{-02}$	0.60000	0.697	$1.4 \cdot 10^{-02}$	0.60000	1.677
$1/5 \cdot 2^{-1}$	$2.1 \cdot 10^{-02}$	0.70000	0.860	$4.3 \cdot 10^{-03}$	0.60000	1.799
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-02}$	0.15000	0.886	$1.2 \cdot 10^{-03}$	0.55000	1.907
$1/5 \cdot 2^{-3}$	$6.4 \cdot 10^{-03}$	0.12500	0.945	$3.3 \cdot 10^{-04}$	0.55000	1.896
$1/5 \cdot 2^{-4}$	$3.3 \cdot 10^{-03}$	0.12500	0.970	$9.0 \cdot 10^{-05}$	0.07500	1.933
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-03}$	0.13125	0.985	$2.3 \cdot 10^{-05}$	0.07500	1.966
$1/5 \cdot 2^{-6}$	$8.6 \cdot 10^{-04}$	0.13125	0.992	$6.0 \cdot 10^{-06}$	0.07500	1.983
$1/5 \cdot 2^{-7}$	$4.3 \cdot 10^{-04}$	0.12968	0.996	$1.5 \cdot 10^{-06}$	0.07500	1.991
$1/5 \cdot 2^{-8}$	$2.1 \cdot 10^{-04}$	0.12968	0.998	$3.8 \cdot 10^{-07}$	0.07500	1.995
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-04}$	0.12968	0.999	$9.6 \cdot 10^{-08}$	0.07500	1.997
$1/5 \cdot 2^{-10}$	$5.4 \cdot 10^{-05}$	0.12968	0.999	$2.4 \cdot 10^{-08}$	0.07480	1.998
$1/5 \cdot 2^{-11}$	$2.7 \cdot 10^{-05}$	0.12978	0.999	$6.0 \cdot 10^{-09}$	0.07480	1.999
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-05}$	0.12973	0.999	$1.5 \cdot 10^{-09}$	0.07480	1.999
$1/5 \cdot 2^{-13}$	$6.8 \cdot 10^{-06}$	0.12973	0.999	$3.7 \cdot 10^{-10}$	0.07478	1.999
$1/5 \cdot 2^{-14}$	$3.4 \cdot 10^{-06}$	0.12973		$9.4 \cdot 10^{-11}$	0.07478	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$4.6 \cdot 10^{-03}$	0.40000	2.509	$1.2 \cdot 10^{-03}$	0.40000	2.843
$1/5 \cdot 2^{-1}$	$8.2 \cdot 10^{-04}$	0.40000	2.707	$1.6 \cdot 10^{-04}$	0.20000	3.116
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-04}$	0.05000	2.584	$1.9 \cdot 10^{-05}$	0.20000	3.273
$1/5 \cdot 2^{-3}$	$2.1 \cdot 10^{-05}$	0.05000	2.713	$2.0 \cdot 10^{-06}$	0.02500	3.377
$1/5 \cdot 2^{-4}$	$3.2 \cdot 10^{-06}$	0.03750	2.859	$1.9 \cdot 10^{-07}$	0.01250	3.662
$1/5 \cdot 2^{-5}$	$4.4 \cdot 10^{-07}$	0.03125	2.919	$1.5 \cdot 10^{-08}$	0.00625	3.709
$1/5 \cdot 2^{-6}$	$5.8 \cdot 10^{-08}$	0.03125	2.959	$1.1 \cdot 10^{-09}$	0.00625	3.811
$1/5 \cdot 2^{-7}$	$7.4 \cdot 10^{-09}$	0.03125	2.979	$8.3 \cdot 10^{-11}$	0.00468	3.882
$1/5 \cdot 2^{-8}$	$9.5 \cdot 10^{-10}$	0.03125	2.989	$5.6 \cdot 10^{-12}$	0.00234	3.905
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-10}$	0.03085	2.994	$3.7 \cdot 10^{-13}$	0.00195	3.946
$1/5 \cdot 2^{-10}$	$1.5 \cdot 10^{-11}$	0.03066	2.997	$2.4 \cdot 10^{-14}$	0.00136	3.963
$1/5 \cdot 2^{-11}$	$1.8 \cdot 10^{-12}$	0.03066	2.998	$1.5 \cdot 10^{-15}$	0.00107	3.973
$1/5 \cdot 2^{-12}$	$2.3 \cdot 10^{-13}$	0.03066	2.999	$1.0 \cdot 10^{-16}$	0.00058	3.981
$1/5 \cdot 2^{-13}$	$2.9 \cdot 10^{-14}$	0.03063		$6.3 \cdot 10^{-18}$	0.00041	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$7.9 \cdot 10^{-04}$	0.80000	3.508	$9.2 \cdot 10^{-04}$	0.80000	3.345
$1/5 \cdot 2^{-1}$	$6.9 \cdot 10^{-05}$	0.40000	3.202	$9.0 \cdot 10^{-05}$	0.40000	3.706
$1/5 \cdot 2^{-2}$	$7.5 \cdot 10^{-06}$	0.20000	3.732	$6.9 \cdot 10^{-06}$	0.20000	3.862
$1/5 \cdot 2^{-3}$	$5.7 \cdot 10^{-07}$	0.10000	4.082	$4.7 \cdot 10^{-07}$	0.10000	3.959
$1/5 \cdot 2^{-4}$	$3.3 \cdot 10^{-08}$	0.05000	4.170	$3.0 \cdot 10^{-08}$	0.05000	3.991
$1/5 \cdot 2^{-5}$	$1.8 \cdot 10^{-09}$	0.02500	4.126	$1.9 \cdot 10^{-09}$	0.02500	3.998
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{-10}$	0.01250	4.063	$1.2 \cdot 10^{-10}$	0.01250	3.999
$1/5 \cdot 2^{-7}$	$6.4 \cdot 10^{-12}$	0.00625	4.024	$7.5 \cdot 10^{-12}$	0.00625	3.999
$1/5 \cdot 2^{-8}$	$3.9 \cdot 10^{-13}$	0.00312	4.007	$4.7 \cdot 10^{-13}$	0.00312	3.999
$1/5 \cdot 2^{-9}$	$2.4 \cdot 10^{-14}$	0.00156	4.001	$2.9 \cdot 10^{-14}$	0.00156	3.999
$1/5 \cdot 2^{-10}$	$1.5 \cdot 10^{-15}$	0.00078	3.999	$1.8 \cdot 10^{-15}$	0.00078	3.999
$1/5 \cdot 2^{-11}$	$9.5 \cdot 10^{-17}$	0.00039	3.999	$1.1 \cdot 10^{-16}$	0.00039	4.000
$1/5 \cdot 2^{-12}$	$5.9 \cdot 10^{-18}$	0.00019		$7.2 \cdot 10^{-18}$	0.00019	

Table 5.45: Implicit Euler, IDeC method on $[0, 1]$ for 5.2a.

h	δ	p	δ	p
$1/5$	$3.5 \cdot 10^{-33}$	0.043	$1.2 \cdot 10^{-33}$	0.601
$1/5 \cdot 2^{-1}$	$3.4 \cdot 10^{-33}$	-0.352	$8.3 \cdot 10^{-34}$	0.322
$1/5 \cdot 2^{-2}$	$4.3 \cdot 10^{-33}$	0.066	$6.6 \cdot 10^{-34}$	0.832
$1/5 \cdot 2^{-3}$	$4.1 \cdot 10^{-33}$	-0.268	$3.7 \cdot 10^{-34}$	0.569
$1/5 \cdot 2^{-4}$	$5.0 \cdot 10^{-33}$	0.149	$2.5 \cdot 10^{-34}$	1.040
$1/5 \cdot 2^{-5}$	$4.5 \cdot 10^{-33}$	-0.257	$1.2 \cdot 10^{-34}$	0.672
$1/5 \cdot 2^{-6}$	$5.4 \cdot 10^{-33}$	0.210	$7.7 \cdot 10^{-35}$	1.165
$1/5 \cdot 2^{-7}$	$4.6 \cdot 10^{-33}$	-0.225	$3.4 \cdot 10^{-35}$	0.747
$1/5 \cdot 2^{-8}$	$5.4 \cdot 10^{-33}$	0.217	$2.0 \cdot 10^{-35}$	1.201
$1/5 \cdot 2^{-9}$	$4.7 \cdot 10^{-33}$	-0.224	$8.9 \cdot 10^{-36}$	0.765
$1/5 \cdot 2^{-10}$	$5.5 \cdot 10^{-33}$	0.221	$5.2 \cdot 10^{-36}$	1.216
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-33}$	-0.222	$2.2 \cdot 10^{-36}$	0.774
$1/5 \cdot 2^{-12}$	$5.5 \cdot 10^{-33}$	0.222	$1.3 \cdot 10^{-36}$	1.220
$1/5 \cdot 2^{-13}$	$4.7 \cdot 10^{-33}$	-0.222	$5.6 \cdot 10^{-37}$	0.776
$1/5 \cdot 2^{-14}$	$5.5 \cdot 10^{-33}$		$3.3 \cdot 10^{-37}$	

h	δ	p	δ	p
$1/5$	$4.6 \cdot 10^{-34}$	0.986	$1.6 \cdot 10^{-34}$	1.212
$1/5 \cdot 2^{-1}$	$2.3 \cdot 10^{-34}$	0.818	$7.3 \cdot 10^{-35}$	1.129
$1/5 \cdot 2^{-2}$	$1.3 \cdot 10^{-34}$	1.402	$3.3 \cdot 10^{-35}$	1.742
$1/5 \cdot 2^{-3}$	$4.9 \cdot 10^{-35}$	1.194	$1.0 \cdot 10^{-35}$	1.534
$1/5 \cdot 2^{-4}$	$2.1 \cdot 10^{-35}$	1.711	$3.4 \cdot 10^{-36}$	2.035
$1/5 \cdot 2^{-5}$	$6.6 \cdot 10^{-36}$	1.381	$8.4 \cdot 10^{-37}$	1.679
$1/5 \cdot 2^{-6}$	$2.5 \cdot 10^{-36}$	1.908	$2.6 \cdot 10^{-37}$	2.176
$1/5 \cdot 2^{-7}$	$6.7 \cdot 10^{-37}$	1.518	$5.8 \cdot 10^{-38}$	1.756
$1/5 \cdot 2^{-8}$	$2.3 \cdot 10^{-37}$	1.996	$1.7 \cdot 10^{-38}$	2.208
$1/5 \cdot 2^{-9}$	$5.9 \cdot 10^{-38}$	1.582	$3.7 \cdot 10^{-39}$	1.770
$1/5 \cdot 2^{-10}$	$1.9 \cdot 10^{-38}$	2.050	$1.0 \cdot 10^{-39}$	2.219
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-39}$	1.624	$2.3 \cdot 10^{-40}$	1.776
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-39}$	2.083	$6.8 \cdot 10^{-41}$	2.221
$1/5 \cdot 2^{-13}$	$3.6 \cdot 10^{-40}$		$1.4 \cdot 10^{-41}$	

h	δ	p	δ	p
$1/5$	$5.7 \cdot 10^{-35}$	1.288	$1.5 \cdot 10^{-35}$	1.220
$1/5 \cdot 2^{-1}$	$2.3 \cdot 10^{-35}$	1.277	$6.5 \cdot 10^{-36}$	1.295
$1/5 \cdot 2^{-2}$	$9.6 \cdot 10^{-36}$	1.902	$2.6 \cdot 10^{-36}$	1.957
$1/5 \cdot 2^{-3}$	$2.5 \cdot 10^{-36}$	1.671	$6.8 \cdot 10^{-37}$	1.717
$1/5 \cdot 2^{-4}$	$8.0 \cdot 10^{-37}$	2.135	$2.0 \cdot 10^{-37}$	2.162
$1/5 \cdot 2^{-5}$	$1.8 \cdot 10^{-37}$	1.744	$4.6 \cdot 10^{-38}$	1.757
$1/5 \cdot 2^{-6}$	$5.4 \cdot 10^{-38}$	2.215	$1.3 \cdot 10^{-38}$	2.220
$1/5 \cdot 2^{-7}$	$1.1 \cdot 10^{-38}$	1.778	$2.9 \cdot 10^{-39}$	1.780
$1/5 \cdot 2^{-8}$	$3.4 \cdot 10^{-39}$	2.220	$8.6 \cdot 10^{-40}$	2.220
$1/5 \cdot 2^{-9}$	$7.4 \cdot 10^{-40}$	1.776	$1.8 \cdot 10^{-40}$	1.777
$1/5 \cdot 2^{-10}$	$2.1 \cdot 10^{-40}$	2.222	$5.4 \cdot 10^{-41}$	2.222
$1/5 \cdot 2^{-11}$	$4.6 \cdot 10^{-41}$	1.777	$1.1 \cdot 10^{-41}$	1.777
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-41}$		$3.3 \cdot 10^{-42}$	

Table 5.46: Implicit Euler, IDeC method at $t = 0$ for 5.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.4 \cdot 10^{-32}$	0.70000	-2.000	$3.9 \cdot 10^{-31}$	0.90000	0.830
$1/5 \cdot 2^{-2}$	$9.8 \cdot 10^{-32}$	0.00000	0.0000	$2.2 \cdot 10^{-31}$	0.00000	-1.700
$1/5 \cdot 2^{-3}$	$9.8 \cdot 10^{-32}$	0.52500	-3.129	$7.2 \cdot 10^{-31}$	0.72500	-1.469
$1/5 \cdot 2^{-4}$	$8.6 \cdot 10^{-31}$	0.01250	0.175	$1.9 \cdot 10^{-30}$	0.13750	-1.398
$1/5 \cdot 2^{-5}$	$7.6 \cdot 10^{-31}$	-0.0000	-0.675	$5.2 \cdot 10^{-30}$	0.01250	0.149
$1/5 \cdot 2^{-6}$	$1.2 \cdot 10^{-30}$	0.69062	-0.021	$4.7 \cdot 10^{-30}$	0.00625	-2.975
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-30}$	0.52343	-3.491	$3.7 \cdot 10^{-29}$	0.00000	0.083
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-29}$	-0.0000	-0.091	$3.5 \cdot 10^{-29}$	0.00078	-0.585
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-29}$	0.00117	-0.580	$5.2 \cdot 10^{-29}$	0.00039	-0.492
$1/5 \cdot 2^{-10}$	$2.2 \cdot 10^{-29}$	0.64824	0.316	$7.4 \cdot 10^{-29}$	0.01582	-0.178
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-29}$	0.25009	-3.643	$8.4 \cdot 10^{-29}$	0.00146	-2.634
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-28}$	0.00004	-0.093	$5.2 \cdot 10^{-28}$	0.00004	-0.776
$1/5 \cdot 2^{-13}$	$2.3 \cdot 10^{-28}$	-0.0000	-0.627	$8.9 \cdot 10^{-28}$	0.00004	-1.370
$1/5 \cdot 2^{-14}$	$3.6 \cdot 10^{-28}$	0.63657		$2.3 \cdot 10^{-27}$	0.00002	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.6 \cdot 10^{-31}$	0.10000	-0.275	$6.4 \cdot 10^{-31}$	0.10000	0.145
$1/5 \cdot 2^{-2}$	$5.6 \cdot 10^{-31}$	0.20000	0.064	$5.7 \cdot 10^{-31}$	0.55000	-0.937
$1/5 \cdot 2^{-3}$	$5.4 \cdot 10^{-31}$	0.00000	-0.649	$1.1 \cdot 10^{-30}$	0.22500	-1.579
$1/5 \cdot 2^{-4}$	$8.5 \cdot 10^{-31}$	0.62500	-0.711	$3.3 \cdot 10^{-30}$	0.01250	0.540
$1/5 \cdot 2^{-5}$	$1.3 \cdot 10^{-30}$	0.06250	-1.056	$2.2 \cdot 10^{-30}$	0.00625	-2.003
$1/5 \cdot 2^{-6}$	$2.8 \cdot 10^{-30}$	0.03437	-2.184	$9.1 \cdot 10^{-30}$	0.13125	-0.569
$1/5 \cdot 2^{-7}$	$1.3 \cdot 10^{-29}$	0.00625	2.094	$1.3 \cdot 10^{-29}$	0.00312	-1.440
$1/5 \cdot 2^{-8}$	$3.0 \cdot 10^{-30}$	0.32890	-0.648	$3.6 \cdot 10^{-29}$	0.00078	0.755
$1/5 \cdot 2^{-9}$	$4.8 \cdot 10^{-30}$	0.41445	-3.038	$2.1 \cdot 10^{-29}$	0.06992	-1.631
$1/5 \cdot 2^{-10}$	$3.9 \cdot 10^{-29}$	0.41777	-1.593	$6.7 \cdot 10^{-29}$	0.22285	-0.918
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-28}$	0.00000	-0.431	$1.2 \cdot 10^{-28}$	0.00878	-0.760
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-28}$	0.43315	0.881	$2.1 \cdot 10^{-28}$	0.00004	-0.796
$1/5 \cdot 2^{-13}$	$8.7 \cdot 10^{-29}$	0.52170		$3.7 \cdot 10^{-28}$	0.00786	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$6.6 \cdot 10^{-31}$	-0.0000	-0.374	$5.1 \cdot 10^{-31}$	0.10000	0.347
$1/5 \cdot 2^{-2}$	$8.6 \cdot 10^{-31}$	0.10000	0.703	$4.0 \cdot 10^{-31}$	0.45000	-0.909
$1/5 \cdot 2^{-3}$	$5.3 \cdot 10^{-31}$	0.07500	-1.341	$7.6 \cdot 10^{-31}$	0.00000	-1.312
$1/5 \cdot 2^{-4}$	$1.3 \cdot 10^{-30}$	0.51250	-1.174	$1.8 \cdot 10^{-30}$	0.01250	0.396
$1/5 \cdot 2^{-5}$	$3.0 \cdot 10^{-30}$	-0.0000	-0.802	$1.4 \cdot 10^{-30}$	0.36875	-1.112
$1/5 \cdot 2^{-6}$	$5.2 \cdot 10^{-30}$	0.21875	-0.614	$3.1 \cdot 10^{-30}$	0.62187	-1.756
$1/5 \cdot 2^{-7}$	$8.0 \cdot 10^{-30}$	0.06406	-0.171	$1.0 \cdot 10^{-29}$	0.00312	-0.991
$1/5 \cdot 2^{-8}$	$9.1 \cdot 10^{-30}$	0.11562	-0.646	$2.0 \cdot 10^{-29}$	0.00078	0.025
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-29}$	0.00039	-2.100	$2.0 \cdot 10^{-29}$	0.00468	-0.602
$1/5 \cdot 2^{-10}$	$6.1 \cdot 10^{-29}$	0.00214	-0.738	$3.1 \cdot 10^{-29}$	0.28730	-2.123
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-28}$	0.00253	-1.360	$1.3 \cdot 10^{-28}$	0.01074	-0.813
$1/5 \cdot 2^{-12}$	$2.6 \cdot 10^{-28}$	0.00249		$2.3 \cdot 10^{-28}$	0.00004	

Table 5.47: Trapezoidal rule, IDeC method on $[0, 1]$ for 4.1a.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$9.2 \cdot 10^{-33}$	-3.736	$1.2 \cdot 10^{-31}$	-1.000
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-31}$	1.321	$2.4 \cdot 10^{-31}$	0.321
$1/5 \cdot 2^{-3}$	$4.9 \cdot 10^{-32}$	-4.000	$1.9 \cdot 10^{-31}$	-3.044
$1/5 \cdot 2^{-4}$	$7.8 \cdot 10^{-31}$	0.356	$1.6 \cdot 10^{-30}$	-1.533
$1/5 \cdot 2^{-5}$	$6.1 \cdot 10^{-31}$	-0.941	$4.7 \cdot 10^{-30}$	-0.022
$1/5 \cdot 2^{-6}$	$1.1 \cdot 10^{-30}$	1.087	$4.7 \cdot 10^{-30}$	-2.985
$1/5 \cdot 2^{-7}$	$5.5 \cdot 10^{-31}$	-4.502	$3.7 \cdot 10^{-29}$	0.190
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-29}$	0.0000	$3.3 \cdot 10^{-29}$	-0.605
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-29}$	-0.245	$5.0 \cdot 10^{-29}$	-0.639
$1/5 \cdot 2^{-10}$	$1.4 \cdot 10^{-29}$	0.748	$7.8 \cdot 10^{-29}$	-0.238
$1/5 \cdot 2^{-11}$	$8.9 \cdot 10^{-30}$	-4.502	$9.2 \cdot 10^{-29}$	-2.432
$1/5 \cdot 2^{-12}$	$2.0 \cdot 10^{-28}$	0.0000	$5.0 \cdot 10^{-28}$	-0.778
$1/5 \cdot 2^{-13}$	$2.0 \cdot 10^{-28}$	-0.127	$8.5 \cdot 10^{-28}$	-1.480
$1/5 \cdot 2^{-14}$	$2.2 \cdot 10^{-28}$		$2.3 \cdot 10^{-27}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$3.6 \cdot 10^{-31}$	4.206	$3.2 \cdot 10^{-31}$	2.700
$1/5 \cdot 2^{-2}$	$2.0 \cdot 10^{-32}$	-4.823	$4.9 \cdot 10^{-32}$	-2.700
$1/5 \cdot 2^{-3}$	$5.6 \cdot 10^{-31}$	0.064	$3.2 \cdot 10^{-31}$	-2.971
$1/5 \cdot 2^{-4}$	$5.4 \cdot 10^{-31}$	1.973	$2.5 \cdot 10^{-30}$	0.971
$1/5 \cdot 2^{-5}$	$1.3 \cdot 10^{-31}$	-4.114	$1.2 \cdot 10^{-30}$	-2.791
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-30}$	-2.312	$8.8 \cdot 10^{-30}$	-0.632
$1/5 \cdot 2^{-7}$	$1.1 \cdot 10^{-29}$	3.195	$1.3 \cdot 10^{-29}$	-1.320
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-30}$	-0.779	$3.4 \cdot 10^{-29}$	1.100
$1/5 \cdot 2^{-9}$	$2.2 \cdot 10^{-30}$	-3.435	$1.6 \cdot 10^{-29}$	-2.098
$1/5 \cdot 2^{-10}$	$2.4 \cdot 10^{-29}$	-2.417	$6.8 \cdot 10^{-29}$	-0.981
$1/5 \cdot 2^{-11}$	$1.2 \cdot 10^{-28}$	0.392	$1.3 \cdot 10^{-28}$	-0.525
$1/5 \cdot 2^{-12}$	$9.7 \cdot 10^{-29}$	0.339	$1.9 \cdot 10^{-28}$	-0.798
$1/5 \cdot 2^{-13}$	$7.7 \cdot 10^{-29}$		$3.3 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$6.6 \cdot 10^{-31}$	3.169	$3.9 \cdot 10^{-31}$	0.192
$1/5 \cdot 2^{-2}$	$7.3 \cdot 10^{-32}$	-0.415	$3.4 \cdot 10^{-31}$	-1.099
$1/5 \cdot 2^{-3}$	$9.8 \cdot 10^{-32}$	-3.285	$7.3 \cdot 10^{-31}$	-0.765
$1/5 \cdot 2^{-4}$	$9.6 \cdot 10^{-31}$	-1.584	$1.2 \cdot 10^{-30}$	0.502
$1/5 \cdot 2^{-5}$	$2.8 \cdot 10^{-30}$	-0.478	$8.8 \cdot 10^{-31}$	-1.187
$1/5 \cdot 2^{-6}$	$4.0 \cdot 10^{-30}$	-0.816	$2.0 \cdot 10^{-30}$	-2.436
$1/5 \cdot 2^{-7}$	$7.0 \cdot 10^{-30}$	-0.401	$1.0 \cdot 10^{-29}$	-0.765
$1/5 \cdot 2^{-8}$	$9.3 \cdot 10^{-30}$	-0.298	$1.8 \cdot 10^{-29}$	0.074
$1/5 \cdot 2^{-9}$	$1.1 \cdot 10^{-29}$	-2.509	$1.7 \cdot 10^{-29}$	-0.277
$1/5 \cdot 2^{-10}$	$6.5 \cdot 10^{-29}$	-0.753	$2.1 \cdot 10^{-29}$	-2.727
$1/5 \cdot 2^{-11}$	$1.1 \cdot 10^{-28}$	-1.120	$1.4 \cdot 10^{-28}$	-0.621
$1/5 \cdot 2^{-12}$	$2.3 \cdot 10^{-28}$		$2.1 \cdot 10^{-28}$	

Table 5.48: Trapezoidal rule, IDeC method at $t = 0$ for 4.1a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.8 \cdot 10^{-03}$	0.60000	2.002	$7.2 \cdot 10^{-06}$	0.60000	4.005
$1/5 \cdot 2^{-2}$	$7.2 \cdot 10^{-04}$	0.60000	1.997	$4.5 \cdot 10^{-07}$	0.60000	3.998
$1/5 \cdot 2^{-3}$	$1.8 \cdot 10^{-04}$	0.57500	2.000	$2.8 \cdot 10^{-08}$	0.57500	4.000
$1/5 \cdot 2^{-4}$	$4.5 \cdot 10^{-05}$	0.57500	2.000	$1.7 \cdot 10^{-09}$	0.57500	4.000
$1/5 \cdot 2^{-5}$	$1.1 \cdot 10^{-05}$	0.57500	1.999	$1.1 \cdot 10^{-10}$	0.57500	3.999
$1/5 \cdot 2^{-6}$	$2.8 \cdot 10^{-06}$	0.57812	2.000	$6.8 \cdot 10^{-12}$	0.57812	4.000
$1/5 \cdot 2^{-7}$	$7.0 \cdot 10^{-07}$	0.57812	1.999	$4.3 \cdot 10^{-13}$	0.57812	3.999
$1/5 \cdot 2^{-8}$	$1.7 \cdot 10^{-07}$	0.57734	2.000	$2.6 \cdot 10^{-14}$	0.57734	4.000
$1/5 \cdot 2^{-9}$	$4.4 \cdot 10^{-08}$	0.57734	2.000	$1.6 \cdot 10^{-15}$	0.57734	4.000
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-08}$	0.57734	2.000	$1.0 \cdot 10^{-16}$	0.57734	4.000
$1/5 \cdot 2^{-11}$	$2.7 \cdot 10^{-09}$	0.57734	2.000	$6.5 \cdot 10^{-18}$	0.57734	4.000
$1/5 \cdot 2^{-12}$	$6.8 \cdot 10^{-10}$	0.57734	2.000	$4.1 \cdot 10^{-19}$	0.57734	4.000
$1/5 \cdot 2^{-13}$	$1.7 \cdot 10^{-10}$	0.57734	1.999	$2.5 \cdot 10^{-20}$	0.57734	3.999
$1/5 \cdot 2^{-14}$	$4.3 \cdot 10^{-11}$	0.57735		$1.6 \cdot 10^{-21}$	0.57735	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.8 \cdot 10^{-08}$	0.60000	6.008	$4.5 \cdot 10^{-11}$	0.60000	8.010
$1/5 \cdot 2^{-2}$	$2.8 \cdot 10^{-10}$	0.60000	5.998	$1.7 \cdot 10^{-13}$	0.60000	7.999
$1/5 \cdot 2^{-3}$	$4.4 \cdot 10^{-12}$	0.57500	6.000	$6.8 \cdot 10^{-16}$	0.57500	8.000
$1/5 \cdot 2^{-4}$	$6.8 \cdot 10^{-14}$	0.57500	6.000	$2.6 \cdot 10^{-18}$	0.57500	8.000
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-15}$	0.57500	5.999	$1.0 \cdot 10^{-20}$	0.57500	8.000
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-17}$	0.57812	6.000	$4.1 \cdot 10^{-23}$	0.57812	8.000
$1/5 \cdot 2^{-7}$	$2.6 \cdot 10^{-19}$	0.57812	5.999	$1.6 \cdot 10^{-25}$	0.57656	7.997
$1/5 \cdot 2^{-8}$	$4.1 \cdot 10^{-21}$	0.57734	6.000	$6.2 \cdot 10^{-28}$	0.58593	4.687
$1/5 \cdot 2^{-9}$	$6.4 \cdot 10^{-23}$	0.57734	5.999	$2.4 \cdot 10^{-29}$	0.08632	−1.000
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-24}$	0.57734	5.998	$4.8 \cdot 10^{-29}$	0.00000	−0.638
$1/5 \cdot 2^{-11}$	$1.5 \cdot 10^{-26}$	0.57802	6.274	$7.5 \cdot 10^{-29}$	0.00087	−1.187
$1/5 \cdot 2^{-12}$	$2.0 \cdot 10^{-28}$	0.45361	1.083	$1.7 \cdot 10^{-28}$	0.00346	−0.909
$1/5 \cdot 2^{-13}$	$9.5 \cdot 10^{-29}$	0.02080		$3.2 \cdot 10^{-28}$	0.00063	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.1 \cdot 10^{-13}$	0.60000	10.013	$2.8 \cdot 10^{-16}$	0.60000	12.016
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-16}$	0.60000	10.000	$6.8 \cdot 10^{-20}$	0.60000	12.000
$1/5 \cdot 2^{-3}$	$1.0 \cdot 10^{-19}$	0.57500	10.000	$1.6 \cdot 10^{-23}$	0.57500	12.001
$1/5 \cdot 2^{-4}$	$1.0 \cdot 10^{-22}$	0.57500	10.000	$4.1 \cdot 10^{-27}$	0.57500	11.059
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-25}$	0.57500	9.985	$1.9 \cdot 10^{-30}$	−0.0000	−1.449
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{-28}$	0.59687	3.977	$5.2 \cdot 10^{-30}$	0.21875	−0.169
$1/5 \cdot 2^{-7}$	$6.4 \cdot 10^{-30}$	0.13906	−0.057	$5.9 \cdot 10^{-30}$	0.00781	−1.709
$1/5 \cdot 2^{-8}$	$6.6 \cdot 10^{-30}$	0.21953	−2.115	$1.9 \cdot 10^{-29}$	0.02265	0.223
$1/5 \cdot 2^{-9}$	$2.8 \cdot 10^{-29}$	0.01523	−0.249	$1.6 \cdot 10^{-29}$	0.05156	−1.203
$1/5 \cdot 2^{-10}$	$3.4 \cdot 10^{-29}$	0.06855	−0.638	$3.8 \cdot 10^{-29}$	0.00214	−1.221
$1/5 \cdot 2^{-11}$	$5.3 \cdot 10^{-29}$	0.16416	−1.406	$8.8 \cdot 10^{-29}$	0.03916	−0.901
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-28}$	0.00419		$1.6 \cdot 10^{-28}$	0.04780	

Table 5.49: Trapezoidal rule, IDeC method on $[0, 1]$ for 4.1b.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$7.3 \cdot 10^{-32}$	0.584	$3.9 \cdot 10^{-31}$	0.093
$1/5 \cdot 2^{-2}$	$4.9 \cdot 10^{-32}$	0.0000	$3.6 \cdot 10^{-31}$	-1.341
$1/5 \cdot 2^{-3}$	$4.9 \cdot 10^{-32}$	0.0000	$9.3 \cdot 10^{-31}$	-1.871
$1/5 \cdot 2^{-4}$	$4.9 \cdot 10^{-32}$	-2.807	$3.4 \cdot 10^{-30}$	1.761
$1/5 \cdot 2^{-5}$	$3.4 \cdot 10^{-31}$	-0.192	$1.0 \cdot 10^{-30}$	-3.166
$1/5 \cdot 2^{-6}$	$3.9 \cdot 10^{-31}$	-0.954	$9.0 \cdot 10^{-30}$	-0.584
$1/5 \cdot 2^{-7}$	$7.6 \cdot 10^{-31}$	-0.976	$1.3 \cdot 10^{-29}$	-0.867
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-30}$	-1.057	$2.4 \cdot 10^{-29}$	-1.015
$1/5 \cdot 2^{-9}$	$3.1 \cdot 10^{-30}$	-1.044	$5.0 \cdot 10^{-29}$	-1.258
$1/5 \cdot 2^{-10}$	$6.4 \cdot 10^{-30}$	-0.646	$1.2 \cdot 10^{-28}$	-0.923
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-29}$	-1.126	$2.2 \cdot 10^{-28}$	-0.901
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-29}$	-0.894	$4.2 \cdot 10^{-28}$	-0.968
$1/5 \cdot 2^{-13}$	$4.1 \cdot 10^{-29}$	-1.015	$8.3 \cdot 10^{-28}$	-0.946
$1/5 \cdot 2^{-14}$	$8.2 \cdot 10^{-29}$		$1.6 \cdot 10^{-27}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.9 \cdot 10^{-31}$	-1.459	$2.4 \cdot 10^{-31}$	-0.137
$1/5 \cdot 2^{-2}$	$5.4 \cdot 10^{-31}$	-1.095	$2.7 \cdot 10^{-31}$	-0.241
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-30}$	1.162	$3.2 \cdot 10^{-31}$	-2.027
$1/5 \cdot 2^{-4}$	$5.1 \cdot 10^{-31}$	-2.000	$1.3 \cdot 10^{-30}$	-0.079
$1/5 \cdot 2^{-5}$	$2.0 \cdot 10^{-30}$	-0.490	$1.3 \cdot 10^{-30}$	-0.567
$1/5 \cdot 2^{-6}$	$2.9 \cdot 10^{-30}$	2.423	$2.0 \cdot 10^{-30}$	1.089
$1/5 \cdot 2^{-7}$	$5.4 \cdot 10^{-31}$	-2.540	$9.6 \cdot 10^{-31}$	-4.063
$1/5 \cdot 2^{-8}$	$3.1 \cdot 10^{-30}$	0.011	$1.6 \cdot 10^{-29}$	-0.474
$1/5 \cdot 2^{-9}$	$3.1 \cdot 10^{-30}$	0.288	$2.2 \cdot 10^{-29}$	-1.123
$1/5 \cdot 2^{-10}$	$2.5 \cdot 10^{-30}$	-2.975	$4.8 \cdot 10^{-29}$	-0.633
$1/5 \cdot 2^{-11}$	$2.0 \cdot 10^{-29}$	-1.018	$7.5 \cdot 10^{-29}$	-1.168
$1/5 \cdot 2^{-12}$	$4.0 \cdot 10^{-29}$	-1.084	$1.6 \cdot 10^{-28}$	-0.932
$1/5 \cdot 2^{-13}$	$8.6 \cdot 10^{-29}$		$3.2 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$2.9 \cdot 10^{-31}$	0.584	$9.8 \cdot 10^{-31}$	1.736
$1/5 \cdot 2^{-2}$	$1.9 \cdot 10^{-31}$	1.415	$2.9 \cdot 10^{-31}$	-0.874
$1/5 \cdot 2^{-3}$	$7.3 \cdot 10^{-32}$	-1.874	$5.4 \cdot 10^{-31}$	-1.447
$1/5 \cdot 2^{-4}$	$2.7 \cdot 10^{-31}$	-3.295	$1.4 \cdot 10^{-30}$	-0.378
$1/5 \cdot 2^{-5}$	$2.6 \cdot 10^{-30}$	2.295	$1.9 \cdot 10^{-30}$	-1.018
$1/5 \cdot 2^{-6}$	$5.4 \cdot 10^{-31}$	-3.529	$3.8 \cdot 10^{-30}$	-0.396
$1/5 \cdot 2^{-7}$	$6.2 \cdot 10^{-30}$	0.731	$5.1 \cdot 10^{-30}$	-1.827
$1/5 \cdot 2^{-8}$	$3.7 \cdot 10^{-30}$	-2.936	$1.8 \cdot 10^{-29}$	0.442
$1/5 \cdot 2^{-9}$	$2.8 \cdot 10^{-29}$	-0.142	$1.3 \cdot 10^{-29}$	-1.478
$1/5 \cdot 2^{-10}$	$3.1 \cdot 10^{-29}$	-0.587	$3.7 \cdot 10^{-29}$	-1.166
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-29}$	-1.561	$8.3 \cdot 10^{-29}$	-0.962
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-28}$		$1.6 \cdot 10^{-28}$	

Table 5.50: Trapezoidal rule, IDeC method at $t = 0$ for 4.1b.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.7 \cdot 10^{-01}$	0.90000	2.115	$4.6 \cdot 10^{-02}$	0.90000	4.314
$1/5 \cdot 2^{-2}$	$6.2 \cdot 10^{-02}$	0.90000	2.026	$2.3 \cdot 10^{-03}$	0.90000	4.072
$1/5 \cdot 2^{-3}$	$1.5 \cdot 10^{-02}$	0.90000	2.006	$1.3 \cdot 10^{-04}$	0.90000	4.017
$1/5 \cdot 2^{-4}$	$3.8 \cdot 10^{-03}$	0.90000	1.999	$8.6 \cdot 10^{-06}$	0.90000	4.001
$1/5 \cdot 2^{-5}$	$9.6 \cdot 10^{-04}$	0.89375	2.000	$5.3 \cdot 10^{-07}$	0.90625	4.001
$1/5 \cdot 2^{-6}$	$2.4 \cdot 10^{-04}$	0.89375	2.000	$3.3 \cdot 10^{-08}$	0.90625	4.000
$1/5 \cdot 2^{-7}$	$6.0 \cdot 10^{-05}$	0.89375	1.999	$2.1 \cdot 10^{-09}$	0.90625	4.000
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-05}$	0.89453	2.000	$1.3 \cdot 10^{-10}$	0.90625	4.000
$1/5 \cdot 2^{-9}$	$3.7 \cdot 10^{-06}$	0.89453	2.000	$8.2 \cdot 10^{-12}$	0.90625	4.000
$1/5 \cdot 2^{-10}$	$9.3 \cdot 10^{-07}$	0.89433	1.999	$5.1 \cdot 10^{-13}$	0.90605	4.000
$1/5 \cdot 2^{-11}$	$2.3 \cdot 10^{-07}$	0.89443	2.000	$3.2 \cdot 10^{-14}$	0.90615	4.000
$1/5 \cdot 2^{-12}$	$5.8 \cdot 10^{-08}$	0.89443	2.000	$2.0 \cdot 10^{-15}$	0.90615	4.000
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-08}$	0.89443	1.999	$1.2 \cdot 10^{-16}$	0.90612	3.999
$1/5 \cdot 2^{-14}$	$3.6 \cdot 10^{-09}$	0.89442		$7.8 \cdot 10^{-18}$	0.90612	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.0 \cdot 10^{-02}$	0.90000	6.525	$2.5 \cdot 10^{-03}$	0.90000	8.735
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-04}$	0.90000	6.120	$5.8 \cdot 10^{-06}$	0.90000	8.169
$1/5 \cdot 2^{-3}$	$1.6 \cdot 10^{-06}$	0.90000	6.022	$2.0 \cdot 10^{-08}$	0.90000	8.032
$1/5 \cdot 2^{-4}$	$2.5 \cdot 10^{-08}$	0.91250	6.007	$7.8 \cdot 10^{-11}$	0.91250	8.010
$1/5 \cdot 2^{-5}$	$3.8 \cdot 10^{-10}$	0.90625	6.001	$3.0 \cdot 10^{-13}$	0.91250	8.002
$1/5 \cdot 2^{-6}$	$6.0 \cdot 10^{-12}$	0.90937	6.000	$1.1 \cdot 10^{-15}$	0.90937	8.000
$1/5 \cdot 2^{-7}$	$9.5 \cdot 10^{-14}$	0.90937	6.000	$4.6 \cdot 10^{-18}$	0.90937	8.000
$1/5 \cdot 2^{-8}$	$1.4 \cdot 10^{-15}$	0.90937	6.000	$1.8 \cdot 10^{-20}$	0.91015	8.000
$1/5 \cdot 2^{-9}$	$2.3 \cdot 10^{-17}$	0.90898	6.000	$7.0 \cdot 10^{-23}$	0.91015	8.000
$1/5 \cdot 2^{-10}$	$3.6 \cdot 10^{-19}$	0.90917	6.000	$2.7 \cdot 10^{-25}$	0.91015	8.056
$1/5 \cdot 2^{-11}$	$5.6 \cdot 10^{-21}$	0.90917	6.000	$1.0 \cdot 10^{-27}$	0.91533	2.339
$1/5 \cdot 2^{-12}$	$8.8 \cdot 10^{-23}$	0.90913	6.000	$2.0 \cdot 10^{-28}$	0.01274	-0.825
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-24}$	0.90915		$3.6 \cdot 10^{-28}$	-0.0000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$6.1 \cdot 10^{-04}$	0.90000	10.941	$1.5 \cdot 10^{-04}$	0.90000	13.145
$1/5 \cdot 2^{-2}$	$3.1 \cdot 10^{-07}$	0.90000	10.216	$1.6 \cdot 10^{-08}$	0.90000	12.264
$1/5 \cdot 2^{-3}$	$2.6 \cdot 10^{-10}$	0.90000	10.042	$3.4 \cdot 10^{-12}$	0.90000	6.970
$1/5 \cdot 2^{-4}$	$2.4 \cdot 10^{-13}$	0.91250	2.358	$2.7 \cdot 10^{-14}$	-0.0000	-0.836
$1/5 \cdot 2^{-5}$	$4.8 \cdot 10^{-14}$	-0.0000	17.657	$4.8 \cdot 10^{-14}$	-0.0000	29.940
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-19}$	0.90937	10.000	$4.7 \cdot 10^{-23}$	0.90937	12.001
$1/5 \cdot 2^{-7}$	$2.2 \cdot 10^{-22}$	0.91093	10.000	$1.1 \cdot 10^{-26}$	0.91093	9.793
$1/5 \cdot 2^{-8}$	$2.2 \cdot 10^{-25}$	0.91015	10.039	$1.2 \cdot 10^{-29}$	0.94609	-0.273
$1/5 \cdot 2^{-9}$	$2.1 \cdot 10^{-28}$	0.90585	2.139	$1.5 \cdot 10^{-29}$	0.38750	-2.071
$1/5 \cdot 2^{-10}$	$4.8 \cdot 10^{-29}$	0.89238	-0.688	$6.5 \cdot 10^{-29}$	0.90605	-0.201
$1/5 \cdot 2^{-11}$	$7.7 \cdot 10^{-29}$	0.00087	-1.102	$7.5 \cdot 10^{-29}$	0.05517	-1.273
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-28}$	0.93901		$1.8 \cdot 10^{-28}$	0.00009	

Table 5.51: Trapezoidal rule, IDeC method on $[0, 1]$ for 4.1c.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	0.0		$7.6 \cdot 10^{-31}$	3.369
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-70}$	-127.27	$7.3 \cdot 10^{-32}$	-2.807
$1/5 \cdot 2^{-3}$	$2.4 \cdot 10^{-32}$	-1.000	$5.1 \cdot 10^{-31}$	-1.000
$1/5 \cdot 2^{-4}$	$4.9 \cdot 10^{-32}$	0.0000	$1.0 \cdot 10^{-30}$	-1.321
$1/5 \cdot 2^{-5}$	$4.9 \cdot 10^{-32}$	-0.584	$2.5 \cdot 10^{-30}$	-0.907
$1/5 \cdot 2^{-6}$	$7.3 \cdot 10^{-32}$	-1.000	$4.8 \cdot 10^{-30}$	-1.637
$1/5 \cdot 2^{-7}$	$1.4 \cdot 10^{-31}$	1.584	$1.5 \cdot 10^{-29}$	-0.780
$1/5 \cdot 2^{-8}$	$4.9 \cdot 10^{-32}$	-4.857	$2.5 \cdot 10^{-29}$	-0.893
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-30}$	-0.271	$4.8 \cdot 10^{-29}$	-1.157
$1/5 \cdot 2^{-10}$	$1.7 \cdot 10^{-30}$	-1.280	$1.0 \cdot 10^{-28}$	-0.819
$1/5 \cdot 2^{-11}$	$4.1 \cdot 10^{-30}$	-0.948	$1.8 \cdot 10^{-28}$	-1.158
$1/5 \cdot 2^{-12}$	$8.0 \cdot 10^{-30}$	-1.064	$4.2 \cdot 10^{-28}$	-0.986
$1/5 \cdot 2^{-13}$	$1.6 \cdot 10^{-29}$	-0.737	$8.3 \cdot 10^{-28}$	-0.954
$1/5 \cdot 2^{-14}$	$2.8 \cdot 10^{-29}$		$1.6 \cdot 10^{-27}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$7.6 \cdot 10^{-31}$	-0.472	$4.9 \cdot 10^{-31}$	-1.000
$1/5 \cdot 2^{-2}$	$1.0 \cdot 10^{-30}$	0.472	$9.8 \cdot 10^{-31}$	0.192
$1/5 \cdot 2^{-3}$	$7.6 \cdot 10^{-31}$	1.369	$8.6 \cdot 10^{-31}$	0.605
$1/5 \cdot 2^{-4}$	$2.9 \cdot 10^{-31}$	-2.969	$5.6 \cdot 10^{-31}$	-1.309
$1/5 \cdot 2^{-5}$	$2.3 \cdot 10^{-30}$	0.015	$1.4 \cdot 10^{-30}$	1.745
$1/5 \cdot 2^{-6}$	$2.2 \cdot 10^{-30}$	0.181	$4.1 \cdot 10^{-31}$	-4.497
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-30}$	0.072	$9.4 \cdot 10^{-30}$	0.272
$1/5 \cdot 2^{-8}$	$1.9 \cdot 10^{-30}$	-2.425	$7.8 \cdot 10^{-30}$	-1.946
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-29}$	0.555	$3.0 \cdot 10^{-29}$	-0.530
$1/5 \cdot 2^{-10}$	$7.0 \cdot 10^{-30}$	-1.633	$4.3 \cdot 10^{-29}$	-1.102
$1/5 \cdot 2^{-11}$	$2.1 \cdot 10^{-29}$	-0.114	$9.3 \cdot 10^{-29}$	-1.101
$1/5 \cdot 2^{-12}$	$2.3 \cdot 10^{-29}$	-0.593	$2.0 \cdot 10^{-28}$	-0.845
$1/5 \cdot 2^{-13}$	$3.5 \cdot 10^{-29}$		$3.6 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$2.4 \cdot 10^{-32}$	-3.807	$6.4 \cdot 10^{-31}$	0.612
$1/5 \cdot 2^{-2}$	$3.4 \cdot 10^{-31}$	-0.192	$4.1 \cdot 10^{-31}$	4.087
$1/5 \cdot 2^{-3}$	$3.9 \cdot 10^{-31}$	-1.700	$2.4 \cdot 10^{-32}$	-6.845
$1/5 \cdot 2^{-4}$	$1.2 \cdot 10^{-30}$	-0.884	$2.8 \cdot 10^{-30}$	0.419
$1/5 \cdot 2^{-5}$	$2.3 \cdot 10^{-30}$	-0.513	$2.1 \cdot 10^{-30}$	0.725
$1/5 \cdot 2^{-6}$	$3.3 \cdot 10^{-30}$	1.705	$1.2 \cdot 10^{-30}$	-2.253
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-30}$	-2.177	$6.1 \cdot 10^{-30}$	-0.817
$1/5 \cdot 2^{-8}$	$4.6 \cdot 10^{-30}$	-2.378	$1.0 \cdot 10^{-29}$	0.060
$1/5 \cdot 2^{-9}$	$2.4 \cdot 10^{-29}$	-0.413	$1.0 \cdot 10^{-29}$	-2.394
$1/5 \cdot 2^{-10}$	$3.2 \cdot 10^{-29}$	-1.252	$5.4 \cdot 10^{-29}$	-0.415
$1/5 \cdot 2^{-11}$	$7.7 \cdot 10^{-29}$	-0.637	$7.2 \cdot 10^{-29}$	-1.331
$1/5 \cdot 2^{-12}$	$1.2 \cdot 10^{-28}$		$1.8 \cdot 10^{-28}$	

Table 5.52: Trapezoidal rule, IDeC method at $t = 0$ for 4.1c.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.3 \cdot 10^{-02}$	-0.0000	1.999	$1.6 \cdot 10^{-04}$	0.10000	3.378
$1/5 \cdot 2^{-2}$	$1.0 \cdot 10^{-02}$	0.00000	1.999	$1.5 \cdot 10^{-05}$	0.05000	3.351
$1/5 \cdot 2^{-3}$	$2.7 \cdot 10^{-03}$	0.00000	1.999	$1.5 \cdot 10^{-06}$	0.02500	3.257
$1/5 \cdot 2^{-4}$	$6.8 \cdot 10^{-04}$	-0.0000	1.999	$1.5 \cdot 10^{-07}$	0.01250	3.161
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-04}$	-0.0000	1.999	$1.7 \cdot 10^{-08}$	0.00625	3.092
$1/5 \cdot 2^{-6}$	$4.2 \cdot 10^{-05}$	0.00000	1.999	$2.0 \cdot 10^{-09}$	0.00312	3.049
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-05}$	0.00000	1.999	$2.5 \cdot 10^{-10}$	0.00156	3.025
$1/5 \cdot 2^{-8}$	$2.6 \cdot 10^{-06}$	-0.0000	1.999	$3.0 \cdot 10^{-11}$	0.00078	3.013
$1/5 \cdot 2^{-9}$	$6.6 \cdot 10^{-07}$	-0.0000	1.999	$3.8 \cdot 10^{-12}$	0.00039	3.006
$1/5 \cdot 2^{-10}$	$1.6 \cdot 10^{-07}$	0.00000	1.999	$4.7 \cdot 10^{-13}$	0.00019	3.003
$1/5 \cdot 2^{-11}$	$4.1 \cdot 10^{-08}$	0.00000	1.999	$5.9 \cdot 10^{-14}$	0.00009	3.001
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-08}$	-0.0000	2.000	$7.4 \cdot 10^{-15}$	0.00004	3.001
$1/5 \cdot 2^{-13}$	$2.6 \cdot 10^{-09}$	-0.0000	1.999	$9.2 \cdot 10^{-16}$	0.00002	3.000
$1/5 \cdot 2^{-14}$	$6.5 \cdot 10^{-10}$	0.00000		$1.1 \cdot 10^{-16}$	0.00001	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$4.6 \cdot 10^{-06}$	0.80000	3.072	$1.0 \cdot 10^{-06}$	0.90000	2.981
$1/5 \cdot 2^{-2}$	$5.5 \cdot 10^{-07}$	0.40000	3.014	$1.3 \cdot 10^{-07}$	0.45000	2.995
$1/5 \cdot 2^{-3}$	$6.8 \cdot 10^{-08}$	0.20000	3.001	$1.7 \cdot 10^{-08}$	0.22500	2.998
$1/5 \cdot 2^{-4}$	$8.5 \cdot 10^{-09}$	0.10000	3.000	$2.1 \cdot 10^{-09}$	0.11250	2.999
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-09}$	0.05000	3.000	$2.7 \cdot 10^{-10}$	0.05625	2.999
$1/5 \cdot 2^{-6}$	$1.3 \cdot 10^{-10}$	0.02500	3.000	$3.3 \cdot 10^{-11}$	0.02812	2.999
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-11}$	0.01250	3.000	$4.2 \cdot 10^{-12}$	0.01406	2.999
$1/5 \cdot 2^{-8}$	$2.0 \cdot 10^{-12}$	0.00625	3.000	$5.2 \cdot 10^{-13}$	0.00703	2.999
$1/5 \cdot 2^{-9}$	$2.6 \cdot 10^{-13}$	0.00312	3.000	$6.6 \cdot 10^{-14}$	0.00351	2.999
$1/5 \cdot 2^{-10}$	$3.2 \cdot 10^{-14}$	0.00156	3.000	$8.2 \cdot 10^{-15}$	0.00175	2.999
$1/5 \cdot 2^{-11}$	$4.0 \cdot 10^{-15}$	0.00078	3.000	$1.0 \cdot 10^{-15}$	0.00087	2.999
$1/5 \cdot 2^{-12}$	$5.0 \cdot 10^{-16}$	0.00039	3.000	$1.2 \cdot 10^{-16}$	0.00043	3.000
$1/5 \cdot 2^{-13}$	$6.3 \cdot 10^{-17}$	0.00019		$1.6 \cdot 10^{-17}$	0.00021	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$3.1 \cdot 10^{-07}$	0.10000	2.979	$6.1 \cdot 10^{-08}$	0.10000	2.969
$1/5 \cdot 2^{-2}$	$4.0 \cdot 10^{-08}$	0.05000	2.994	$7.8 \cdot 10^{-09}$	0.05000	2.990
$1/5 \cdot 2^{-3}$	$5.0 \cdot 10^{-09}$	0.02500	2.998	$9.9 \cdot 10^{-10}$	0.02500	2.997
$1/5 \cdot 2^{-4}$	$6.2 \cdot 10^{-10}$	0.01250	2.999	$1.2 \cdot 10^{-10}$	0.01250	2.999
$1/5 \cdot 2^{-5}$	$7.8 \cdot 10^{-11}$	0.00625	2.999	$1.5 \cdot 10^{-11}$	0.00625	2.999
$1/5 \cdot 2^{-6}$	$9.8 \cdot 10^{-12}$	0.00312	2.999	$1.9 \cdot 10^{-12}$	0.00312	2.999
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-12}$	0.00156	2.999	$2.4 \cdot 10^{-13}$	0.00156	2.999
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-13}$	0.00078	2.999	$3.0 \cdot 10^{-14}$	0.00078	2.999
$1/5 \cdot 2^{-9}$	$1.9 \cdot 10^{-14}$	0.00039	2.999	$3.7 \cdot 10^{-15}$	0.00039	2.999
$1/5 \cdot 2^{-10}$	$2.4 \cdot 10^{-15}$	0.00019	2.999	$4.7 \cdot 10^{-16}$	0.00019	2.999
$1/5 \cdot 2^{-11}$	$3.0 \cdot 10^{-16}$	0.00009	2.999	$5.9 \cdot 10^{-17}$	0.00009	2.999
$1/5 \cdot 2^{-12}$	$3.7 \cdot 10^{-17}$	0.00004		$7.4 \cdot 10^{-18}$	0.00004	

Table 5.53: Trapezoidal rule, IDeC method on $[0, 1]$ for 4.1d.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$4.3 \cdot 10^{-02}$	1.999	$1.5 \cdot 10^{-04}$	3.999
$1/5 \cdot 2^{-2}$	$1.0 \cdot 10^{-02}$	1.999	$9.5 \cdot 10^{-06}$	3.999
$1/5 \cdot 2^{-3}$	$2.7 \cdot 10^{-03}$	1.999	$5.9 \cdot 10^{-07}$	3.999
$1/5 \cdot 2^{-4}$	$6.8 \cdot 10^{-04}$	1.999	$3.7 \cdot 10^{-08}$	3.999
$1/5 \cdot 2^{-5}$	$1.7 \cdot 10^{-04}$	1.999	$2.3 \cdot 10^{-09}$	3.999
$1/5 \cdot 2^{-6}$	$4.2 \cdot 10^{-05}$	1.999	$1.4 \cdot 10^{-10}$	3.999
$1/5 \cdot 2^{-7}$	$1.0 \cdot 10^{-05}$	1.999	$9.0 \cdot 10^{-12}$	3.999
$1/5 \cdot 2^{-8}$	$2.6 \cdot 10^{-06}$	1.999	$5.6 \cdot 10^{-13}$	3.999
$1/5 \cdot 2^{-9}$	$6.6 \cdot 10^{-07}$	1.999	$3.5 \cdot 10^{-14}$	3.999
$1/5 \cdot 2^{-10}$	$1.6 \cdot 10^{-07}$	1.999	$2.2 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-11}$	$4.1 \cdot 10^{-08}$	1.999	$1.3 \cdot 10^{-16}$	3.999
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-08}$	2.000	$8.6 \cdot 10^{-18}$	4.000
$1/5 \cdot 2^{-13}$	$2.6 \cdot 10^{-09}$	1.999	$5.4 \cdot 10^{-19}$	3.999
$1/5 \cdot 2^{-14}$	$6.5 \cdot 10^{-10}$		$3.3 \cdot 10^{-20}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$3.4 \cdot 10^{-07}$	5.999	$5.4 \cdot 10^{-11}$	5.029
$1/5 \cdot 2^{-2}$	$5.3 \cdot 10^{-09}$	5.999	$1.6 \cdot 10^{-12}$	7.624
$1/5 \cdot 2^{-3}$	$8.3 \cdot 10^{-11}$	5.999	$8.4 \cdot 10^{-15}$	7.918
$1/5 \cdot 2^{-4}$	$1.3 \cdot 10^{-12}$	5.999	$3.5 \cdot 10^{-17}$	7.980
$1/5 \cdot 2^{-5}$	$2.0 \cdot 10^{-14}$	5.999	$1.3 \cdot 10^{-19}$	7.995
$1/5 \cdot 2^{-6}$	$3.1 \cdot 10^{-16}$	5.999	$5.4 \cdot 10^{-22}$	7.998
$1/5 \cdot 2^{-7}$	$4.9 \cdot 10^{-18}$	5.999	$2.1 \cdot 10^{-24}$	7.999
$1/5 \cdot 2^{-8}$	$7.7 \cdot 10^{-20}$	5.999	$8.3 \cdot 10^{-27}$	8.876
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-21}$	5.999	$1.7 \cdot 10^{-29}$	-3.285
$1/5 \cdot 2^{-10}$	$1.8 \cdot 10^{-23}$	5.999	$1.7 \cdot 10^{-28}$	-0.269
$1/5 \cdot 2^{-11}$	$2.9 \cdot 10^{-25}$	6.069	$2.0 \cdot 10^{-28}$	1.881
$1/5 \cdot 2^{-12}$	$4.4 \cdot 10^{-27}$	6.951	$5.6 \cdot 10^{-29}$	-1.665
$1/5 \cdot 2^{-13}$	$3.5 \cdot 10^{-29}$		$1.7 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$6.1 \cdot 10^{-10}$	9.936	$6.1 \cdot 10^{-10}$	9.936
$1/5 \cdot 2^{-2}$	$6.3 \cdot 10^{-13}$	9.983	$6.3 \cdot 10^{-13}$	9.983
$1/5 \cdot 2^{-3}$	$6.2 \cdot 10^{-16}$	9.995	$6.2 \cdot 10^{-16}$	9.995
$1/5 \cdot 2^{-4}$	$6.0 \cdot 10^{-19}$	9.998	$6.0 \cdot 10^{-19}$	9.998
$1/5 \cdot 2^{-5}$	$5.9 \cdot 10^{-22}$	9.999	$5.9 \cdot 10^{-22}$	9.999
$1/5 \cdot 2^{-6}$	$5.8 \cdot 10^{-25}$	9.959	$5.8 \cdot 10^{-25}$	9.950
$1/5 \cdot 2^{-7}$	$5.8 \cdot 10^{-28}$	5.988	$5.8 \cdot 10^{-28}$	8.824
$1/5 \cdot 2^{-8}$	$9.1 \cdot 10^{-30}$	0.710	$1.2 \cdot 10^{-30}$	-3.219
$1/5 \cdot 2^{-9}$	$5.6 \cdot 10^{-30}$	-4.202	$1.2 \cdot 10^{-29}$	-3.316
$1/5 \cdot 2^{-10}$	$1.0 \cdot 10^{-28}$	-0.723	$1.2 \cdot 10^{-28}$	-0.776
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-28}$	3.039	$2.0 \cdot 10^{-28}$	3.310
$1/5 \cdot 2^{-12}$	$2.0 \cdot 10^{-29}$		$2.0 \cdot 10^{-29}$	

Table 5.54: Trapezoidal rule, IDeC method at $t = 0$ for 4.1d.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.0 \cdot 10^{-02}$	0.70000	2.007	$2.0 \cdot 10^{-04}$	0.70000	4.006
$1/5 \cdot 2^{-2}$	$5.0 \cdot 10^{-03}$	0.70000	2.001	$1.2 \cdot 10^{-05}$	0.65000	3.999
$1/5 \cdot 2^{-3}$	$1.2 \cdot 10^{-03}$	0.67500	1.999	$7.9 \cdot 10^{-07}$	0.67500	4.000
$1/5 \cdot 2^{-4}$	$3.1 \cdot 10^{-04}$	0.68750	2.000	$4.9 \cdot 10^{-08}$	0.67500	3.999
$1/5 \cdot 2^{-5}$	$7.8 \cdot 10^{-05}$	0.68750	2.000	$3.0 \cdot 10^{-09}$	0.66875	4.000
$1/5 \cdot 2^{-6}$	$1.9 \cdot 10^{-05}$	0.68750	1.999	$1.9 \cdot 10^{-10}$	0.66875	4.000
$1/5 \cdot 2^{-7}$	$4.9 \cdot 10^{-06}$	0.68593	2.000	$1.2 \cdot 10^{-11}$	0.67031	4.000
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-06}$	0.68593	2.000	$7.5 \cdot 10^{-13}$	0.66953	3.999
$1/5 \cdot 2^{-9}$	$3.0 \cdot 10^{-07}$	0.68593	1.999	$4.7 \cdot 10^{-14}$	0.66992	4.000
$1/5 \cdot 2^{-10}$	$7.6 \cdot 10^{-08}$	0.68613	2.000	$2.9 \cdot 10^{-15}$	0.66992	4.000
$1/5 \cdot 2^{-11}$	$1.9 \cdot 10^{-08}$	0.68613	2.000	$1.8 \cdot 10^{-16}$	0.66992	4.000
$1/5 \cdot 2^{-12}$	$4.7 \cdot 10^{-09}$	0.68613	2.000	$1.1 \cdot 10^{-17}$	0.66992	4.000
$1/5 \cdot 2^{-13}$	$1.1 \cdot 10^{-09}$	0.68613	1.999	$7.1 \cdot 10^{-19}$	0.66989	3.999
$1/5 \cdot 2^{-14}$	$2.9 \cdot 10^{-10}$	0.68613		$4.4 \cdot 10^{-20}$	0.66990	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.5 \cdot 10^{-06}$	0.60000	6.036	$1.3 \cdot 10^{-08}$	0.70000	5.910
$1/5 \cdot 2^{-2}$	$2.3 \cdot 10^{-08}$	0.65000	5.999	$2.2 \cdot 10^{-10}$	0.35000	5.143
$1/5 \cdot 2^{-3}$	$3.6 \cdot 10^{-10}$	0.62500	5.818	$6.3 \cdot 10^{-12}$	0.17500	5.012
$1/5 \cdot 2^{-4}$	$6.4 \cdot 10^{-12}$	0.01250	5.004	$1.9 \cdot 10^{-13}$	0.08750	5.001
$1/5 \cdot 2^{-5}$	$2.0 \cdot 10^{-13}$	0.00625	5.000	$6.1 \cdot 10^{-15}$	0.04375	5.000
$1/5 \cdot 2^{-6}$	$6.2 \cdot 10^{-15}$	0.00312	4.999	$1.9 \cdot 10^{-16}$	0.02187	5.000
$1/5 \cdot 2^{-7}$	$1.9 \cdot 10^{-16}$	0.00156	4.999	$6.0 \cdot 10^{-18}$	0.01093	5.000
$1/5 \cdot 2^{-8}$	$6.1 \cdot 10^{-18}$	0.00078	4.999	$1.8 \cdot 10^{-19}$	0.00546	5.000
$1/5 \cdot 2^{-9}$	$1.9 \cdot 10^{-19}$	0.00039	4.999	$5.8 \cdot 10^{-21}$	0.00273	5.000
$1/5 \cdot 2^{-10}$	$5.9 \cdot 10^{-21}$	0.00019	4.999	$1.8 \cdot 10^{-22}$	0.00136	5.000
$1/5 \cdot 2^{-11}$	$1.8 \cdot 10^{-22}$	0.00009	4.999	$5.7 \cdot 10^{-24}$	0.00068	5.000
$1/5 \cdot 2^{-12}$	$5.8 \cdot 10^{-24}$	0.00004	5.000	$1.7 \cdot 10^{-25}$	0.00034	5.000
$1/5 \cdot 2^{-13}$	$1.8 \cdot 10^{-25}$	0.00002		$5.6 \cdot 10^{-27}$	0.00017	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.6 \cdot 10^{-09}$	0.10000	5.029	$5.2 \cdot 10^{-10}$	0.10000	5.069
$1/5 \cdot 2^{-2}$	$5.0 \cdot 10^{-11}$	0.05000	5.002	$1.5 \cdot 10^{-11}$	0.05000	4.996
$1/5 \cdot 2^{-3}$	$1.5 \cdot 10^{-12}$	0.02500	5.000	$4.8 \cdot 10^{-13}$	0.02500	4.998
$1/5 \cdot 2^{-4}$	$4.9 \cdot 10^{-14}$	0.01250	5.000	$1.5 \cdot 10^{-14}$	0.01250	4.999
$1/5 \cdot 2^{-5}$	$1.5 \cdot 10^{-15}$	0.00625	5.000	$4.7 \cdot 10^{-16}$	0.00625	4.999
$1/5 \cdot 2^{-6}$	$4.8 \cdot 10^{-17}$	0.00312	5.000	$1.4 \cdot 10^{-17}$	0.00312	4.999
$1/5 \cdot 2^{-7}$	$1.5 \cdot 10^{-18}$	0.00156	5.000	$4.6 \cdot 10^{-19}$	0.00156	4.999
$1/5 \cdot 2^{-8}$	$4.7 \cdot 10^{-20}$	0.00078	5.000	$1.4 \cdot 10^{-20}$	0.00078	4.999
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-21}$	0.00039	5.000	$4.5 \cdot 10^{-22}$	0.00039	4.999
$1/5 \cdot 2^{-10}$	$4.6 \cdot 10^{-23}$	0.00019	5.000	$1.4 \cdot 10^{-23}$	0.00019	4.999
$1/5 \cdot 2^{-11}$	$1.4 \cdot 10^{-24}$	0.00009	5.000	$4.4 \cdot 10^{-25}$	0.00009	4.999
$1/5 \cdot 2^{-12}$	$4.4 \cdot 10^{-26}$	0.00004		$1.3 \cdot 10^{-26}$	0.00004	

Table 5.55: Trapezoidal rule, IDeC method on $[0, 1]$ for 4.1e.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$5.0 \cdot 10^{-03}$	1.998	$2.9 \cdot 10^{-05}$	3.998
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-03}$	1.999	$1.8 \cdot 10^{-06}$	3.999
$1/5 \cdot 2^{-3}$	$3.1 \cdot 10^{-04}$	1.999	$1.1 \cdot 10^{-07}$	3.999
$1/5 \cdot 2^{-4}$	$7.9 \cdot 10^{-05}$	1.999	$7.3 \cdot 10^{-09}$	3.999
$1/5 \cdot 2^{-5}$	$1.9 \cdot 10^{-05}$	1.999	$4.5 \cdot 10^{-10}$	3.999
$1/5 \cdot 2^{-6}$	$4.9 \cdot 10^{-06}$	1.999	$2.8 \cdot 10^{-11}$	3.999
$1/5 \cdot 2^{-7}$	$1.2 \cdot 10^{-06}$	1.999	$1.7 \cdot 10^{-12}$	3.999
$1/5 \cdot 2^{-8}$	$3.1 \cdot 10^{-07}$	1.999	$1.1 \cdot 10^{-13}$	3.999
$1/5 \cdot 2^{-9}$	$7.7 \cdot 10^{-08}$	1.999	$6.9 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-10}$	$1.9 \cdot 10^{-08}$	1.999	$4.3 \cdot 10^{-16}$	3.999
$1/5 \cdot 2^{-11}$	$4.8 \cdot 10^{-09}$	1.999	$2.7 \cdot 10^{-17}$	3.999
$1/5 \cdot 2^{-12}$	$1.2 \cdot 10^{-09}$	2.000	$1.7 \cdot 10^{-18}$	4.000
$1/5 \cdot 2^{-13}$	$3.0 \cdot 10^{-10}$	1.999	$1.0 \cdot 10^{-19}$	3.999
$1/5 \cdot 2^{-14}$	$7.5 \cdot 10^{-11}$		$6.6 \cdot 10^{-21}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$8.0 \cdot 10^{-08}$	5.999	$2.2 \cdot 10^{-11}$	5.763
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{-09}$	5.999	$4.2 \cdot 10^{-13}$	7.598
$1/5 \cdot 2^{-3}$	$1.9 \cdot 10^{-11}$	5.999	$2.1 \cdot 10^{-15}$	7.913
$1/5 \cdot 2^{-4}$	$3.0 \cdot 10^{-13}$	5.999	$9.0 \cdot 10^{-18}$	7.979
$1/5 \cdot 2^{-5}$	$4.7 \cdot 10^{-15}$	5.999	$3.5 \cdot 10^{-20}$	7.994
$1/5 \cdot 2^{-6}$	$7.4 \cdot 10^{-17}$	5.999	$1.4 \cdot 10^{-22}$	7.998
$1/5 \cdot 2^{-7}$	$1.1 \cdot 10^{-18}$	5.999	$5.4 \cdot 10^{-25}$	7.996
$1/5 \cdot 2^{-8}$	$1.8 \cdot 10^{-20}$	5.999	$2.1 \cdot 10^{-27}$	7.990
$1/5 \cdot 2^{-9}$	$2.8 \cdot 10^{-22}$	5.999	$8.4 \cdot 10^{-30}$	-1.433
$1/5 \cdot 2^{-10}$	$4.4 \cdot 10^{-24}$	5.999	$2.2 \cdot 10^{-29}$	-1.150
$1/5 \cdot 2^{-11}$	$6.9 \cdot 10^{-26}$	5.999	$5.0 \cdot 10^{-29}$	-1.670
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-27}$	3.560	$1.6 \cdot 10^{-28}$	-0.786
$1/5 \cdot 2^{-13}$	$9.2 \cdot 10^{-29}$		$2.7 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.6 \cdot 10^{-10}$	9.934	$1.6 \cdot 10^{-10}$	9.934
$1/5 \cdot 2^{-2}$	$1.7 \cdot 10^{-13}$	9.983	$1.7 \cdot 10^{-13}$	9.983
$1/5 \cdot 2^{-3}$	$1.6 \cdot 10^{-16}$	9.995	$1.6 \cdot 10^{-16}$	9.995
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-19}$	9.998	$1.6 \cdot 10^{-19}$	9.998
$1/5 \cdot 2^{-5}$	$1.6 \cdot 10^{-22}$	9.999	$1.6 \cdot 10^{-22}$	9.999
$1/5 \cdot 2^{-6}$	$1.5 \cdot 10^{-25}$	9.951	$1.5 \cdot 10^{-25}$	9.942
$1/5 \cdot 2^{-7}$	$1.5 \cdot 10^{-28}$	2.849	$1.6 \cdot 10^{-28}$	4.408
$1/5 \cdot 2^{-8}$	$2.2 \cdot 10^{-29}$	0.030	$7.5 \cdot 10^{-30}$	-0.787
$1/5 \cdot 2^{-9}$	$2.1 \cdot 10^{-29}$	-1.512	$1.3 \cdot 10^{-29}$	-1.500
$1/5 \cdot 2^{-10}$	$6.1 \cdot 10^{-29}$	-0.830	$3.6 \cdot 10^{-29}$	-0.592
$1/5 \cdot 2^{-11}$	$1.0 \cdot 10^{-28}$	-0.438	$5.5 \cdot 10^{-29}$	-1.431
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-28}$		$1.4 \cdot 10^{-28}$	

Table 5.56: Trapezoidal rule, IDeC method at $t = 0$ for 4.1e.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.8 \cdot 10^{-01}$	0.90000	2.114	$4.7 \cdot 10^{-02}$	0.90000	4.314
$1/5 \cdot 2^{-2}$	$6.4 \cdot 10^{-02}$	0.90000	2.026	$2.3 \cdot 10^{-03}$	0.90000	4.072
$1/5 \cdot 2^{-3}$	$1.5 \cdot 10^{-02}$	0.90000	2.005	$1.4 \cdot 10^{-04}$	0.90000	4.017
$1/5 \cdot 2^{-4}$	$3.9 \cdot 10^{-03}$	0.88750	1.999	$8.7 \cdot 10^{-06}$	0.90000	4.001
$1/5 \cdot 2^{-5}$	$9.9 \cdot 10^{-04}$	0.89375	2.000	$5.4 \cdot 10^{-07}$	0.90625	4.001
$1/5 \cdot 2^{-6}$	$2.4 \cdot 10^{-04}$	0.89375	2.000	$3.4 \cdot 10^{-08}$	0.90625	4.000
$1/5 \cdot 2^{-7}$	$6.1 \cdot 10^{-05}$	0.89375	2.000	$2.1 \cdot 10^{-09}$	0.90625	4.000
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-05}$	0.89375	1.999	$1.3 \cdot 10^{-10}$	0.90625	4.000
$1/5 \cdot 2^{-9}$	$3.8 \cdot 10^{-06}$	0.89335	2.000	$8.3 \cdot 10^{-12}$	0.90585	4.000
$1/5 \cdot 2^{-10}$	$9.6 \cdot 10^{-07}$	0.89335	2.000	$5.2 \cdot 10^{-13}$	0.90605	4.000
$1/5 \cdot 2^{-11}$	$2.4 \cdot 10^{-07}$	0.89335	2.000	$3.2 \cdot 10^{-14}$	0.90605	4.000
$1/5 \cdot 2^{-12}$	$6.0 \cdot 10^{-08}$	0.89340	2.000	$2.0 \cdot 10^{-15}$	0.90600	4.000
$1/5 \cdot 2^{-13}$	$1.5 \cdot 10^{-08}$	0.89338	1.999	$1.2 \cdot 10^{-16}$	0.90600	3.999
$1/5 \cdot 2^{-14}$	$3.7 \cdot 10^{-09}$	0.89339		$7.9 \cdot 10^{-18}$	0.90601	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$1.0 \cdot 10^{-02}$	0.90000	6.526	$2.5 \cdot 10^{-03}$	0.90000	8.735
$1/5 \cdot 2^{-2}$	$1.1 \cdot 10^{-04}$	0.90000	6.120	$5.9 \cdot 10^{-06}$	0.90000	8.169
$1/5 \cdot 2^{-3}$	$1.6 \cdot 10^{-06}$	0.90000	6.022	$2.0 \cdot 10^{-08}$	0.90000	8.032
$1/5 \cdot 2^{-4}$	$2.5 \cdot 10^{-08}$	0.91250	6.007	$7.8 \cdot 10^{-11}$	0.91250	8.010
$1/5 \cdot 2^{-5}$	$3.9 \cdot 10^{-10}$	0.90625	6.001	$3.0 \cdot 10^{-13}$	0.91250	8.002
$1/5 \cdot 2^{-6}$	$6.1 \cdot 10^{-12}$	0.90937	6.000	$1.1 \cdot 10^{-15}$	0.90937	8.000
$1/5 \cdot 2^{-7}$	$9.6 \cdot 10^{-14}$	0.90937	6.000	$4.6 \cdot 10^{-18}$	0.90937	8.000
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-15}$	0.90937	6.000	$1.8 \cdot 10^{-20}$	0.91015	8.000
$1/5 \cdot 2^{-9}$	$2.3 \cdot 10^{-17}$	0.90898	6.000	$7.1 \cdot 10^{-23}$	0.91015	8.000
$1/5 \cdot 2^{-10}$	$3.6 \cdot 10^{-19}$	0.90917	6.000	$2.7 \cdot 10^{-25}$	0.91015	8.139
$1/5 \cdot 2^{-11}$	$5.7 \cdot 10^{-21}$	0.90908	6.000	$9.8 \cdot 10^{-28}$	0.91503	2.125
$1/5 \cdot 2^{-12}$	$8.9 \cdot 10^{-23}$	0.90913	6.000	$2.2 \cdot 10^{-28}$	0.89218	-0.400
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-24}$	0.90917		$2.9 \cdot 10^{-28}$	0.00957	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$6.2 \cdot 10^{-04}$	0.90000	10.941	$1.5 \cdot 10^{-04}$	0.90000	13.146
$1/5 \cdot 2^{-2}$	$3.1 \cdot 10^{-07}$	0.90000	10.216	$1.6 \cdot 10^{-08}$	0.90000	12.264
$1/5 \cdot 2^{-3}$	$2.6 \cdot 10^{-10}$	0.90000	10.042	$3.4 \cdot 10^{-12}$	0.90000	6.983
$1/5 \cdot 2^{-4}$	$2.5 \cdot 10^{-13}$	0.91250	2.372	$2.7 \cdot 10^{-14}$	-0.0000	-0.836
$1/5 \cdot 2^{-5}$	$4.8 \cdot 10^{-14}$	-0.0000	17.643	$4.8 \cdot 10^{-14}$	-0.0000	29.927
$1/5 \cdot 2^{-6}$	$2.3 \cdot 10^{-19}$	0.90937	10.000	$4.7 \cdot 10^{-23}$	0.90937	12.007
$1/5 \cdot 2^{-7}$	$2.3 \cdot 10^{-22}$	0.91093	10.000	$1.1 \cdot 10^{-26}$	0.91093	8.944
$1/5 \cdot 2^{-8}$	$2.2 \cdot 10^{-25}$	0.91015	9.997	$2.3 \cdot 10^{-29}$	0.94453	-0.431
$1/5 \cdot 2^{-9}$	$2.2 \cdot 10^{-28}$	0.91445	1.166	$3.1 \cdot 10^{-29}$	0.94101	-1.621
$1/5 \cdot 2^{-10}$	$9.8 \cdot 10^{-29}$	0.92167	0.312	$9.7 \cdot 10^{-29}$	0.88847	-0.085
$1/5 \cdot 2^{-11}$	$7.9 \cdot 10^{-29}$	0.92490	-1.472	$1.0 \cdot 10^{-28}$	0.89511	-1.218
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-28}$	0.87539		$2.4 \cdot 10^{-28}$	0.92138	

Table 5.57: Trapezoidal rule, IDeC method on $[0, 1]$ for 4.1f.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$8.4 \cdot 10^{-03}$	1.999	$1.2 \cdot 10^{-05}$	3.999
$1/5 \cdot 2^{-2}$	$2.1 \cdot 10^{-03}$	1.999	$8.0 \cdot 10^{-07}$	3.999
$1/5 \cdot 2^{-3}$	$5.2 \cdot 10^{-04}$	1.999	$5.0 \cdot 10^{-08}$	3.999
$1/5 \cdot 2^{-4}$	$1.3 \cdot 10^{-04}$	1.999	$3.1 \cdot 10^{-09}$	3.999
$1/5 \cdot 2^{-5}$	$3.2 \cdot 10^{-05}$	1.999	$1.9 \cdot 10^{-10}$	3.999
$1/5 \cdot 2^{-6}$	$8.2 \cdot 10^{-06}$	1.999	$1.2 \cdot 10^{-11}$	3.999
$1/5 \cdot 2^{-7}$	$2.0 \cdot 10^{-06}$	1.999	$7.7 \cdot 10^{-13}$	3.999
$1/5 \cdot 2^{-8}$	$5.1 \cdot 10^{-07}$	1.999	$4.8 \cdot 10^{-14}$	3.999
$1/5 \cdot 2^{-9}$	$1.2 \cdot 10^{-07}$	1.999	$3.0 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-10}$	$3.2 \cdot 10^{-08}$	1.999	$1.8 \cdot 10^{-16}$	3.999
$1/5 \cdot 2^{-11}$	$8.0 \cdot 10^{-09}$	1.999	$1.1 \cdot 10^{-17}$	3.999
$1/5 \cdot 2^{-12}$	$2.0 \cdot 10^{-09}$	2.000	$7.3 \cdot 10^{-19}$	4.000
$1/5 \cdot 2^{-13}$	$5.0 \cdot 10^{-10}$	1.999	$4.5 \cdot 10^{-20}$	3.999
$1/5 \cdot 2^{-14}$	$1.2 \cdot 10^{-10}$		$2.8 \cdot 10^{-21}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.1 \cdot 10^{-08}$	5.999	$1.1 \cdot 10^{-11}$	9.384
$1/5 \cdot 2^{-2}$	$1.7 \cdot 10^{-10}$	5.999	$1.7 \cdot 10^{-14}$	8.851
$1/5 \cdot 2^{-3}$	$2.7 \cdot 10^{-12}$	5.999	$3.8 \cdot 10^{-17}$	8.332
$1/5 \cdot 2^{-4}$	$4.2 \cdot 10^{-14}$	5.999	$1.1 \cdot 10^{-19}$	8.097
$1/5 \cdot 2^{-5}$	$6.6 \cdot 10^{-16}$	5.999	$4.3 \cdot 10^{-22}$	8.025
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{-17}$	5.999	$1.6 \cdot 10^{-24}$	8.005
$1/5 \cdot 2^{-7}$	$1.6 \cdot 10^{-19}$	5.999	$6.4 \cdot 10^{-27}$	7.715
$1/5 \cdot 2^{-8}$	$2.5 \cdot 10^{-21}$	6.000	$3.0 \cdot 10^{-29}$	0.641
$1/5 \cdot 2^{-9}$	$3.9 \cdot 10^{-23}$	5.999	$1.9 \cdot 10^{-29}$	1.774
$1/5 \cdot 2^{-10}$	$6.1 \cdot 10^{-25}$	5.997	$5.7 \cdot 10^{-30}$	-3.454
$1/5 \cdot 2^{-11}$	$9.6 \cdot 10^{-27}$	7.563	$6.2 \cdot 10^{-29}$	-1.732
$1/5 \cdot 2^{-12}$	$5.1 \cdot 10^{-29}$	-1.809	$2.0 \cdot 10^{-28}$	-0.501
$1/5 \cdot 2^{-13}$	$1.7 \cdot 10^{-28}$		$2.9 \cdot 10^{-28}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$9.3 \cdot 10^{-12}$	9.882	$9.3 \cdot 10^{-12}$	9.882
$1/5 \cdot 2^{-2}$	$9.9 \cdot 10^{-15}$	9.970	$9.9 \cdot 10^{-15}$	9.970
$1/5 \cdot 2^{-3}$	$9.8 \cdot 10^{-18}$	9.992	$9.8 \cdot 10^{-18}$	9.992
$1/5 \cdot 2^{-4}$	$9.7 \cdot 10^{-21}$	9.998	$9.7 \cdot 10^{-21}$	9.998
$1/5 \cdot 2^{-5}$	$9.4 \cdot 10^{-24}$	9.999	$9.4 \cdot 10^{-24}$	9.999
$1/5 \cdot 2^{-6}$	$9.2 \cdot 10^{-27}$	10.245	$9.2 \cdot 10^{-27}$	9.966
$1/5 \cdot 2^{-7}$	$7.6 \cdot 10^{-30}$	-1.029	$9.2 \cdot 10^{-30}$	-0.791
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-29}$	0.032	$1.6 \cdot 10^{-29}$	-0.179
$1/5 \cdot 2^{-9}$	$1.5 \cdot 10^{-29}$	0.149	$1.8 \cdot 10^{-29}$	0.686
$1/5 \cdot 2^{-10}$	$1.3 \cdot 10^{-29}$	-1.463	$1.1 \cdot 10^{-29}$	-1.290
$1/5 \cdot 2^{-11}$	$3.7 \cdot 10^{-29}$	-2.176	$2.7 \cdot 10^{-29}$	-2.687
$1/5 \cdot 2^{-12}$	$1.7 \cdot 10^{-28}$		$1.7 \cdot 10^{-28}$	

Table 5.58: Trapezoidal rule, IDeC method at $t = 0$ for 4.1f.

Finally, we discuss possible modifications of implicit schemes for (5.1) that avoid
the evaluation of the right-hand side at $t \approx$ EPS. The heuristic of evaluating at
a point close to the singularity seems to perform reasonably well, nonetheless we
look for a more elegant and intuitively more correct way to implement implicit
methods. We restrict our attention to the implicit Euler method and only give
two examples illustrating our observations.

A natural modification of the scheme is to freeze the Jacobian in the last step.
This means that for the computation of the numerical solution in the last step,
we do not use the exact system matrix (or Jacobian in the Newton iteration
for nonlinear problems), but a perturbed matrix which is the same matrix that
was used in the previous step. It is clear that using a perturbed matrix for
the linear or nonlinear system of equations should not introduce a big error.
Indeed, the asymptotic behavior of the solution is the same as in our original
version of the implicit Euler method for the tested problems. This is observed
both for the maximal error on $[0, 1]$ and the error at $t = 0$. To illustrate this
observation, we give the results for Example 4.1e in Table 5.63 and Table 5.64.
The order sequences coincide with those observed in Tables 5.39 and 5.40.

The second modification of the implicit Euler method we examined was to use
one step of the explicit Euler scheme in the last step instead of the implicit
scheme. This did not improve the quality of the scheme, however. On the
contrary, the order sequences we observed was either the same as for the original
scheme or the modification even caused a reduction in the maximal attainable
convergence order. We give the results for Example 4.1f, which demonstrate
such an order reduction, see Tables 5.65 and 5.66. The results prove the method
to be less robust as compared to the results given in Tables 5.41 and 5.42.

We conclude that the best alternative of implementing implicit methods for
problems (5.1) is to use an evaluation of the right-hand side of (5.1a) at $t \approx$ EPS
instead of the evaluation at $t = 0$. Using a frozen Jacobian or an explicit method
to compute the last step does not yield an improvement. However, the resulting
procedures are still faced with the same order reductions (as compared to the
classical order sequences observed for regular problems) as explicit methods and
cannot be used reliably.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	1.6	0.50000	1.888	$3.1 \cdot 10^{-01}$	0.30000	3.420
$1/5 \cdot 2^{-2}$	$4.4 \cdot 10^{-01}$	0.50000	1.979	$2.9 \cdot 10^{-02}$	0.55000	3.888
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-01}$	0.50000	1.995	$1.9 \cdot 10^{-03}$	0.57500	3.980
$1/5 \cdot 2^{-4}$	$2.8 \cdot 10^{-02}$	0.50000	1.998	$1.2 \cdot 10^{-04}$	0.57500	3.995
$1/5 \cdot 2^{-5}$	$7.0 \cdot 10^{-03}$	0.50000	1.999	$7.8 \cdot 10^{-06}$	0.57500	3.998
$1/5 \cdot 2^{-6}$	$1.7 \cdot 10^{-03}$	0.50312	1.999	$4.9 \cdot 10^{-07}$	0.57500	3.999
$1/5 \cdot 2^{-7}$	$4.4 \cdot 10^{-04}$	0.50156	1.999	$3.0 \cdot 10^{-08}$	0.57656	3.999
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-04}$	0.50234	1.999	$1.9 \cdot 10^{-09}$	0.57656	3.999
$1/5 \cdot 2^{-9}$	$2.7 \cdot 10^{-05}$	0.50195	1.999	$1.2 \cdot 10^{-10}$	0.57656	3.999
$1/5 \cdot 2^{-10}$	$6.9 \cdot 10^{-06}$	0.50195	1.999	$7.5 \cdot 10^{-12}$	0.57656	3.999
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-06}$	0.50205	1.999	$4.7 \cdot 10^{-13}$	0.57656	3.999
$1/5 \cdot 2^{-12}$	$4.3 \cdot 10^{-07}$	0.50200	2.000	$2.9 \cdot 10^{-14}$	0.57656	4.000
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-07}$	0.50200	1.999	$1.8 \cdot 10^{-15}$	0.57656	3.999
$1/5 \cdot 2^{-14}$	$2.6 \cdot 10^{-08}$	0.50201		$1.1 \cdot 10^{-16}$	0.57655	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$9.1 \cdot 10^{-02}$	0.90000	5.311	$5.8 \cdot 10^{-02}$	0.90000	7.793
$1/5 \cdot 2^{-2}$	$2.3 \cdot 10^{-03}$	0.35000	5.775	$2.6 \cdot 10^{-04}$	0.45000	8.018
$1/5 \cdot 2^{-3}$	$4.2 \cdot 10^{-05}$	0.37500	5.956	$1.0 \cdot 10^{-06}$	0.42500	6.226
$1/5 \cdot 2^{-4}$	$6.7 \cdot 10^{-07}$	0.38750	5.990	$1.3 \cdot 10^{-08}$	0.11250	4.242
$1/5 \cdot 2^{-5}$	$1.0 \cdot 10^{-08}$	0.38750	4.775	$7.1 \cdot 10^{-10}$	0.05625	4.063
$1/5 \cdot 2^{-6}$	$3.8 \cdot 10^{-10}$	0.00312	4.018	$4.2 \cdot 10^{-11}$	0.02812	4.012
$1/5 \cdot 2^{-7}$	$2.4 \cdot 10^{-11}$	0.00156	4.000	$2.6 \cdot 10^{-12}$	0.01406	4.002
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-12}$	0.00078	3.999	$1.6 \cdot 10^{-13}$	0.00703	4.000
$1/5 \cdot 2^{-9}$	$9.3 \cdot 10^{-14}$	0.00039	3.999	$1.0 \cdot 10^{-14}$	0.00351	4.000
$1/5 \cdot 2^{-10}$	$5.8 \cdot 10^{-15}$	0.00019	3.999	$6.4 \cdot 10^{-16}$	0.00175	4.000
$1/5 \cdot 2^{-11}$	$3.6 \cdot 10^{-16}$	0.00009	3.999	$4.0 \cdot 10^{-17}$	0.00087	4.000
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-17}$	0.00004	4.000	$2.5 \cdot 10^{-18}$	0.00043	4.000
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-18}$	0.00002		$1.5 \cdot 10^{-19}$	0.00021	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$6.2 \cdot 10^{-02}$	0.90000	9.337	$6.8 \cdot 10^{-02}$	0.90000	9.884
$1/5 \cdot 2^{-2}$	$9.6 \cdot 10^{-05}$	0.45000	10.414	$7.2 \cdot 10^{-05}$	0.45000	10.279
$1/5 \cdot 2^{-3}$	$7.0 \cdot 10^{-08}$	0.02500	4.115	$5.7 \cdot 10^{-08}$	0.97500	5.744
$1/5 \cdot 2^{-4}$	$4.0 \cdot 10^{-09}$	0.01250	4.012	$1.0 \cdot 10^{-09}$	0.01250	4.024
$1/5 \cdot 2^{-5}$	$2.5 \cdot 10^{-10}$	0.00625	4.000	$6.6 \cdot 10^{-11}$	0.00625	4.002
$1/5 \cdot 2^{-6}$	$1.5 \cdot 10^{-11}$	0.00312	3.999	$4.1 \cdot 10^{-12}$	0.00312	3.999
$1/5 \cdot 2^{-7}$	$9.8 \cdot 10^{-13}$	0.00156	3.999	$2.5 \cdot 10^{-13}$	0.00156	3.999
$1/5 \cdot 2^{-8}$	$6.1 \cdot 10^{-14}$	0.00078	3.999	$1.6 \cdot 10^{-14}$	0.00078	3.999
$1/5 \cdot 2^{-9}$	$3.8 \cdot 10^{-15}$	0.00039	3.999	$1.0 \cdot 10^{-15}$	0.00039	3.999
$1/5 \cdot 2^{-10}$	$2.4 \cdot 10^{-16}$	0.00019	3.999	$6.3 \cdot 10^{-17}$	0.00019	3.999
$1/5 \cdot 2^{-11}$	$1.5 \cdot 10^{-17}$	0.00009	3.999	$3.9 \cdot 10^{-18}$	0.00009	3.999
$1/5 \cdot 2^{-12}$	$9.3 \cdot 10^{-19}$	0.00004		$2.4 \cdot 10^{-19}$	0.00004	

Table 5.59: Trapezoidal rule, IDeC method on $[0, 1]$ for 4.2a.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$2.7 \cdot 10^{-01}$	2.071	$4.2 \cdot 10^{-02}$	10.923
$1/5 \cdot 2^{-2}$	$6.4 \cdot 10^{-02}$	2.029	$2.1 \cdot 10^{-05}$	-0.454
$1/5 \cdot 2^{-3}$	$1.5 \cdot 10^{-02}$	2.008	$2.9 \cdot 10^{-05}$	3.772
$1/5 \cdot 2^{-4}$	$3.9 \cdot 10^{-03}$	2.002	$2.1 \cdot 10^{-06}$	3.956
$1/5 \cdot 2^{-5}$	$9.8 \cdot 10^{-04}$	2.000	$1.4 \cdot 10^{-07}$	3.990
$1/5 \cdot 2^{-6}$	$2.4 \cdot 10^{-04}$	2.000	$8.8 \cdot 10^{-09}$	3.997
$1/5 \cdot 2^{-7}$	$6.1 \cdot 10^{-05}$	2.000	$5.5 \cdot 10^{-10}$	3.999
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-05}$	2.000	$3.4 \cdot 10^{-11}$	3.999
$1/5 \cdot 2^{-9}$	$3.8 \cdot 10^{-06}$	2.000	$2.1 \cdot 10^{-12}$	3.999
$1/5 \cdot 2^{-10}$	$9.5 \cdot 10^{-07}$	2.000	$1.3 \cdot 10^{-13}$	3.999
$1/5 \cdot 2^{-11}$	$2.3 \cdot 10^{-07}$	2.000	$8.4 \cdot 10^{-15}$	3.999
$1/5 \cdot 2^{-12}$	$5.9 \cdot 10^{-08}$	2.000	$5.2 \cdot 10^{-16}$	4.000
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-08}$	1.999	$3.2 \cdot 10^{-17}$	3.999
$1/5 \cdot 2^{-14}$	$3.7 \cdot 10^{-09}$		$2.0 \cdot 10^{-18}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$3.0 \cdot 10^{-02}$	6.786	$2.0 \cdot 10^{-02}$	10.318
$1/5 \cdot 2^{-2}$	$2.7 \cdot 10^{-04}$	6.168	$1.6 \cdot 10^{-05}$	8.367
$1/5 \cdot 2^{-3}$	$3.7 \cdot 10^{-06}$	6.215	$4.9 \cdot 10^{-08}$	5.387
$1/5 \cdot 2^{-4}$	$5.0 \cdot 10^{-08}$	6.218	$1.1 \cdot 10^{-09}$	5.840
$1/5 \cdot 2^{-5}$	$6.8 \cdot 10^{-10}$	6.248	$2.0 \cdot 10^{-11}$	5.956
$1/5 \cdot 2^{-6}$	$9.0 \cdot 10^{-12}$	6.299	$3.3 \cdot 10^{-13}$	5.986
$1/5 \cdot 2^{-7}$	$1.1 \cdot 10^{-13}$	6.377	$5.2 \cdot 10^{-15}$	5.995
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-15}$	6.513	$8.2 \cdot 10^{-17}$	5.998
$1/5 \cdot 2^{-9}$	$1.5 \cdot 10^{-17}$	6.803	$1.2 \cdot 10^{-18}$	5.999
$1/5 \cdot 2^{-10}$	$1.3 \cdot 10^{-19}$	7.974	$2.0 \cdot 10^{-20}$	5.999
$1/5 \cdot 2^{-11}$	$5.3 \cdot 10^{-22}$	5.050	$3.1 \cdot 10^{-22}$	5.999
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-23}$	4.667	$4.9 \cdot 10^{-24}$	5.998
$1/5 \cdot 2^{-13}$	$6.3 \cdot 10^{-25}$		$7.6 \cdot 10^{-26}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.8 \cdot 10^{-02}$	9.933	$1.8 \cdot 10^{-02}$	9.915
$1/5 \cdot 2^{-2}$	$1.8 \cdot 10^{-05}$	12.140	$1.9 \cdot 10^{-05}$	15.216
$1/5 \cdot 2^{-3}$	$4.1 \cdot 10^{-09}$	6.615	$5.0 \cdot 10^{-10}$	3.681
$1/5 \cdot 2^{-4}$	$4.2 \cdot 10^{-11}$	6.778	$3.9 \cdot 10^{-11}$	5.924
$1/5 \cdot 2^{-5}$	$3.8 \cdot 10^{-13}$	6.445	$6.4 \cdot 10^{-13}$	6.005
$1/5 \cdot 2^{-6}$	$4.3 \cdot 10^{-15}$	6.172	$1.0 \cdot 10^{-14}$	6.003
$1/5 \cdot 2^{-7}$	$6.0 \cdot 10^{-17}$	6.055	$1.5 \cdot 10^{-16}$	6.000
$1/5 \cdot 2^{-8}$	$9.1 \cdot 10^{-19}$	6.016	$2.4 \cdot 10^{-18}$	6.000
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-20}$	6.004	$3.8 \cdot 10^{-20}$	6.000
$1/5 \cdot 2^{-10}$	$2.2 \cdot 10^{-22}$	6.001	$5.9 \cdot 10^{-22}$	6.000
$1/5 \cdot 2^{-11}$	$3.4 \cdot 10^{-24}$	5.999	$9.2 \cdot 10^{-24}$	6.000
$1/5 \cdot 2^{-12}$	$5.3 \cdot 10^{-26}$		$1.4 \cdot 10^{-25}$	

Table 5.60: Trapezoidal rule, IDeC method at $t = 0$ for 4.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.1 \cdot 10^{-03}$	0.20000	1.964	$2.0 \cdot 10^{-05}$	0.20000	3.940
$1/5 \cdot 2^{-2}$	$5.6 \cdot 10^{-04}$	0.25000	1.995	$1.3 \cdot 10^{-06}$	0.25000	3.860
$1/5 \cdot 2^{-3}$	$1.4 \cdot 10^{-04}$	0.22500	1.996	$9.2 \cdot 10^{-08}$	0.22500	3.958
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{-05}$	0.23750	1.999	$5.9 \cdot 10^{-09}$	0.01250	3.635
$1/5 \cdot 2^{-5}$	$8.8 \cdot 10^{-06}$	0.23750	1.999	$4.7 \cdot 10^{-10}$	0.00625	3.808
$1/5 \cdot 2^{-6}$	$2.2 \cdot 10^{-06}$	0.23750	1.999	$3.4 \cdot 10^{-11}$	0.00312	3.894
$1/5 \cdot 2^{-7}$	$5.5 \cdot 10^{-07}$	0.23750	1.999	$2.2 \cdot 10^{-12}$	0.00156	3.936
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-07}$	0.23671	1.999	$1.4 \cdot 10^{-13}$	0.00156	3.943
$1/5 \cdot 2^{-9}$	$3.4 \cdot 10^{-08}$	0.23671	1.999	$9.7 \cdot 10^{-15}$	0.00078	3.965
$1/5 \cdot 2^{-10}$	$8.6 \cdot 10^{-09}$	0.23691	1.999	$6.2 \cdot 10^{-16}$	0.00058	3.975
$1/5 \cdot 2^{-11}$	$2.1 \cdot 10^{-09}$	0.23691	1.999	$3.9 \cdot 10^{-17}$	0.00029	3.986
$1/5 \cdot 2^{-12}$	$5.3 \cdot 10^{-10}$	0.23686	2.000	$2.5 \cdot 10^{-18}$	0.00014	3.987
$1/5 \cdot 2^{-13}$	$1.3 \cdot 10^{-10}$	0.23686	1.999	$1.5 \cdot 10^{-19}$	0.00024	3.988
$1/5 \cdot 2^{-14}$	$3.3 \cdot 10^{-11}$	0.23687		$9.9 \cdot 10^{-21}$	0.00012	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$5.2 \cdot 10^{-07}$	0.80000	3.781	$8.0 \cdot 10^{-08}$	0.10000	3.831
$1/5 \cdot 2^{-2}$	$3.8 \cdot 10^{-08}$	0.40000	3.840	$5.6 \cdot 10^{-09}$	0.45000	3.939
$1/5 \cdot 2^{-3}$	$2.6 \cdot 10^{-09}$	0.20000	3.855	$3.6 \cdot 10^{-10}$	0.22500	3.982
$1/5 \cdot 2^{-4}$	$1.8 \cdot 10^{-10}$	0.01250	3.894	$2.3 \cdot 10^{-11}$	0.11250	3.994
$1/5 \cdot 2^{-5}$	$1.2 \cdot 10^{-11}$	0.00625	3.949	$1.4 \cdot 10^{-12}$	0.05625	3.998
$1/5 \cdot 2^{-6}$	$8.0 \cdot 10^{-13}$	0.00312	3.975	$9.1 \cdot 10^{-14}$	0.02812	3.999
$1/5 \cdot 2^{-7}$	$5.1 \cdot 10^{-14}$	0.00156	3.987	$5.7 \cdot 10^{-15}$	0.01406	3.999
$1/5 \cdot 2^{-8}$	$3.2 \cdot 10^{-15}$	0.00078	3.993	$3.5 \cdot 10^{-16}$	0.00703	3.999
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-16}$	0.00039	3.996	$2.2 \cdot 10^{-17}$	0.00351	3.999
$1/5 \cdot 2^{-10}$	$1.2 \cdot 10^{-17}$	0.00019	3.998	$1.4 \cdot 10^{-18}$	0.00175	3.999
$1/5 \cdot 2^{-11}$	$7.9 \cdot 10^{-19}$	0.00009	3.999	$8.7 \cdot 10^{-20}$	0.00087	3.999
$1/5 \cdot 2^{-12}$	$4.9 \cdot 10^{-20}$	0.00004	3.999	$5.4 \cdot 10^{-21}$	0.00043	4.000
$1/5 \cdot 2^{-13}$	$3.1 \cdot 10^{-21}$	0.00002		$3.4 \cdot 10^{-22}$	0.00021	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5 \cdot 2^{-1}$	$2.8 \cdot 10^{-08}$	0.10000	3.827	$7.0 \cdot 10^{-09}$	0.10000	3.770
$1/5 \cdot 2^{-2}$	$2.0 \cdot 10^{-09}$	0.05000	3.919	$5.1 \cdot 10^{-10}$	0.05000	3.902
$1/5 \cdot 2^{-3}$	$1.3 \cdot 10^{-10}$	0.02500	3.961	$3.4 \cdot 10^{-11}$	0.02500	3.954
$1/5 \cdot 2^{-4}$	$8.5 \cdot 10^{-12}$	0.01250	3.981	$2.2 \cdot 10^{-12}$	0.01250	3.977
$1/5 \cdot 2^{-5}$	$5.3 \cdot 10^{-13}$	0.00625	3.990	$1.4 \cdot 10^{-13}$	0.00625	3.989
$1/5 \cdot 2^{-6}$	$3.3 \cdot 10^{-14}$	0.00312	3.995	$8.9 \cdot 10^{-15}$	0.00312	3.994
$1/5 \cdot 2^{-7}$	$2.1 \cdot 10^{-15}$	0.00156	3.997	$5.5 \cdot 10^{-16}$	0.00156	3.997
$1/5 \cdot 2^{-8}$	$1.3 \cdot 10^{-16}$	0.00078	3.998	$3.5 \cdot 10^{-17}$	0.00078	3.998
$1/5 \cdot 2^{-9}$	$8.3 \cdot 10^{-18}$	0.00039	3.999	$2.1 \cdot 10^{-18}$	0.00039	3.999
$1/5 \cdot 2^{-10}$	$5.2 \cdot 10^{-19}$	0.00019	3.999	$1.3 \cdot 10^{-19}$	0.00019	3.999
$1/5 \cdot 2^{-11}$	$3.2 \cdot 10^{-20}$	0.00009	3.999	$8.5 \cdot 10^{-21}$	0.00009	3.999
$1/5 \cdot 2^{-12}$	$2.0 \cdot 10^{-21}$	0.00004		$5.3 \cdot 10^{-22}$	0.00004	

Table 5.61: Trapezoidal rule, IDeC method on $[0, 1]$ for 5.2a.

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$2.0 \cdot 10^{-34}$	0.912	$2.9 \cdot 10^{-36}$	0.682
$1/5 \cdot 2^{-2}$	$1.0 \cdot 10^{-34}$	1.234	$1.8 \cdot 10^{-36}$	1.735
$1/5 \cdot 2^{-3}$	$4.5 \cdot 10^{-35}$	0.829	$5.5 \cdot 10^{-37}$	1.576
$1/5 \cdot 2^{-4}$	$2.5 \cdot 10^{-35}$	1.203	$1.8 \cdot 10^{-37}$	2.067
$1/5 \cdot 2^{-5}$	$1.1 \cdot 10^{-35}$	0.771	$4.4 \cdot 10^{-38}$	1.696
$1/5 \cdot 2^{-6}$	$6.4 \cdot 10^{-36}$	1.225	$1.3 \cdot 10^{-38}$	2.183
$1/5 \cdot 2^{-7}$	$2.7 \cdot 10^{-36}$	0.781	$3.0 \cdot 10^{-39}$	1.758
$1/5 \cdot 2^{-8}$	$1.6 \cdot 10^{-36}$	1.221	$8.9 \cdot 10^{-40}$	2.208
$1/5 \cdot 2^{-9}$	$6.9 \cdot 10^{-37}$	0.777	$1.9 \cdot 10^{-40}$	1.770
$1/5 \cdot 2^{-10}$	$4.0 \cdot 10^{-37}$	1.222	$5.6 \cdot 10^{-41}$	2.218
$1/5 \cdot 2^{-11}$	$1.7 \cdot 10^{-37}$	0.777	$1.2 \cdot 10^{-41}$	1.775
$1/5 \cdot 2^{-12}$	$1.0 \cdot 10^{-37}$	1.222	$3.5 \cdot 10^{-42}$	2.221
$1/5 \cdot 2^{-13}$	$4.3 \cdot 10^{-38}$	0.777	$7.6 \cdot 10^{-43}$	1.776
$1/5 \cdot 2^{-14}$	$2.5 \cdot 10^{-38}$		$2.2 \cdot 10^{-43}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$1.1 \cdot 10^{-36}$	1.519	$2.4 \cdot 10^{-37}$	1.628
$1/5 \cdot 2^{-2}$	$4.1 \cdot 10^{-37}$	2.062	$8.0 \cdot 10^{-38}$	2.101
$1/5 \cdot 2^{-3}$	$9.9 \cdot 10^{-38}$	1.745	$1.8 \cdot 10^{-38}$	1.761
$1/5 \cdot 2^{-4}$	$2.9 \cdot 10^{-38}$	2.161	$5.5 \cdot 10^{-39}$	2.169
$1/5 \cdot 2^{-5}$	$6.6 \cdot 10^{-39}$	1.750	$1.2 \cdot 10^{-39}$	1.753
$1/5 \cdot 2^{-6}$	$1.9 \cdot 10^{-39}$	2.214	$3.6 \cdot 10^{-40}$	2.216
$1/5 \cdot 2^{-7}$	$4.2 \cdot 10^{-40}$	1.776	$7.8 \cdot 10^{-41}$	1.777
$1/5 \cdot 2^{-8}$	$1.2 \cdot 10^{-40}$	2.218	$2.2 \cdot 10^{-41}$	2.218
$1/5 \cdot 2^{-9}$	$2.6 \cdot 10^{-41}$	1.775	$4.8 \cdot 10^{-42}$	1.776
$1/5 \cdot 2^{-10}$	$7.7 \cdot 10^{-42}$	2.221	$1.4 \cdot 10^{-42}$	2.222
$1/5 \cdot 2^{-11}$	$1.6 \cdot 10^{-42}$	1.777	$3.0 \cdot 10^{-43}$	1.777
$1/5 \cdot 2^{-12}$	$4.8 \cdot 10^{-43}$	2.222	$8.9 \cdot 10^{-44}$	2.222
$1/5 \cdot 2^{-13}$	$1.0 \cdot 10^{-43}$		$1.9 \cdot 10^{-44}$	

h	δ	p	δ	p
$1/5 \cdot 2^{-1}$	$4.4 \cdot 10^{-38}$	1.641	$6.3 \cdot 10^{-39}$	1.588
$1/5 \cdot 2^{-2}$	$1.4 \cdot 10^{-38}$	2.106	$2.1 \cdot 10^{-39}$	2.089
$1/5 \cdot 2^{-3}$	$3.2 \cdot 10^{-39}$	1.764	$4.9 \cdot 10^{-40}$	1.756
$1/5 \cdot 2^{-4}$	$9.6 \cdot 10^{-40}$	2.170	$1.4 \cdot 10^{-40}$	2.166
$1/5 \cdot 2^{-5}$	$2.1 \cdot 10^{-40}$	1.754	$3.2 \cdot 10^{-41}$	1.752
$1/5 \cdot 2^{-6}$	$6.3 \cdot 10^{-41}$	2.216	$9.6 \cdot 10^{-42}$	2.215
$1/5 \cdot 2^{-7}$	$1.3 \cdot 10^{-41}$	1.777	$2.0 \cdot 10^{-42}$	1.777
$1/5 \cdot 2^{-8}$	$3.9 \cdot 10^{-42}$	2.219	$6.0 \cdot 10^{-43}$	2.218
$1/5 \cdot 2^{-9}$	$8.5 \cdot 10^{-43}$	1.776	$1.3 \cdot 10^{-43}$	1.776
$1/5 \cdot 2^{-10}$	$2.5 \cdot 10^{-43}$	2.222	$3.8 \cdot 10^{-44}$	2.221
$1/5 \cdot 2^{-11}$	$5.3 \cdot 10^{-44}$	1.777	$8.1 \cdot 10^{-45}$	1.777
$1/5 \cdot 2^{-12}$	$1.5 \cdot 10^{-44}$		$2.3 \cdot 10^{-45}$	

Table 5.62: Trapezoidal rule, IDeC method at $t = 0$ for 5.2a.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$8.5 \cdot 10^{-01}$	0.60000	0.767	$2.5 \cdot 10^{-01}$	0.80000	1.512
$1/5 \cdot 2^{-1}$	$5.0 \cdot 10^{-01}$	0.70000	0.905	$9.0 \cdot 10^{-02}$	0.70000	1.757
$1/5 \cdot 2^{-2}$	$2.6 \cdot 10^{-01}$	0.70000	0.949	$2.6 \cdot 10^{-02}$	0.75000	1.874
$1/5 \cdot 2^{-3}$	$1.3 \cdot 10^{-01}$	0.70000	0.974	$7.2 \cdot 10^{-03}$	0.75000	1.932
$1/5 \cdot 2^{-4}$	$7.0 \cdot 10^{-02}$	0.70000	0.986	$1.9 \cdot 10^{-03}$	0.73750	1.967
$1/5 \cdot 2^{-5}$	$3.5 \cdot 10^{-02}$	0.70000	0.993	$4.8 \cdot 10^{-04}$	0.73750	1.983
$1/5 \cdot 2^{-6}$	$1.7 \cdot 10^{-02}$	0.70312	0.996	$1.2 \cdot 10^{-04}$	0.74062	1.991
$1/5 \cdot 2^{-7}$	$9.0 \cdot 10^{-03}$	0.70312	0.998	$3.1 \cdot 10^{-05}$	0.74062	1.995
$1/5 \cdot 2^{-8}$	$4.5 \cdot 10^{-03}$	0.70312	0.999	$7.7 \cdot 10^{-06}$	0.73984	1.997
$1/5 \cdot 2^{-9}$	$2.2 \cdot 10^{-03}$	0.70273	0.999	$1.9 \cdot 10^{-06}$	0.73984	1.998
$1/5 \cdot 2^{-10}$	$1.1 \cdot 10^{-03}$	0.70292	0.999	$4.8 \cdot 10^{-07}$	0.74003	1.999
$1/5 \cdot 2^{-11}$	$5.6 \cdot 10^{-04}$	0.70283	0.999	$1.2 \cdot 10^{-07}$	0.74003	1.999
$1/5 \cdot 2^{-12}$	$2.8 \cdot 10^{-04}$	0.70288	0.999	$3.0 \cdot 10^{-08}$	0.74003	1.999
$1/5 \cdot 2^{-13}$	$1.4 \cdot 10^{-04}$	0.70285	0.999	$7.6 \cdot 10^{-09}$	0.74006	1.999
$1/5 \cdot 2^{-14}$	$7.0 \cdot 10^{-05}$	0.70286		$1.9 \cdot 10^{-09}$	0.74006	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$9.6 \cdot 10^{-02}$	0.80000	2.301	$3.9 \cdot 10^{-02}$	0.80000	3.097
$1/5 \cdot 2^{-1}$	$1.9 \cdot 10^{-02}$	0.70000	2.602	$4.5 \cdot 10^{-03}$	0.70000	3.471
$1/5 \cdot 2^{-2}$	$3.2 \cdot 10^{-03}$	0.75000	2.798	$4.1 \cdot 10^{-04}$	0.75000	3.732
$1/5 \cdot 2^{-3}$	$4.6 \cdot 10^{-04}$	0.75000	2.895	$3.0 \cdot 10^{-05}$	0.75000	3.861
$1/5 \cdot 2^{-4}$	$6.2 \cdot 10^{-05}$	0.75000	2.947	$2.1 \cdot 10^{-06}$	0.75000	3.929
$1/5 \cdot 2^{-5}$	$8.1 \cdot 10^{-06}$	0.75000	2.973	$1.3 \cdot 10^{-07}$	0.74375	3.964
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{-06}$	0.74687	2.986	$8.9 \cdot 10^{-09}$	0.74375	3.982
$1/5 \cdot 2^{-7}$	$1.3 \cdot 10^{-07}$	0.74687	2.993	$5.6 \cdot 10^{-10}$	0.74531	3.991
$1/5 \cdot 2^{-8}$	$1.6 \cdot 10^{-08}$	0.74765	2.996	$3.5 \cdot 10^{-11}$	0.74531	3.995
$1/5 \cdot 2^{-9}$	$2.0 \cdot 10^{-09}$	0.74726	2.998	$2.2 \cdot 10^{-12}$	0.74531	3.997
$1/5 \cdot 2^{-10}$	$2.5 \cdot 10^{-10}$	0.74746	2.999	$1.3 \cdot 10^{-13}$	0.74511	3.998
$1/5 \cdot 2^{-11}$	$3.2 \cdot 10^{-11}$	0.74736	2.999	$8.7 \cdot 10^{-15}$	0.74511	3.999
$1/5 \cdot 2^{-12}$	$4.0 \cdot 10^{-12}$	0.74736	2.999	$5.4 \cdot 10^{-16}$	0.74516	3.999
$1/5 \cdot 2^{-13}$	$5.0 \cdot 10^{-13}$	0.74738		$3.4 \cdot 10^{-17}$	0.74516	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.6 \cdot 10^{-02}$	0.80000	3.946	$8.0 \cdot 10^{-03}$	0.80000	4.763
$1/5 \cdot 2^{-1}$	$1.0 \cdot 10^{-03}$	0.70000	4.353	$2.9 \cdot 10^{-04}$	0.80000	5.017
$1/5 \cdot 2^{-2}$	$5.3 \cdot 10^{-05}$	0.70000	4.668	$9.0 \cdot 10^{-06}$	0.70000	5.408
$1/5 \cdot 2^{-3}$	$2.0 \cdot 10^{-06}$	0.72500	4.835	$2.1 \cdot 10^{-07}$	0.72500	4.845
$1/5 \cdot 2^{-4}$	$7.3 \cdot 10^{-08}$	0.73750	4.918	$7.4 \cdot 10^{-09}$	0.05000	4.018
$1/5 \cdot 2^{-5}$	$2.4 \cdot 10^{-09}$	0.74375	4.959	$4.5 \cdot 10^{-10}$	0.02500	4.001
$1/5 \cdot 2^{-6}$	$7.7 \cdot 10^{-11}$	0.74687	4.978	$2.8 \cdot 10^{-11}$	0.01250	4.000
$1/5 \cdot 2^{-7}$	$2.4 \cdot 10^{-12}$	0.74062	4.519	$1.7 \cdot 10^{-12}$	0.00625	4.000
$1/5 \cdot 2^{-8}$	$1.0 \cdot 10^{-13}$	0.00312	3.979	$1.1 \cdot 10^{-13}$	0.00312	4.000
$1/5 \cdot 2^{-9}$	$6.8 \cdot 10^{-15}$	0.00156	3.989	$7.0 \cdot 10^{-15}$	0.00156	4.000
$1/5 \cdot 2^{-10}$	$4.2 \cdot 10^{-16}$	0.00078	3.994	$4.3 \cdot 10^{-16}$	0.00078	4.000
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-17}$	0.00039	3.997	$2.7 \cdot 10^{-17}$	0.00039	4.000
$1/5 \cdot 2^{-12}$	$1.6 \cdot 10^{-18}$	0.00019		$1.7 \cdot 10^{-18}$	0.00019	

Table 5.63: Implicit Euler, frozen Jacobian, IDeC method on $[0, 1]$ for 4.1e.

h	δ	p	δ	p
$1/5$	$7.0 \cdot 10^{-02}$	0.806	$4.9 \cdot 10^{-02}$	1.849
$1/5 \cdot 2^{-1}$	$4.0 \cdot 10^{-02}$	0.911	$1.3 \cdot 10^{-02}$	1.923
$1/5 \cdot 2^{-2}$	$2.1 \cdot 10^{-02}$	0.957	$3.6 \cdot 10^{-03}$	1.961
$1/5 \cdot 2^{-3}$	$1.1 \cdot 10^{-02}$	0.979	$9.2 \cdot 10^{-04}$	1.981
$1/5 \cdot 2^{-4}$	$5.5 \cdot 10^{-03}$	0.989	$2.3 \cdot 10^{-04}$	1.990
$1/5 \cdot 2^{-5}$	$2.8 \cdot 10^{-03}$	0.994	$5.9 \cdot 10^{-05}$	1.995
$1/5 \cdot 2^{-6}$	$1.4 \cdot 10^{-03}$	0.997	$1.4 \cdot 10^{-05}$	1.997
$1/5 \cdot 2^{-7}$	$7.0 \cdot 10^{-04}$	0.998	$3.7 \cdot 10^{-06}$	1.998
$1/5 \cdot 2^{-8}$	$3.5 \cdot 10^{-04}$	0.999	$9.3 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-9}$	$1.7 \cdot 10^{-04}$	0.999	$2.3 \cdot 10^{-07}$	1.999
$1/5 \cdot 2^{-10}$	$8.8 \cdot 10^{-05}$	0.999	$5.8 \cdot 10^{-08}$	1.999
$1/5 \cdot 2^{-11}$	$4.4 \cdot 10^{-05}$	0.999	$1.4 \cdot 10^{-08}$	2.000
$1/5 \cdot 2^{-12}$	$2.2 \cdot 10^{-05}$	0.999	$3.6 \cdot 10^{-09}$	1.999
$1/5 \cdot 2^{-13}$	$1.1 \cdot 10^{-05}$	0.999	$9.0 \cdot 10^{-10}$	1.999
$1/5 \cdot 2^{-14}$	$5.5 \cdot 10^{-06}$		$2.2 \cdot 10^{-10}$	

h	δ	p	δ	p
$1/5$	$1.4 \cdot 10^{-02}$	2.798	$4.7 \cdot 10^{-03}$	4.022
$1/5 \cdot 2^{-1}$	$2.1 \cdot 10^{-03}$	2.906	$2.9 \cdot 10^{-04}$	4.042
$1/5 \cdot 2^{-2}$	$2.8 \cdot 10^{-04}$	2.956	$1.7 \cdot 10^{-05}$	4.029
$1/5 \cdot 2^{-3}$	$3.6 \cdot 10^{-05}$	2.978	$1.0 \cdot 10^{-06}$	4.016
$1/5 \cdot 2^{-4}$	$4.5 \cdot 10^{-06}$	2.989	$6.6 \cdot 10^{-08}$	4.008
$1/5 \cdot 2^{-5}$	$5.7 \cdot 10^{-07}$	2.994	$4.1 \cdot 10^{-09}$	4.004
$1/5 \cdot 2^{-6}$	$7.2 \cdot 10^{-08}$	2.997	$2.5 \cdot 10^{-10}$	4.002
$1/5 \cdot 2^{-7}$	$9.0 \cdot 10^{-09}$	2.998	$1.6 \cdot 10^{-11}$	4.001
$1/5 \cdot 2^{-8}$	$1.1 \cdot 10^{-09}$	2.999	$1.0 \cdot 10^{-12}$	4.000
$1/5 \cdot 2^{-9}$	$1.4 \cdot 10^{-10}$	2.999	$6.2 \cdot 10^{-14}$	4.000
$1/5 \cdot 2^{-10}$	$1.7 \cdot 10^{-11}$	2.999	$3.9 \cdot 10^{-15}$	4.000
$1/5 \cdot 2^{-11}$	$2.2 \cdot 10^{-12}$	3.000	$2.4 \cdot 10^{-16}$	4.000
$1/5 \cdot 2^{-12}$	$2.7 \cdot 10^{-13}$	2.999	$1.5 \cdot 10^{-17}$	3.999
$1/5 \cdot 2^{-13}$	$3.4 \cdot 10^{-14}$		$9.5 \cdot 10^{-19}$	

h	δ	p	δ	p
$1/5$	$1.5 \cdot 10^{-03}$	4.888	$1.5 \cdot 10^{-03}$	4.888
$1/5 \cdot 2^{-1}$	$5.2 \cdot 10^{-05}$	4.967	$5.2 \cdot 10^{-05}$	4.967
$1/5 \cdot 2^{-2}$	$1.6 \cdot 10^{-06}$	4.989	$1.6 \cdot 10^{-06}$	4.989
$1/5 \cdot 2^{-3}$	$5.2 \cdot 10^{-08}$	4.996	$5.2 \cdot 10^{-08}$	4.996
$1/5 \cdot 2^{-4}$	$1.6 \cdot 10^{-09}$	4.998	$1.6 \cdot 10^{-09}$	4.998
$1/5 \cdot 2^{-5}$	$5.1 \cdot 10^{-11}$	4.999	$5.1 \cdot 10^{-11}$	4.999
$1/5 \cdot 2^{-6}$	$1.6 \cdot 10^{-12}$	4.999	$1.6 \cdot 10^{-12}$	4.999
$1/5 \cdot 2^{-7}$	$5.0 \cdot 10^{-14}$	4.999	$5.0 \cdot 10^{-14}$	4.999
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-15}$	4.999	$1.5 \cdot 10^{-15}$	4.999
$1/5 \cdot 2^{-9}$	$4.9 \cdot 10^{-17}$	4.999	$4.9 \cdot 10^{-17}$	4.999
$1/5 \cdot 2^{-10}$	$1.5 \cdot 10^{-18}$	4.999	$1.5 \cdot 10^{-18}$	4.999
$1/5 \cdot 2^{-11}$	$4.7 \cdot 10^{-20}$	5.000	$4.7 \cdot 10^{-20}$	5.000
$1/5 \cdot 2^{-12}$	$1.4 \cdot 10^{-21}$		$1.4 \cdot 10^{-21}$	

Table 5.64: Implicit Euler, frozen Jacobian, IDeC method at $t = 0$ for 4.1e.

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$1.3 \cdot 10^{+03}$	-0.0000	5.447	$1.0 \cdot 10^{+05}$	-0.0000	5.429
$1/5 \cdot 2^{-1}$	$3.1 \cdot 10^{+01}$	-0.0000	3.918	$2.4 \cdot 10^{+03}$	-0.0000	3.917
$1/5 \cdot 2^{-2}$	2.0	0.00000	2.234	$1.6 \cdot 10^{+02}$	0.00000	2.744
$1/5 \cdot 2^{-3}$	$4.4 \cdot 10^{-01}$	0.90000	0.932	$2.4 \cdot 10^{+01}$	0.00000	2.392
$1/5 \cdot 2^{-4}$	$2.3 \cdot 10^{-01}$	0.88750	0.963	4.5	-0.0000	2.209
$1/5 \cdot 2^{-5}$	$1.1 \cdot 10^{-01}$	0.89375	0.982	$9.9 \cdot 10^{-01}$	-0.0000	2.109
$1/5 \cdot 2^{-6}$	$5.9 \cdot 10^{-02}$	0.89687	0.991	$2.3 \cdot 10^{-01}$	0.00000	2.055
$1/5 \cdot 2^{-7}$	$3.0 \cdot 10^{-02}$	0.89531	0.995	$5.5 \cdot 10^{-02}$	0.00000	2.028
$1/5 \cdot 2^{-8}$	$1.5 \cdot 10^{-02}$	0.89609	0.997	$1.3 \cdot 10^{-02}$	-0.0000	2.014
$1/5 \cdot 2^{-9}$	$7.5 \cdot 10^{-03}$	0.89648	0.998	$3.3 \cdot 10^{-03}$	-0.0000	2.007
$1/5 \cdot 2^{-10}$	$3.7 \cdot 10^{-03}$	0.89628	0.999	$8.3 \cdot 10^{-04}$	0.00000	2.003
$1/5 \cdot 2^{-11}$	$1.8 \cdot 10^{-03}$	0.89638	0.999	$2.0 \cdot 10^{-04}$	0.00000	2.001
$1/5 \cdot 2^{-12}$	$9.4 \cdot 10^{-04}$	0.89638	0.999	$5.2 \cdot 10^{-05}$	-0.0000	2.000
$1/5 \cdot 2^{-13}$	$4.7 \cdot 10^{-04}$	0.89641	0.999	$1.3 \cdot 10^{-05}$	-0.0000	2.000
$1/5 \cdot 2^{-14}$	$2.3 \cdot 10^{-04}$	0.89641		$3.2 \cdot 10^{-06}$	0.00000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$8.0 \cdot 10^{+06}$	-0.0000	5.429	$6.1 \cdot 10^{+08}$	-0.0000	5.429
$1/5 \cdot 2^{-1}$	$1.8 \cdot 10^{+05}$	-0.0000	3.917	$1.4 \cdot 10^{+07}$	-0.0000	3.917
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{+04}$	0.00000	2.744	$9.5 \cdot 10^{+05}$	0.00000	2.744
$1/5 \cdot 2^{-3}$	$1.8 \cdot 10^{+03}$	0.00000	2.392	$1.4 \cdot 10^{+05}$	0.00000	2.392
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{+02}$	-0.0000	2.209	$2.7 \cdot 10^{+04}$	-0.0000	2.209
$1/5 \cdot 2^{-5}$	$7.6 \cdot 10^{+01}$	-0.0000	2.109	$5.8 \cdot 10^{+03}$	-0.0000	2.109
$1/5 \cdot 2^{-6}$	$1.7 \cdot 10^{+01}$	0.00000	2.055	$1.3 \cdot 10^{+03}$	0.00000	2.055
$1/5 \cdot 2^{-7}$	4.2	0.00000	2.028	$3.2 \cdot 10^{+02}$	0.00000	2.028
$1/5 \cdot 2^{-8}$	1.0	-0.0000	2.014	$7.9 \cdot 10^{+01}$	-0.0000	2.014
$1/5 \cdot 2^{-9}$	$2.5 \cdot 10^{-01}$	-0.0000	2.007	$1.9 \cdot 10^{+01}$	-0.0000	2.007
$1/5 \cdot 2^{-10}$	$6.4 \cdot 10^{-02}$	0.00000	2.003	4.9	0.00000	2.003
$1/5 \cdot 2^{-11}$	$1.5 \cdot 10^{-02}$	0.00000	2.001	1.2	0.00000	2.001
$1/5 \cdot 2^{-12}$	$3.9 \cdot 10^{-03}$	-0.0000	2.000	$3.0 \cdot 10^{-01}$	-0.0000	2.000
$1/5 \cdot 2^{-13}$	$9.9 \cdot 10^{-04}$	-0.0000		$7.6 \cdot 10^{-02}$	-0.0000	

h	δ	$t^{\max}$	p	δ	$t^{\max}$	p
$1/5$	$4.7 \cdot 10^{+10}$	-0.0000	5.429	$3.6 \cdot 10^{+12}$	-0.0000	5.429
$1/5 \cdot 2^{-1}$	$1.1 \cdot 10^{+09}$	-0.0000	3.917	$8.4 \cdot 10^{+10}$	-0.0000	3.917
$1/5 \cdot 2^{-2}$	$7.3 \cdot 10^{+07}$	0.00000	2.744	$5.6 \cdot 10^{+09}$	0.00000	2.744
$1/5 \cdot 2^{-3}$	$1.0 \cdot 10^{+07}$	0.00000	2.392	$8.3 \cdot 10^{+08}$	0.00000	2.392
$1/5 \cdot 2^{-4}$	$2.0 \cdot 10^{+06}$	-0.0000	2.209	$1.5 \cdot 10^{+08}$	-0.0000	2.209
$1/5 \cdot 2^{-5}$	$4.4 \cdot 10^{+05}$	-0.0000	2.109	$3.4 \cdot 10^{+07}$	-0.0000	2.109
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{+05}$	0.00000	2.055	$7.9 \cdot 10^{+06}$	0.00000	2.055
$1/5 \cdot 2^{-7}$	$2.5 \cdot 10^{+04}$	0.00000	2.028	$1.9 \cdot 10^{+06}$	0.00000	2.028
$1/5 \cdot 2^{-8}$	$6.1 \cdot 10^{+03}$	-0.0000	2.014	$4.7 \cdot 10^{+05}$	-0.0000	2.014
$1/5 \cdot 2^{-9}$	$1.5 \cdot 10^{+03}$	-0.0000	2.007	$1.1 \cdot 10^{+05}$	-0.0000	2.007
$1/5 \cdot 2^{-10}$	$3.7 \cdot 10^{+02}$	0.00000	2.003	$2.8 \cdot 10^{+04}$	0.00000	2.003
$1/5 \cdot 2^{-11}$	$9.4 \cdot 10^{+01}$	0.00000	2.001	$7.2 \cdot 10^{+03}$	0.00000	2.001
$1/5 \cdot 2^{-12}$	$2.3 \cdot 10^{+01}$	-0.0000		$1.8 \cdot 10^{+03}$	-0.0000	

Table 5.65: Implicit Euler, last step explicit, IDeC method on $[0, 1]$ for 4.1f.

h	δ	p	δ	p
$1/5$	$1.3 \cdot 10^{+03}$	5.447	$1.0 \cdot 10^{+05}$	5.429
$1/5 \cdot 2^{-1}$	$3.1 \cdot 10^{+01}$	3.918	$2.4 \cdot 10^{+03}$	3.917
$1/5 \cdot 2^{-2}$	2.0	2.743	$1.6 \cdot 10^{+02}$	2.744
$1/5 \cdot 2^{-3}$	$3.0 \cdot 10^{-01}$	2.392	$2.4 \cdot 10^{+01}$	2.392
$1/5 \cdot 2^{-4}$	$5.9 \cdot 10^{-02}$	1.942	4.5	2.209
$1/5 \cdot 2^{-5}$	$1.5 \cdot 10^{-02}$	0.920	$9.9 \cdot 10^{-01}$	2.109
$1/5 \cdot 2^{-6}$	$8.1 \cdot 10^{-03}$	0.965	$2.3 \cdot 10^{-01}$	2.055
$1/5 \cdot 2^{-7}$	$4.1 \cdot 10^{-03}$	0.983	$5.5 \cdot 10^{-02}$	2.028
$1/5 \cdot 2^{-8}$	$2.1 \cdot 10^{-03}$	0.992	$1.3 \cdot 10^{-02}$	2.014
$1/5 \cdot 2^{-9}$	$1.0 \cdot 10^{-03}$	0.996	$3.3 \cdot 10^{-03}$	2.007
$1/5 \cdot 2^{-10}$	$5.2 \cdot 10^{-04}$	0.998	$8.3 \cdot 10^{-04}$	2.003
$1/5 \cdot 2^{-11}$	$2.6 \cdot 10^{-04}$	0.999	$2.0 \cdot 10^{-04}$	2.001
$1/5 \cdot 2^{-12}$	$1.3 \cdot 10^{-04}$	0.999	$5.2 \cdot 10^{-05}$	2.000
$1/5 \cdot 2^{-13}$	$6.6 \cdot 10^{-05}$	0.999	$1.3 \cdot 10^{-05}$	2.000
$1/5 \cdot 2^{-14}$	$3.3 \cdot 10^{-05}$		$3.2 \cdot 10^{-06}$	

h	δ	p	δ	p
$1/5$	$8.0 \cdot 10^{+06}$	5.429	$6.1 \cdot 10^{+08}$	5.429
$1/5 \cdot 2^{-1}$	$1.8 \cdot 10^{+05}$	3.917	$1.4 \cdot 10^{+07}$	3.917
$1/5 \cdot 2^{-2}$	$1.2 \cdot 10^{+04}$	2.744	$9.5 \cdot 10^{+05}$	2.744
$1/5 \cdot 2^{-3}$	$1.8 \cdot 10^{+03}$	2.392	$1.4 \cdot 10^{+05}$	2.392
$1/5 \cdot 2^{-4}$	$3.5 \cdot 10^{+02}$	2.209	$2.7 \cdot 10^{+04}$	2.209
$1/5 \cdot 2^{-5}$	$7.6 \cdot 10^{+01}$	2.109	$5.8 \cdot 10^{+03}$	2.109
$1/5 \cdot 2^{-6}$	$1.7 \cdot 10^{+01}$	2.055	$1.3 \cdot 10^{+03}$	2.055
$1/5 \cdot 2^{-7}$	4.2	2.028	$3.2 \cdot 10^{+02}$	2.028
$1/5 \cdot 2^{-8}$	1.0	2.014	$7.9 \cdot 10^{+01}$	2.014
$1/5 \cdot 2^{-9}$	$2.5 \cdot 10^{-01}$	2.007	$1.9 \cdot 10^{+01}$	2.007
$1/5 \cdot 2^{-10}$	$6.4 \cdot 10^{-02}$	2.003	4.9	2.003
$1/5 \cdot 2^{-11}$	$1.5 \cdot 10^{-02}$	2.001	1.2	2.001
$1/5 \cdot 2^{-12}$	$3.9 \cdot 10^{-03}$	2.000	$3.0 \cdot 10^{-01}$	2.000
$1/5 \cdot 2^{-13}$	$9.9 \cdot 10^{-04}$		$7.6 \cdot 10^{-02}$	

h	δ	p	δ	p
$1/5$	$4.7 \cdot 10^{+10}$	5.429	$3.6 \cdot 10^{+12}$	5.429
$1/5 \cdot 2^{-1}$	$1.1 \cdot 10^{+09}$	3.917	$8.4 \cdot 10^{+10}$	3.917
$1/5 \cdot 2^{-2}$	$7.3 \cdot 10^{+07}$	2.744	$5.6 \cdot 10^{+09}$	2.744
$1/5 \cdot 2^{-3}$	$1.0 \cdot 10^{+07}$	2.392	$8.3 \cdot 10^{+08}$	2.392
$1/5 \cdot 2^{-4}$	$2.0 \cdot 10^{+06}$	2.209	$1.5 \cdot 10^{+08}$	2.209
$1/5 \cdot 2^{-5}$	$4.4 \cdot 10^{+05}$	2.109	$3.4 \cdot 10^{+07}$	2.109
$1/5 \cdot 2^{-6}$	$1.0 \cdot 10^{+05}$	2.055	$7.9 \cdot 10^{+06}$	2.055
$1/5 \cdot 2^{-7}$	$2.5 \cdot 10^{+04}$	2.028	$1.9 \cdot 10^{+06}$	2.028
$1/5 \cdot 2^{-8}$	$6.1 \cdot 10^{+03}$	2.014	$4.7 \cdot 10^{+05}$	2.014
$1/5 \cdot 2^{-9}$	$1.5 \cdot 10^{+03}$	2.007	$1.1 \cdot 10^{+05}$	2.007
$1/5 \cdot 2^{-10}$	$3.7 \cdot 10^{+02}$	2.003	$2.8 \cdot 10^{+04}$	2.003
$1/5 \cdot 2^{-11}$	$9.4 \cdot 10^{+01}$	2.001	$7.2 \cdot 10^{+03}$	2.001
$1/5 \cdot 2^{-12}$	$2.3 \cdot 10^{+01}$		$1.8 \cdot 10^{+03}$	

Table 5.66: Implicit Euler, last step explicit, IDeC method at $t = 0$ for 4.1f.

6 Conclusions

In this report, we investigated the application of Iterated Defect Correction to both singular initial value problems posed at $t = 0$ and singular terminal value problems posed at $t = 1$. Four low order one-step schemes, the explicit and implicit Euler method, the box scheme and the trapezoidal rule, were used as basic methods.

For initial value problems, IDeC was found to have the full classical convergence order if the implicit Euler method served as basic scheme. This result was also shown theoretically in [19]. The box scheme on the other hand shows order reductions. These appear to be caused by unsmooth inhomogeneous terms in the right-hand side of the variational equations, which are used in the computation of asymptotic expansions of the global error. For the trapezoidal rule and the (computationally cheap) explicit Euler method, similar order reductions appear. In these cases, however, the asymptotic error expansions cannot be computed explicitly. A comparison of IDeC with Richardson extrapolation suggests, however, that the order reductions for the latter two methods are again caused by an insuffient length of the error expansion.

For terminal value problems, we are confronted with another feature of singular problems. Here, the spectral properties of the coefficient matrix M determine the smoothness of the solutions of the variational equations. Thus, even if the solution of a terminal value problem is smooth, no satisfactory error expansion need exist. This is apparently the reason why IDeC based on the explicit Euler method fails for terminal value problems. Even though the basic convergence order of 1 holds under certain restrictions on the spectrum of M and the variational equations have the same structure as for the implicit Euler method for initial value problems, the error expansions are often too short to enable a satisfactory performance of IDeC. For the box scheme, both the structure of the variational equations and the inherent unsmoothness of solutions of the terminal value problems appear responsible for the failure of IDeC to improve the convergence order in the classical manner.

For implicit methods, an additional difficulty occurs. The evaluation of the right-hand side of the differential equation is not possible at $t = 0$. The standard modification of replacing this evaluation by the value of $z'(0)$, which restores convergence in the case of initial value problems, is not feasible in this case. If we use the value of the right-hand side at a point $t \approx \text{EPS}$ instead, the implicit Euler method performs similarly as the explicit Euler method, and the same holds for the trapezoidal rule and the box scheme. However, in the last step the convergence is better for the implicit methods. Alternative modifications of the scheme in the last step were not found to improve the results obtained with the evaluation at $t \approx \text{EPS}$.

7   Acknowledgements

List of Figures

List of Tables

References

[1] U. ASCHER, R. MATTHEIJ, AND R. RUSSELL, *Numerical Solution of Boundary Value Problems for Ordinary Differential Equations*, Prentice-Hall, Englewood Cliffs, NJ, 1988.

[2] W. AUZINGER, O. KOCH, P. KOFLER, AND E. WEINMÜLLER, *The application of shooting to singular boundary value problems*, Techn. Rep. Nr. 126/99, Inst. for Appl. Math. and Numer. Anal., Vienna Univ. of Technology, Austria, 1999. Available at `http://fsmat.at/~othmar/research.html`.

[3] J. CASH AND H. SILVA, *On the numerical solution of a class of singular two-point boundary value problems*, J. Comput. Appl. Math., 45 (1993), pp. 91–102.

[4] C. CHAN AND Y. HON, *A constructive solution for a generalized Thomas-Fermi theory of ionized atoms*, Quart. Appl. Math., 45 (1987), pp. 591–599.

[5] M. DRMOTA, R. SCHEIDL, H. TROGER, AND E. WEINMÜLLER, *On the imperfection sensitivity of complete spherical shells*, Comp. Mech., 2 (1987), pp. 63–74.

[6] R. FRANK, *The method of Iterated Defect Correction and its application to two-point boundary value problems, Part I*, Numer. Math., 25 (1976), pp. 409–419.

[7] ——, *The method of Iterated Defect Correction and its application to two-point boundary value problems, Part II*, Numer. Math., 27 (1977), pp. 407–420.

[8] R. FRANK AND C. ÜBERHUBER, *Iterated Defect Correction for Runge-Kutta methods*, Techn. Rep. Nr. 14/75, Inst. for Appl. Math. and Numer. Anal., Vienna Univ. of Technolgy, Austria, 1975.

[9] F. FROMMLET AND E. WEINMÜLLER, *Asymptotic error expansions for singular boundary value problems*, Math. Models Methods Appl. Sci., 11 (2001), pp. 71–85.

[10] M. GRÄFF AND E. WEINMÜLLER, *Schätzungen des lokalen Diskretisierungsfehlers bei singulären Anfangswertproblemen*, Techn. Rep. Nr. 66/86, Inst. for Appl. Math. and Numer. Anal., Vienna Univ. of Technolgy, Austria, 1986.

[11] F. D. HOOG AND R. WEISS, *Difference methods for boundary value problems with a singularity of the first kind*, SIAM J. Numer. Anal., 13 (1976), pp. 775–813.

[12] ——, *The application of linear multistep methods to singular initial value problems*, Math. Comp., 32 (1977), pp. 676–690.

[13] ——, *The application of Runge-Kutta schemes to singular initial value problems*, Math. Comp., 44 (1985), pp. 93–103.

[14] O. KOCH, *Numerische Lösung singulärer Anfangswertprobleme zweiter Ordnung*, Ph.D. Thesis, Inst. for Appl. Math. and Numer. Anal., Vienna Univ. of Technology, Austria, 1998. Available at `http://fsmat.at/~othmar/research.html`.

[15] O. KOCH, P. KOFLER, AND E. WEINMÜLLER, *Analysis of singular initial and terminal value problems*, Techn. Rep. Nr. 125/99, Inst. for Appl. Math. and Numer. Anal., Vienna Univ. of Technology, Austria, 1999. Available at `http://fsmat.at/~othmar/research.html`.

[16] ——, *The implicit Euler method for the numerical solution of singular initial value problems*, Appl. Num. Math., 34 (2000), pp. 231–252.

[17] ——, *Initial value problems for systems of ordinary first and second order differential equations with a singularity of the first kind*, Analysis, 21 (2001), pp. 373–389.

[18] O. KOCH AND E. WEINMÜLLER, *The convergence of shooting methods for singular boundary value problems.* To appear in Math. Comp. Also available as ANUM Preprint Nr. 9/01 at `http://www.math.tuwien.ac.at/~inst115/preprints.htm`.

[19] ——, *Iterated Defect Correction for the solution of singular initial value problems*, SIAM J. Numer. Anal., 38 (2001), pp. 1784–1799.

[20] P. KOFLER, *Theorie und numerische Lösung singulärer Anfangswertprobleme gewöhnlicher Differentialgleichungen mit der Singularität erster Art*, Ph. D. Thesis, Inst. for Appl. Math. and Numer. Anal., Vienna Univ. of Technology, Austria, 1998.

[21] D. M. McCLUNG AND A. I. MEARS, *Dry-flowing avalanche run-up and run-out*, J. Glaciol., 41 (1995), pp. 359–369.

[22] S. PARTER, M. STEIN, AND P. STEIN, *On the multiplicity of solutions of a differential equation arising in chemical reactor theory*, Techn. Rep. Nr. 194, Dept. Computer Sciences, Univ. of Wisconsin, 1973.

[23] R. RUSSEL AND L. SHAMPINE, *Numerical methods for singular boundary value problems*, SIAM J. Numer. Anal., 12 (1975), pp. 13–36.

[24] H. J. STETTER, *Analysis of Discretization Methods for Ordinary Differential Equations*, Springer-Verlag, Berlin-Heidelberg-New York, 1973.

[25] J. STOER, *Numerische Mathematik 1*, Springer-Verlag, Berlin-Heidelberg-New York, 8$^{\text{th}}$ ed., 1999.

[26] E. WEINMÜLLER, *A difference method for a singular boundary value problem of second order*, Math. Comp., 42 (1984), pp. 441–464.

[27] H. WERNER AND H. ARNDT, *Gewöhnliche Differentialgleichungen: eine Einführung in Theorie und Praxis*, Springer-Verlag, Berlin-Heidelberg, 1986.

[28] P. ZADUNAISKY, *On the estimation of errors propagated in the numerical integration of ODEs*, Numer. Math., 27 (1976), pp. 21–39.